

CAREER POINT

Study Material for Pre foundation Class 9

Prepared by Career Point Kota Experts

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Class IX

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English [Set-1]

- ◆ Diary writing
- ◆ Article Writing
- ◆ Story Writing
- ◆ Integrated Grammar
- ◆ How I Taught My Grandmother to Read
- ◆ The Pot Maker
- ◆ Winds of Change
- ◆ Vitamin-M
- ◆ The World of Limitless Possibilities
- ◆ Twin Melodies
- ◆ Carrier of Words
- ◆ Follow That Dream
- ◆ Bharat Our Land
- ◆ Gifts of Grace : Honouring Our Vocations
- ◆ Canvas of Soil
- ◆ I Cannot Remember My Mother
- ◆ Nine Gold Medals
- ◆ A Friend Found In Music
- ◆ Words
- ◆ Believe In Yourself

English [Set-2]

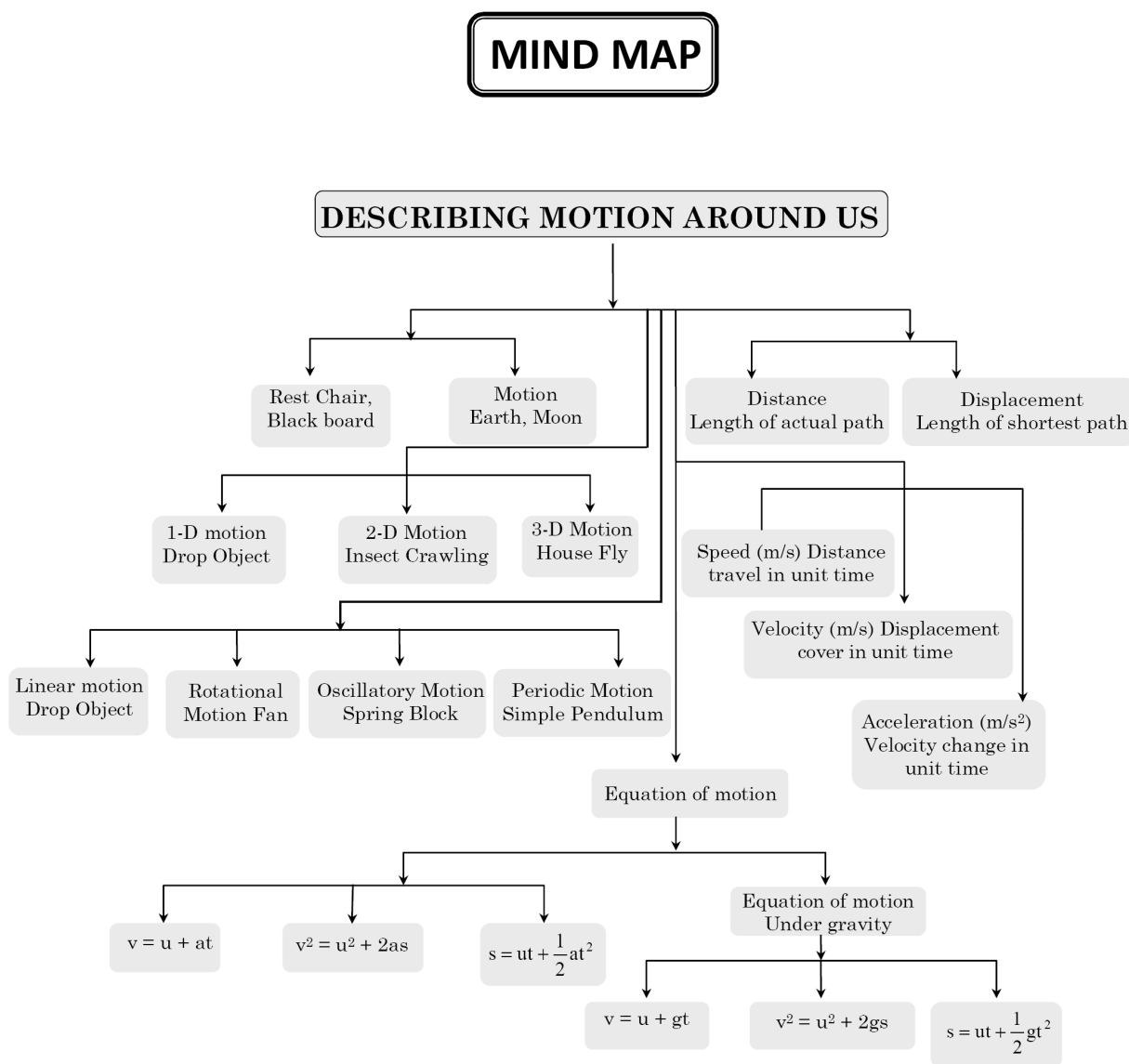
- ◆ Noun
- ◆ Pronoun
- ◆ Adjective
- ◆ Adverb
- ◆ Determiners
- ◆ Connectors
- ◆ Prepositions
- ◆ Tense
- ◆ Passivization
- ◆ Reported speech
- ◆ Modals
- ◆ Subject verb concord
- ◆ Conditionals
- ◆ Error spotting
- ◆ Comprehension
- ◆ Fillers
- ◆ Cloze test
- ◆ Synonyms & antonyms
- ◆ Idioms and phrases
- ◆ Phrasal verbs
- ◆ One word substitution
- ◆ Spelling

Note to the Students

Career Point offers this Class 9 Study Package to support complete learning for school curriculum as well as Olympiad and foundation-level exams. This sample represents our set of nine books: **Physics**, **Chemistry**, **Biology**, two **Mathematics** books, two **Social Science** books, two **English** books and **Mental Ability**. Each book provides clear concepts, illustrative examples and ample practice to strengthen understanding and build analytical skills. The material is designed to develop a strong academic base and prepare students with confidence for higher classes and future competitive studies.

Mind Map

Each chapter contains many articles (Concepts, Theories etc.). Mind map interconnect all these articles logically. By this student can understand whole chapter articles interconnectivity clearly in a single picture frame.



Theory & Concepts

Each chapter consist of exhaustive theory which gives conceptual clarity and command over topics. Appropriate explanation of theory with the help of images, diagrams, flowcharts, mind maps, info graphics, and tables.

DESCRIBING MOTION AROUND US

Physics

It is the branch of science in which we observe, measure and describe nature and natural phenomena.

◆ Mechanics

It is the branch of physics which deals with the study of objects in the condition of rest or motion. It is divided into two parts

(i) Statics

(ii) Kinematics and Dynamics

(i) **Statics:** It deals with the study of objects at rest or in equilibrium, even when they are under the action of several forces.

(ii) **Kinematics and Dynamics:**

Kinematics: It deals with the study of motion of objects without considering the cause of motion.

e.g.: equations of motion

Competitive Level

Competitive level is specially designed for competition exam requirements and to better understanding the concepts, well explained theory, clearly explained formulas with good number of quality examples are given in this.

COMPETITIVE LEVEL

- **Representation of a vector:** A vector is represented by a directed line segment drawn in the given direction on a certain scale. The length of line shows its magnitude and arrow shows direction.

Tail —————→ head

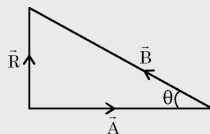
(symbolic representation)

In Chapter Example

To understand the application of concepts, there is **in chapter solved example** are given. It contains large variety of all types of solved examples with explanation to ensure understanding the application of concepts.

Ex.5 The resultant of two velocity vectors $\vec{A}$ and $\vec{B}$ is perpendicular to $\vec{A}$. Magnitude of Resultant $\vec{R}$ is equal to half magnitude of $\vec{B}$. Find the angle between $\vec{A}$ and $\vec{B}$?

Sol. Since $\vec{R}$ is perpendicular to $\vec{A}$. Figure shows the three vectors $\vec{A}$, $\vec{B}$ and $\vec{R}$.



angle between $\vec{A}$ and $\vec{B}$ is $\pi - \theta$

$$\sin \theta = \frac{R}{B} = \frac{R}{2B} = \frac{1}{2}$$

$$\Rightarrow \theta = 30^\circ$$

$\Rightarrow$ angle between A and B is 150° .

Practice Exercises

Includes three sets of exercises covering all the topics. Helps the students to assess their strengths and weaknesses and work on them accordingly. Separate exercises for subjective as well as objective questions and previous year competitive exams questions (NTSE, Olympiads)

EXERCISE-1

➤ Very Short Answer Type Questions

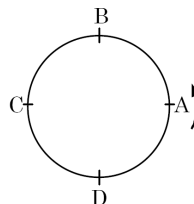
- Q.1** A ball is thrown up with a certain velocity. It attains a height of 40 m and comes back to the thrower. Find the distance and magnitude of displacement.
- Q.2** What is the S.I. unit of displacement?
- Q.3** A horse runs a distance of 1200 m in 3 min and 20 s. What is the speed of the horse?
- Q.4** What is the S.I. unit of velocity?
- Q.5** What is the S.I. unit of acceleration?

➤ Short Answer Type Questions – Type I

- Q.6** Distance and displacement are equal in some cases. Give reasons.

➤ Short Answer Type Questions – Type II

- Q.11** Give three examples to explain that motion is relative.
- Q.12** Which of the following is true for displacement?
(i) It cannot be zero.
(ii) Its magnitude is greater than the distance travelled by the object.
- Q.13** A particle moves along a circle of radius R as shown in figure. It starts from A and moves in anti-clockwise direction.



Calculate the distance travelled and displacement.

EXERCISE-2

- Q.1** ABC is the shortest path length between the two points and ADC is the actual path length. Then which of the two corresponds to displacement?
(A) ADC (B) ABC
(C) Can't say (D) None of these
- Q.2** Rest and motion both are
(A) Relative terms
(B) Absolute terms
(C) Can't say
(D) None of these
- Q.3** An object has travelled 10 km in 15 minutes, its displacement will be
(A) 10 km
(B) zero
(C) More than 10 km
(D) Cannot be predicted
- Q.4** Which of the following does not need direction to be defined completely
- Q.8** From the top of a tower, a particle is projected upwards and it reaches the ground after 5 s. The initial velocity of the particle is 12 m/s, the height of the tower is
(A) 55 m (B) 65 m
(C) 75 m (D) 85 m
- Q.9** A train passes over a 400 m long bridge. If the speed of the train is 30 m/s and the train takes 20 s to cross the bridge, then the length of the train is
(A) 400 m (B) 600 m
(C) 800 m (D) 200 m
- Q.10** If a body covers a distance d with velocity v_1 and another distance d with same velocity v_2 , then average velocity for the whole journey would be equal to
(A) $\frac{2v_1v_2}{v_1 + v_2}$ (B) $\frac{v_1v_2}{v_1 + v_2}$
(C) $\frac{v_1v_2}{2v_1 + v_2}$ (D) $\frac{2(v_1v_2)}{v_1v_2}$

SOLUTIONS

EXERCISE-1

➤ Very Short Answer Type Questions

Sol.1 40 m.



Displacement = 0
Distance = 40m + 40m = 80m

Sol.2 Metre

Sol.3 Speed = $\frac{\text{distance}}{\text{time}} = \frac{1200}{200} = 6 \text{ m/sec.}$

Sol.4 m/sec

Sol.5 m/sec²

(i) Distance = Displacement = Area A₁
= 100 m.

(ii) Displacement = Area A₁ + Area A₂
= 100 – 12.5 = 87.5 m

Distance = (Area A₁) + (Area A₂)
= 100 + 12.5 = 112.5 m.

➤ Short Answer Type Questions – Type II

Sol.11 A train is moving on the track, the passenger are seated, will be stationary w.r.t. to each other but in moving condition w.r.t. to station.

Sol.12 Both are incorrect.

Sol.13 (i) A → B

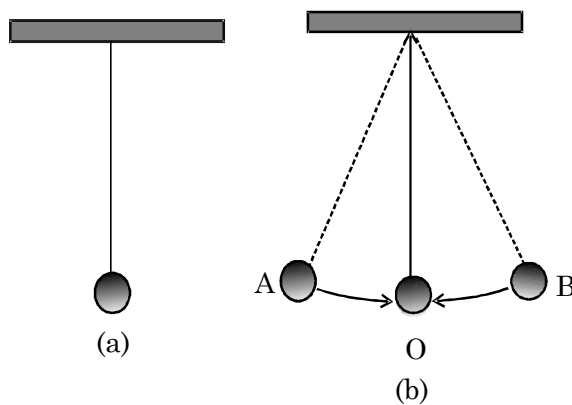
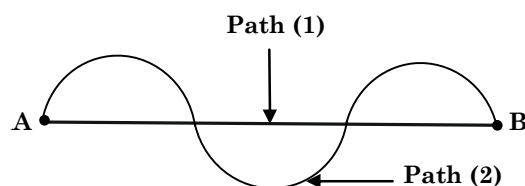
$$\text{Distance} = \frac{1}{4}(2\pi R) = \frac{\pi}{2}R$$

PHYSICS

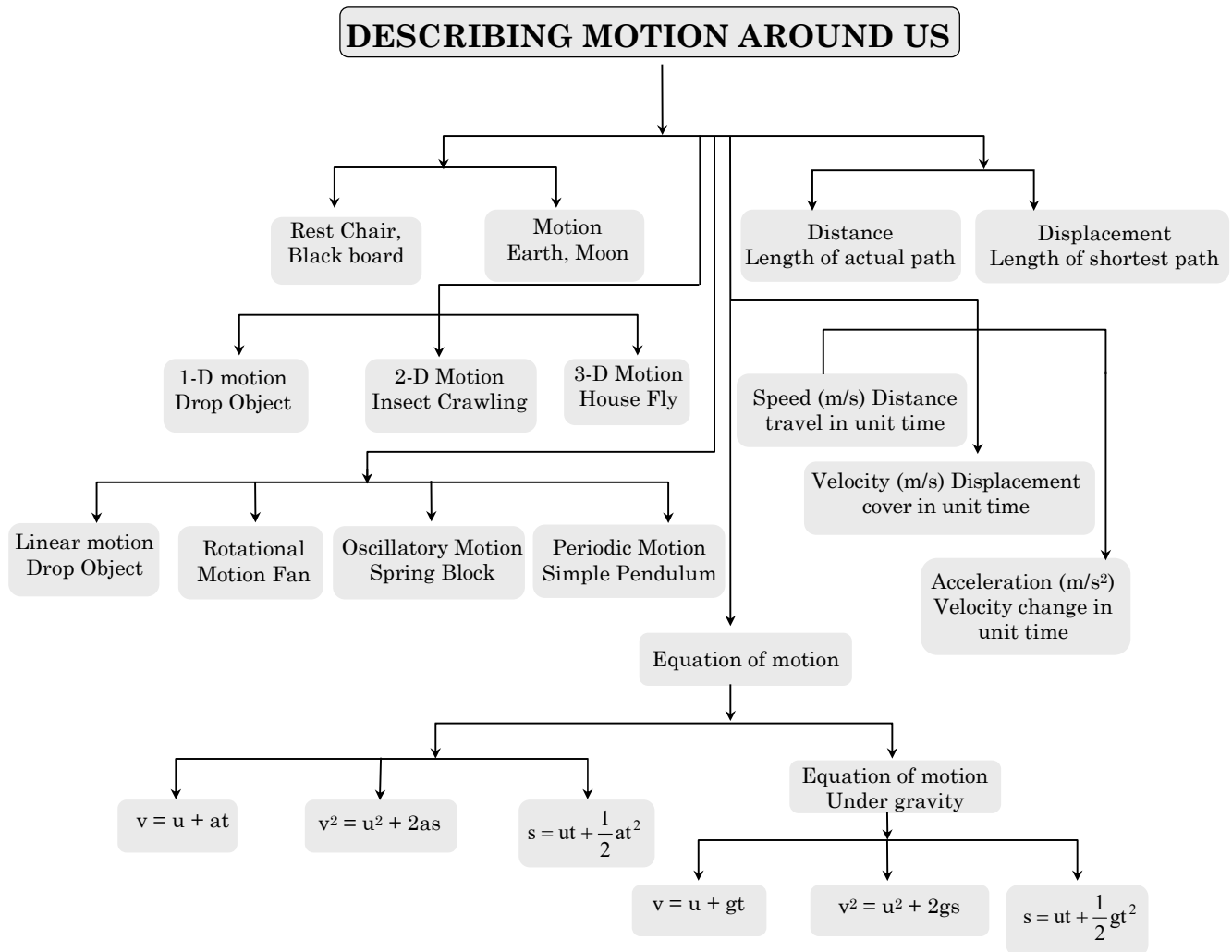
DESCRIBING MOTION AROUND US

Chapter Outline

- ✧ Physics
- ✧ Rest and Motion
- ✧ Types of motion
- ✧ Scalar and Vector Quantity
- ✧ Distance and displacement
- ✧ Speed & Velocity
- ✧ Uniform and non uniform motion
- ✧ Acceleration
- ✧ Graphical representation of motion
- ✧ Equation of uniformly accelerated motion
- ✧ Circular motion



MIND MAP



DESCRIBING MOTION AROUND US

Physics

It is the branch of science in which we observe, measure and describe nature and natural phenomena.

◆ Mechanics

It is the branch of physics which deals with the study of objects in the condition of rest or motion. It is divided into two parts

(i) Statics (ii) Kinematics and Dynamics

(i) Statics: It deals with the study of objects at rest or in equilibrium, even when they are under the action of several forces.

(ii) Kinematics and Dynamics:

Kinematics: It deals with the study of motion of objects without considering the cause of motion.

e.g.: equations of motion

Dynamics: It deals with the study of motion of objects with considering the cause of motion.

e.g.: Newton's laws of motion

Rest & Motion

◆ Rest

An object is said to be at rest if it does not change its position w.r.t. its surroundings with the passage of time.

e.g.: The chair, black board, table in the class room are at rest w.r.t. the students.

◆ Motion

A body is said to be in motion if its position changes continuously w.r.t. the surroundings (or with respect to an observer) with the passage of time.

e.g.: A car moving on the road will be in motion w.r.t. to the person standing on the road

Rest and motion are relative terms, there is nothing like absolute motion or rest.

e.g.: A train is moving on the track, the passengers are seated, will be stationary with respect to each other but in moving condition with respect to station.

Therefore, all the motions are relative. There is nothing like absolute motion.

To study the motion of an object, following points are essential:

◆ Concept of a Point Object

In mechanics while studying the motion of an object, sometimes its dimensions are not important and the object may be treated as a point object without much error. When the size of the object is much less in comparison to the distance covered by the object then the object is considered as a point object.

e.g.: Earth can be considered as a point object for studying its motion around the sun. Because length of the path covered by the earth in one revolution is very large in comparison to the size of earth, so that earth can be considered as a point object.

◆ Frame of Reference

A fixed point or a fixed object with respect to which the given body changes its position is known as reference point or origin

To locate the position of object we need a frame of reference. A convenient way to set up a frame of reference is to choose three mutually perpendicular axis and name them x-y-z axis. The co-ordinates (x, y, z) of the particle then specify the position of object w.r.t. that frame. If any one or more co-ordinates change with time, then we say that the object is moving w.r.t. this frame.

Note: We need Frame of reference to define dimensions of motion.

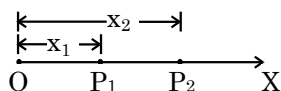
Types of Motion

◆ Motion in 1-D

If only one of the three co-ordinates specifying the position of object changes w.r.t. time then its motion is known as 1D motion. In such a case the object moves along a straight line. This motion is also known as rectilinear or linear motion.

e.g.:

- (i) Motion of train along straight railway track.
- (ii) An object falling freely under gravity.
- (iii) When a particle moves from P_1 to P_2 along a straight line path only the x-co-ordinate changes.



◆ Motion in 2-D

If two of the three co-ordinates specifying the position of object changes w.r.t. time, then the motion of object is called two dimensional. In such a motion the object moves in a plane.

e.g.:

- (i) Motion of queen on carom board.
- (ii) An insect crawling on the floor of the room.
- (iii) Motion of object in horizontal and vertical circles etc.
- (iv) Motion of planets around the sun.
- (v) A car moving along a zig-zag path on a level road.

◆ Motion in 3-D

If all the three co-ordinates specifying the position of object changes w.r.t. time, then the motion of object is called 3-D. In such a motion the object moves in a space.

e.g.:

- (i) A bird or kite flying in the sky (Also kite).
- (ii) Random motion of gas molecules.
- (iii) Motion of an aeroplane in space.

◆ Linear Motion (or Translatory Motion)

The straight line motion is called linear motion.

e.g.: The motion of a car moving on straight road, a running person, a stone being dropped, motion of a train on a straight track

◆ Rotational Motion

Motion of a body around a fixed axis is called rotational motion.

e.g.: The motion of an electric fan, motion of earth about its own axis.

◆ Oscillatory Motion

The to and fro periodic motion of a body around a fixed point is called oscillatory motion.

e.g.: The motion of a simple pendulum, a body suspended from a spring.

Scalar and Vector Quantity

Physical quantities (i.e. quantities of physics) can be divided into two types:

◆ Scalar quantity

Any physical quantity, which can be completely specified by its magnitude, is known as scalar quantity or a scalar.

e.g.: Charge, distance, area, speed, time, temperature, density, volume, work, power, energy, pressure, potential etc.

◆ Vector quantity

The quantity which can be determined by its magnitude and direction and also can be added or subtracted by vector algebra, is called a vector quantity.

e.g.: Displacement, velocity, acceleration, force, momentum, weight, electric field etc.

◆ Difference between scalar and vector

Scalar	Vector
1. They have magnitude only.	1. They have magnitude as well as direction.
2. They are added or subtracted arithmetically like $3 \text{ kg} + 5 \text{ kg} = 8 \text{ kg}$	2. They are added or subtracted by the process of vector addition.

COMPETITIVE LEVEL

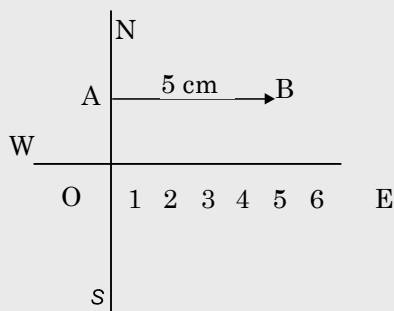
- **Representation of a vector:** A vector is represented by a directed line segment drawn in the given direction on a certain scale. The length of line shows its magnitude and arrow shows direction.

Tail —————> head

(symbolic representation)

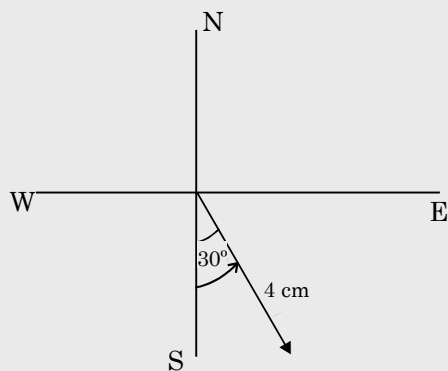
Ex.1 Represent a displacement of 50 m towards east.

Sol. Take the scale 10 m = 1 cm



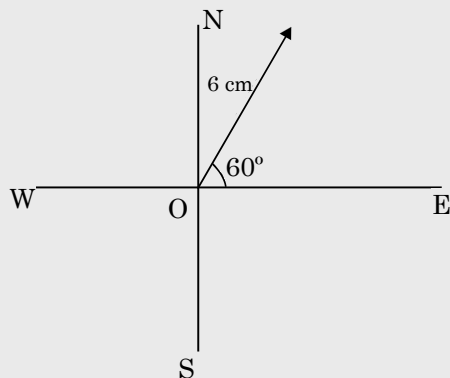
Ex.2 Represent a velocity of 20 km/h towards 30° east of south.

Sol. Take scale 5 km/h = 1 cm.



Ex.3 Represent on graph 6 m displacement, 60° north-east (north of east)

Sol. Take scale 1 m = 1 cm



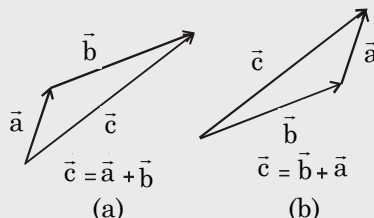
◆ Addition of Vectors

Two or more vectors are added by following laws:

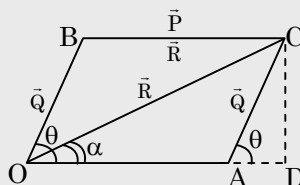
- (i) triangle law
- (ii) parallelogram law
- (iii) polygon law

- **Triangle law of vector addition:** If two vectors are represented both in magnitude and direction by the two sides of a triangle taken in the same order, then the resultant of these two vectors is represented in magnitude and direction by the third side of the triangle taken in the opposite order.

Given two vectors $\vec{a}$ and $\vec{b}$, put the tail of $\vec{b}$ at the head of $\vec{a}$, then the sum of $\vec{a}$ and $\vec{b}$ is defined as the vector $\vec{c}$ drawn from the tail of $\vec{a}$ to the head of $\vec{b}$.



- **Parallelogram Law of Vector Addition Statement:** If two vectors acting simultaneously at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, then the diagonal of the parallelogram passing through that point represents the resultant in magnitude and direction.
- **Analytical approach to parallelogram law of vector addition:** Let the two vectors $\vec{P}$ and $\vec{Q}$ be represented in magnitude and direction by the adjacent sides $\vec{OA}$ and $\vec{OB}$ of the parallelogram OACB. Suppose the angle between the vectors is θ , i.e. $\angle AOB = \theta$. According to parallelogram law of vector addition, the diagonal represents the resultant $\vec{R}(\vec{OC})$ in magnitude and direction. Suppose $\vec{R}$ makes an angle α with $\vec{P}$ i.e. $\angle AOC = \alpha$



Magnitude of Resultant:

$$\therefore R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

Direction of resultant:

$$\therefore \tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$$

Ex.4 Find the resultant of two forces each having magnitude F_0 , and angle between them is θ .

Sol. $F_{\text{Resultant}}^2 = F_0^2 + F_0^2 + 2F_0^2 \cos \theta$

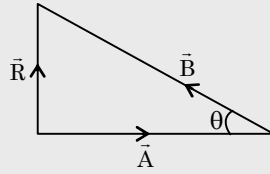
$$= 2F_0^2 (1 + \cos \theta) = 2F_0^2 \times 2 \cos^2 \frac{\theta}{2}$$

$$2 \times 2 \cos^2 \frac{\theta}{2}$$

$$F_{\text{resultant}} = 2F_0 \cos \frac{\theta}{2}$$

Ex.5 The resultant of two velocity vectors $\vec{A}$ and $\vec{B}$ is perpendicular to $\vec{A}$. Magnitude of Resultant $\vec{R}$ is equal to half magnitude of $\vec{B}$. Find the angle between $\vec{A}$ and $\vec{B}$?

Sol. Since $\vec{R}$ is perpendicular to $\vec{A}$. Figure shows the three vectors $\vec{A}$, $\vec{B}$ and $\vec{R}$.



angle between $\vec{A}$ and $\vec{B}$ is $\pi - \theta$

$$\sin \theta = \frac{R}{B} = \frac{R}{2B} = \frac{1}{2}$$

$$\Rightarrow \theta = 30^\circ$$

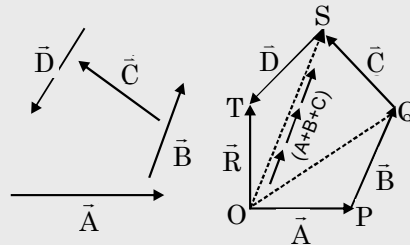
$\Rightarrow$ angle between A and B is 150° .

◆ Polygon Law of Vector Addition

If a number of vectors are represented both in magnitude and direction by the sides of a polygon taken in the same order, then the resultant vector is represented both in magnitude and direction by the closing side of the polygon taken in the opposite order.

Let the number of vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$ etc. be acting in different directions as shown in figure.

To find their resultant vector, coincide the tail of $\vec{B}$ with the head of $\vec{A}$, tail of $\vec{C}$ with the head of $\vec{B}$ and tail of $\vec{D}$ with the head of $\vec{C}$. Then the single vector drawn from the tail of $\vec{A}$ to head of $\vec{D}$.

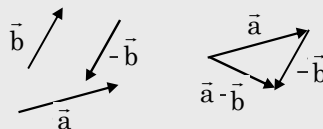


will thus, it is clear that the vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$ are represented in magnitude and direction by the sides $\overrightarrow{OP}$, $\overrightarrow{PQ}$, $\overrightarrow{QS}$ and $\overrightarrow{ST}$ of an open polygon taken in the same order, then their resultant vector $\vec{R}$ will be represented in magnitude and direction by the closing side $\overrightarrow{OT}$ of the polygon taken in opposite order. This method of finding the resultant is called polygon law of vectors.

- **Subtraction of Vectors:** The negative of a vector is defined as a vector of same magnitude but opposite direction.



The Subtraction of a vector $\vec{b}$ from another vector $\vec{a}$ is defined as the addition of $-\vec{b}$ to $\vec{a}$, as shown in figure

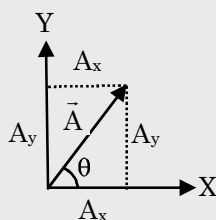


- **Resolution of A Vector:** It is the process of splitting a single vector into two or more vectors in different directions which together produce the same effect as is produced by the single vector alone. The vectors into which the given single vector is split are called component vectors. Infact, the resolution of a vector is just opposite to composition of vectors.

A vector can have infinite component vectors but for simplicity a vector is resolved into two or three mutually perpendicular components

- **Resolution into mutually Perpendicular Vectors in a plane (2D Resolution):** The figure shows resolution of a vector $\vec{A}$ into two oblique vectors $\vec{A}_x$ and $\vec{A}_y$.

Using the elementary knowledge of trigonometry the vectors $\vec{A}_x$ and $\vec{A}_y$ are given by $\vec{A}_x = \vec{A} \cos \theta$, $\vec{A}_y = \vec{A} \sin \theta$



Resolution of a vector into mutually perpendicular components

◆ Scalar Product

The scalar product or dot product of any two vectors $\vec{A}$ and $\vec{B}$, denoted as $\vec{A} \cdot \vec{B}$ (read $\vec{A}$ dot $\vec{B}$) is defined as the product of their magnitude with cosine of angle

between them. Thus, $\vec{A} \cdot \vec{B} = AB \cos \theta$ {here θ is the angle between the vectors}

If the scalar product of two nonzero vectors vanishes then the vectors are perpendicular.

The scalar product of a vector by itself is termed as self dot product and is given by

$$(\vec{A})^2 = \vec{A} \cdot \vec{A} = AA \cos \theta = AA \cos 0^\circ = A^2$$

$$\Rightarrow A = \sqrt{\vec{A} \cdot \vec{A}}$$

In case of unit vector $\hat{n}$,

$$\hat{n} \cdot \hat{n} = 1 \times 1 \times \cos 0^\circ = 1$$

$$\Rightarrow \hat{n} \cdot \hat{n} = \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

In case of orthogonal unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$;

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

$$\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) = [A_x B_x + A_y B_y + A_z B_z]$$

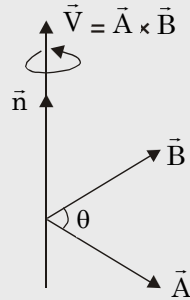
◆ Vector Product

The vector product or cross product of any two vectors $\vec{A}$ and $\vec{B}$, denoted as $\vec{A} \times \vec{B}$ (read $\vec{A}$ cross $\vec{B}$) is defined as:

$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$$

Here θ is the angle between the vectors and the direction $\hat{n}$ is given by the right-hand-thumb rule.

- **Right-Hand-Thumb Rule:** To find the direction of $\hat{n}$, draw the two vectors $\vec{A}$ and $\vec{B}$ with both the tails coinciding. Now place your stretched right palm perpendicular to the plane of $\vec{A}$ and $\vec{B}$ in such a way that the fingers are along the vector $\vec{A}$ and when the fingers are closed they go towards $\vec{B}$. The direction of the thumb gives the direction of.



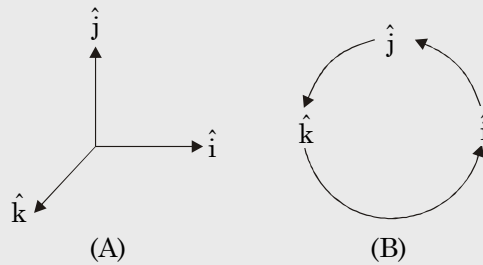
$$\vec{A} \times \vec{A} = AA \sin 0^\circ \hat{n} = \vec{0}.$$

In case of unit vector $\hat{n}$, $\hat{n} \times \hat{n} = \vec{0}$

$$\Rightarrow \hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \vec{0}$$

In case of orthogonal unit vectors $\hat{i}, \hat{j}, \hat{k}$

in accordance with right-hand-thumb-rule, $\hat{i} \times \hat{j} = \hat{k}$ $\hat{j} \times \hat{k} = \hat{i}$ $\hat{k} \times \hat{i} = \hat{j}$



$$\text{In terms of components, } \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{j} \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k} \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \hat{i} (A_y B_z - A_z B_y) - \hat{j} (A_x B_z - A_z B_x) + \hat{k} (A_x B_y - A_y B_x)$$

The magnitude of area of the parallelogram formed by the adjacent sides of vector and equal to $|\vec{A} \times \vec{B}|$.

Ex.6 If the Vectors $\vec{P} = a\hat{i} + a\hat{j} + 3\hat{k}$ and $\vec{Q} = a\hat{i} - 2\hat{j} - \hat{k}$ are perpendicular to each other. Find the value of a ?

Sol. If vectors $\vec{P}$ and $\vec{Q}$ are perpendicular

$$\Rightarrow \vec{P} \cdot \vec{Q} = 0$$

$$\Rightarrow (a\hat{i} + a\hat{j} + 3\hat{k}) \cdot (a\hat{i} - 2\hat{j} - \hat{k}) = 0$$

$$\Rightarrow a^2 - 2a - 3 = 0$$

$$\Rightarrow a^2 - 3a + a - 3 = 0$$

$$\Rightarrow a(a - 3) + 1(a - 3) = 0$$

$$\Rightarrow a = -1, 3$$

Ex.7 Two vectors $\vec{A}$ and $\vec{B}$ are inclined to each other at an angle θ . Find a unit vector which is perpendicular to both $\vec{A}$ and $\vec{B}$.

Sol. $\vec{A} \times \vec{B} = AB \sin \theta \Rightarrow \hat{n} = \frac{\vec{A} \times \vec{B}}{AB \sin \theta}$

here $\hat{n}$ is perpendicular to both $\vec{A}$ and $\vec{B}$.

Ex.8 Find $\vec{A} \times \vec{B}$, if $\vec{A} = \hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{B} = 2\hat{i} - \hat{j} + 2\hat{k}$

Sol. $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 4 \\ 2 & -1 & 2 \end{vmatrix}$

$$= \hat{i}(-4 - (-4)) - \hat{j}(2 - 12) + \hat{k}(-1 - (-6)) = 10\hat{j} + 5\hat{k}$$

Distance and Displacement

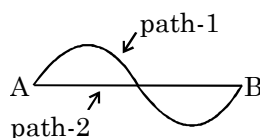
◆ Distance:

It is the actual length of path covered by a moving particle. It is a scalar quantity. Its S.I. unit is metre (m).

◆ Displacement:

It is the shortest distance between the initial and final position of the particle. It is a vector quantity. Its S.I. unit is metre (m).

e.g.: Consider a body moving from a point A to a point B along the path shown in figure. Then total length of path covered is called distance (path-1). While the length of straight line AB in the direction from A to B is called displacement (path-2).

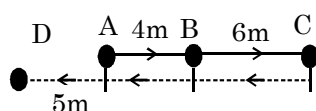


Note : If a body travels in such a way that it comes back to its starting position, then the displacement is zero. However, distance travelled is never zero in case of moving body.

◆ **Some important points:**

- (i) When an object moves towards right from origin, its displacement consider as positive.
- (ii) When an object moves towards left from origin its displacement consider as negative.
- (iii) When an object remains stationary or it moves first towards right and then an equal distance towards left, its displacement is zero.
- (iv) Shifting origin causes no change in displacement.
- (v) If body moves along the circumference of the circle of radius r then distance travelled by it is given by $2\pi r$ and displacement is given by zero, for one complete revolution.

Ex.9 A body starts from A and moves according to given figure. (Body retraces the path after C then reaches to D)



The distance and displacement are as follows for different path

Sol.

Path	Distance	Displacement
AB	4m	4m
ABC	10m	10m
ABCB	16m	4m
ABCA	20m	0m
ABCAD	25m	-5m

◆ **Difference between distance and displacement**

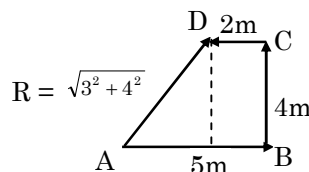
Distance	Displacement
1. Distance is the length of the path actually traveled by a body in any direction.	1. Displacement is the shortest distance between the initial and the final positions of a body in the direction of the point of the final position.
2. Distance between two given points depends upon the path chosen.	2. Displacement between two points is measured by the straight path between the points.
3. Distance is always positive.	3. Displacement may be positive as well as negative and even zero.
4. Distance is a scalar quantity.	4. Displacement is a vector quantity.
5. Distance will never decrease.	5. Displacement may decrease.

Ex.10 A person travels a distance of 5 m towards east, then 4 m towards north and then 2 m towards west.

- Calculate the total distance travelled.
- Calculate the resultant displacement.

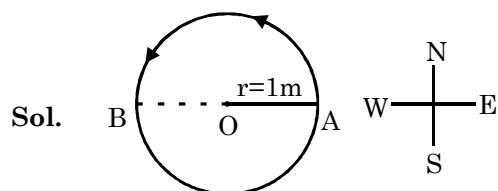
Sol. (i) Total distance travelled by the person = 5 m + 4 m + 2 m = 11 m

- The resultant displacement is calculated by joining the initial position A to the final position D. Hence, the displacement of the person = 5m towards AD.



Ex.11 A person moves in a circular path centered at O. He starts from A and reaches diametrically opposite point B. Then find:

- distance between A and B
- displacement between A and B



- Distance = Length of actual circular path from A to B = Half the circumference

$$\text{i.e. Distance} = \frac{2\pi r}{2} = \pi r$$

$$r = 1\text{ m}$$

$$\therefore \text{Distance} = \pi \text{ m}$$

- Displacement = 2r along west = 2m along west

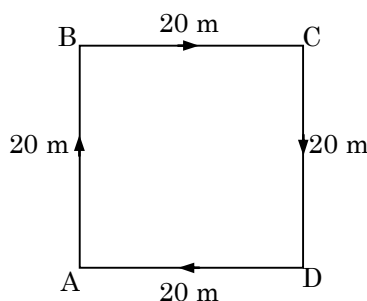
Ex.12 What does the odometer of an automobile measure?

Sol. The odometer of an automobile measures the distance covered by an automobile.

Ex.13 An object has moved through a distance. Can it have zero displacement? If yes, support your answer with an example.

Sol. Yes, an object that has moved through a distance can have zero displacement. Displacement is the shortest measurable distance between the initial and the final position of an object. An object which has covered a distance can have zero displacement, if it comes back to its starting point, i.e., the initial position.

Consider the following situation: A man is walking in a square park of length 20 m (as shown in the following figure). He starts walking from point A and after moving along all the corners of the park (point B, C, D), he again comes back to the same point, i.e., A.



In this case, the total distance covered by the man is $20\text{ m} + 20\text{ m} + 20\text{ m} + 20\text{ m} = 80\text{ m}$. However, his displacement is zero because the shortest distance between his initial and final position is zero.

Ex.14 A farmer moves along the boundary of a square field of side 10 m in 40 s . What will be the magnitude of displacement of the farmer at the end of 2 minutes 20 seconds?

Sol. The farmer takes 40 s to cover $4 \times 10 = 40\text{ m}$.

In 2 min and 20 s (140 s), he will cover a distance $\frac{40}{40} \times 140 = 140\text{ m}$

Therefore, the farmer completes $\frac{140}{40} = 3.5$ rounds (3 complete rounds and a half round) of the field in 2 min and 20 s.

That means, after 2 min 20 s, the farmer will be at the opposite end of the starting point.

Now, there can be two extreme cases.

Case I: Starting point is a corner point of the field.

In this case, the farmer will be at the diagonally opposite corner of the field after 2 min 20 s.

Therefore, the displacement will be equal to the diagonal of the field.

Hence, the displacement will be $\sqrt{10^2 + 10^2} = 14.1\text{ m}$

Case II: Starting point is the middle point of any side of the field.

In this case the farmer will be at the middle point of the opposite side of the field after 2 min 20 s.

Therefore, the displacement will be equal to the side of the field, i.e., 10 m .

For any other starting point, the displacement will be between 14.1 m and 10 m .

Speed & Velocity

◆ Speed

The distance travelled by a body in unit time is called its speed. Therefore, $\text{speed} = \frac{\text{Distance}}{\text{Time}}$ or $s = \frac{d}{t}$

S.I. unit of speed or average speed is m/sec . It is a scalar quantity.

◆ Types of Speed

- **Average Speed:** For an object moving with variable speed, it is the total distance travelled by the object divided by the total time taken to cover that distance.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

- (i) Let initial speed of an object is v_1 , final speed is v_2 and acceleration is constant, then

$$\text{average speed} = \frac{v_1 + v_2}{2}$$

- (ii) A body covers a distance s_1 in time t_1 , s_2 in time t_2 and s_3 in time t_3 .

$$\text{Then, average speed, } v_{av} = \frac{s_1 + s_2 + s_3}{t_1 + t_2 + t_3}$$

- (iii) A body travels with speed v_1 for a time t_1 , v_2 for time t_2 and v_3 for the time t_3 .

$$\text{Then, average speed, } v_{av} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3}{t_1 + t_2 + t_3}$$

$$s_1 = v_1 t_1, s_2 = v_2 t_2 \text{ and } s_3 = v_3 t_3$$

$$\text{if } t_1 = t_2 = t_3 = t$$

$$v_{av} = \frac{t(v_1 + v_2 + v_3)}{3t}$$

$$v_{av} = \frac{(v_1 + v_2 + v_3)}{3}$$

- (iv) A body covers a distance s_1 with speed v_1 , s_2 with speed v_2 and s_3 with speed v_3 .

$$\text{Then, average speed, } v_{avg} = \frac{(s_1 + s_2 + s_3)}{\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3}}$$

$$t_1 = \frac{s_1}{v_1}, t_2 = \frac{s_2}{v_2}, t_3 = \frac{s_3}{v_3}$$

- (v) A boy goes from home to school with speed v_1 and come back to home with speed v_2 .

Here distance covered by the boy is same

Time taken by the boy, while traveling from home to school,

$$t_1 = \frac{s}{v_1}$$

and time taken by the boy, while traveling from school to home,

$$t_2 = \frac{s}{v_2}$$

$$\text{Then, average speed, } v_{av} = \frac{s + s}{t_1 + t_2} = \frac{2s}{\frac{s}{v_1} + \frac{s}{v_2}}$$

$$v_{av} = \frac{2v_1 v_2}{v_1 + v_2}$$

- (vi) If an object covers $1/3^{\text{rd}}$ distance with speed u , next $1/3^{\text{rd}}$ with speed v and last $1/3^{\text{rd}}$ distance

$$\text{with speed } w, \text{ then } v_{avg} = \frac{3uvw}{uv + vw + wu}$$

- **Uniform Speed (or Constant Speed):** When an object covers equal distance in equal intervals of time, it is said to move with uniform speed. e.g.: A car moves 10 m in every one second so its motion is uniform.
- **Variable Speed (Non-Uniform Speed):** If a body covers unequal distance in equal intervals of time, its motion is said to be non-uniform. e.g.: Falling of an apple from a tree, a cyclist moving on a rough road, an athlete running a race, vehicle starting from rest, the motion of a freely falling body etc.
- **Instantaneous speed:** The speed of object at a particular instant is called instantaneous speed.

COMPETITIVE LEVEL

The limiting value of average speed when the time interval approaches zero, Thus,

$$\text{Instantaneous speed} = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

◆ Velocity

It is defined as the rate of change of displacement.

$$\text{Therefore, velocity} = \frac{\text{displacement}}{\text{time}}$$

or it is the distance travelled in unit time in a given direction.

$$\text{Velocity} = \frac{\text{Distance travelled in a given direction}}{\text{time taken}}$$

S.I. unit of velocity is m/s. It is a vector quantity.

(Magnitude of the velocity is known as speed)

Note:

- To convert m/s into km/h we multiply by 18/5
- To convert km/h into m/s we multiply by 5/18

◆ Types of Velocity

- **Uniform Velocity (Constant Velocity):** If a body covers equal distance in equal intervals of time in a given direction then it is said to be moving with constant velocity.
- **Non-Uniform Velocity:** When a body does not cover equal distances in equal intervals of time, in a given direction (in this case speed is not constant), then it is known as non uniform velocity.

In uniform circular motion speed is constant but velocity is not constant.

- **Average Velocity:** It is defined as the ratio of total displacement to the total time taken for this displacement. It is denoted by $\bar{v}_{av}$ or $\bar{v}$. It is a vector quantity.

$$\therefore \text{Average velocity} = \frac{\text{Total displacement}}{\text{total time}}$$

$$\text{i.e. } \bar{v} = \frac{s}{t}$$

It is a vector quantity, and its direction is in the direction of displacement.

Note:

- If for a straight line motion displacement is positive then $\bar{v}$ is positive.
- If displacement is negative then $\bar{v}$ is also negative.

- **Instantaneous Velocity:** The velocity of an object at a particular instant of time is called instantaneous velocity. It is a vector quantity.

COMPETITIVE LEVEL

It is equal to the limiting value of average velocity of the object when the time interval approaches zero, thus Instantaneous velocity $\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{s}}{\Delta t} = \frac{d\vec{s}}{dt}$

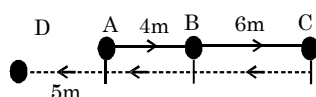
◆ More about Speed and Velocity:

- The instantaneous velocity in magnitude is equal to instantaneous speed.
- A particle may have constant speed but variable velocity. In uniform circular motion speed remains constant while velocity changes because of change in direction in motion.
- If particle is moving along a straight line without changing the direction, its average velocity will be equal to its average speed. Otherwise average velocity will be less than average speed.

$$\left| \frac{\text{Average velocity}}{\text{average speed}} \right| \leq 1$$

Ex.15 A body starts from A and moves according to given figure. Time for each interval is:

$$t_{AB} = 2s, t_{BC} = 3s, t_{CB} = 2s, t_{BA} = 3s, t_{AD} = 4s$$



Find the distance, displacement, speed and velocity for path AB, ABC, ABCB, ABCA, ABCAD.

Sol. The distance, displacement, speed and velocity are as follows for different paths.

Path	Distance	Displacement	Speed	Velocity
AB	4m	4m	4/2 m/s	4/2 m/s
ABC	10m	10m	10/5 m/s	10/5 m/s
ABCB	16m	4m	16/7 m/s	4/7 m/s
ABCA	20m	0m	20/10 m/s	0/10 m/s
ABCAD	25m	-5m	25/14 m/s	-5/14 m/s

Ex.16 Distinguish between speed and velocity.

Sol. Difference between speed and velocity:

Speed	Velocity
1. It is the rate of change of position of an object.	1. It is the rate of change of position of an object in a specific direction.
2. $\text{Speed} = \frac{\text{distance travelled}}{\text{time}}$	2. $\text{Velocity} = \frac{\text{displacement}}{\text{time}}$
3. It is a scalar quantity.	3. It is a vector quantity
4. Speed will always be positive	4. It will be positive or negative depending on the direction of motion.
5. For moving body, it will never be zero.	5. It may be zero.

Ex.17 What does the path of an object look like when it is in uniform motion?

Sol. An object having uniform motion has a straight line path.

Ex.18 During an experiment, a signal from a spaceship reached the ground station in five minutes. What is the distance of the spaceship from the ground station? The signal travels at the speed of light, that is, $3 \times 10^8 \text{ m s}^{-1}$.

Sol. Time taken by the signal to reach the ground station from the spaceship = 5 min = $5 \times 60 = 300 \text{ s}$
Speed of the signal = $3 \times 10^8 \text{ m/s}$

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

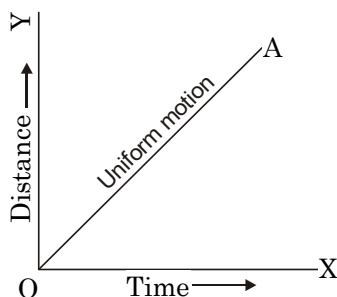
$$\therefore \text{Distance travelled} = \text{Speed} \times \text{Time taken} = 3 \times 10^8 \times 300 = 9 \times 10^{10}$$

Hence, the distance of the spaceship from the ground station is $9 \times 10^{10} \text{ m}$.

Uniform and Non-Uniform Motion

◆ Uniform Motion

A body has a uniform motion if it travels equal distances in equal intervals of time, no matter how small these time intervals may be. For example, a car running at a constant speed say, 10 metre per second, will cover equal distances of 10 metre every second, so its motion will be uniform. Please note that the distance-time graph for uniform motion is a straight line (As shown in the figure).



◆ Non-Uniform Motion

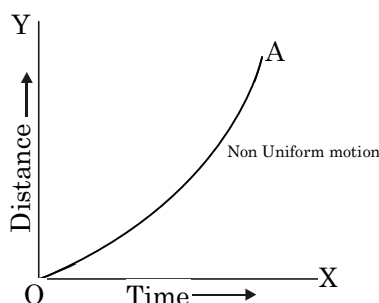
A body has a non-uniform motion if it travels unequal distances in equal intervals of time. For example, if we drop a ball from the roof of a building, we will find that it covers unequal distances in equal intervals of time. It covers:

4.9 metre in the 1st second,

14.7 metre in the 2nd second,

24.5 metre in the 3rd second, and so on.

Thus, a freely falling ball covers smaller distance in the initial '1 second' interval and larger distance in the later '1 second' interval. From this discussion we conclude that the motion of a freely falling body is an example of non-uniform motion. The motion of a train starting from the railway station is also an example of non-uniform motion. This is because when the train starts from a station, it moves a very small distance in the 'first' second. The train moves a little more distance in the '2nd' second and so on. And when the train approaches the next station, the distance travelled by it per second decreases.

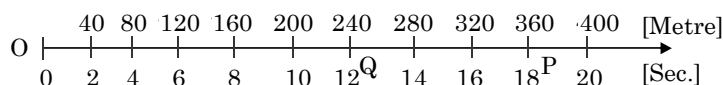


Please note that the distance-time graph for a body having non-uniform motion is a curved line (As shown in the figure). Thus, in order to find out whether a body has uniform motion or non-uniform motion, we should draw the distance-time graph for it. If the distance time graph is a straight line, the motion will be uniform and if the distance-time graph is a curved line, the motion will be non-uniform. It should be noted that non-uniform motion is also called accelerated motion.

◆ Features of Uniform Motion

- (i) The velocity in uniform motion does not depend on the choice of origin.
- (ii) The velocity in uniform motion does not depend on the choice of the time interval.
- (iii) For uniform motion along a straight line in the same direction, the magnitude of the displacement is equal to the actual distance covered by the object.
- (iv) The velocity is positive if the object is moving towards the right of the origin and negative if the object is moving towards the left of the origin.
- (v) For an object in uniform motion no force is required to maintain its motion.
- (vi) In uniform motion, the instantaneous velocity is equal to the average velocity at all times because velocity remains constant at each instant, at each point of the path.

Ex.19 A car is moving along x-axis. As shown in figure it moves from O to P in 18 s and returns from P to Q in 6 second. What is the average velocity and average speed of the car in going from (i) O to P and (ii) from O to P and back to Q.



Sol. (i) Average velocity = $\frac{\text{Displacement}}{\text{time interval}} = \frac{360}{18} = 20 \text{ ms}^{-1}$

Average speed = $\frac{\text{path length}}{\text{time interval}} = \frac{360}{18} = 20 \text{ ms}^{-1}$

(ii) From O to P and back to Q

Average velocity = $\frac{\text{OQ}}{18 + 6} = \frac{240\text{m}}{24} = 10 \text{ ms}^{-1}$

Average speed = $\frac{\text{path length}}{\text{time interval}}$
 $= \frac{\text{OP} + \text{PQ}}{18 + 6} = \frac{360 + 120}{24} = 20 \text{ ms}^{-1}$

Ex.20 A car covers the 1st half of the distance between two places at a speed of 40 km h⁻¹ and the 2nd half with 60 km h⁻¹. What is the average speed of the car?

Sol. Suppose the total distance covered is 2S.

Then time taken to cover the distance 'S' with speed 40 km/h,

$$t_1 = \frac{S}{40} \text{ h}$$

Time taken to cover the next distance 'S' with speed 60 km/h,

$$t_2 = \frac{S}{60} \text{ h}$$

$$V_{av} = \frac{\text{total distance}}{\text{total time}} = \frac{S + S}{\left(\frac{S}{40} + \frac{S}{60}\right)}$$

$$V_{av} = \frac{2S}{\left(\frac{3S + 2S}{120}\right)} = \frac{2S}{5S} \times 120$$

$$\Rightarrow V_{av} = 48 \text{ km/h}$$

Ex.21 A non-stop bus goes from one station to another station with a speed of 54 km/h, the same bus returns from the second station to the first station with a speed of 36 km/h. Find the average speed of the bus for the entire journey.

Sol. Suppose the distance between the stations is S. Time taken in reaching from one station to another station.

$$t_1 = \frac{S}{54} \text{ h}$$

Time taken in returning back,

$$t_2 = \frac{S}{36} \text{ h}$$

Total time $t = t_1 + t_2$

$$t = \frac{S}{54} + \frac{S}{36} = \frac{2S + 3S}{108} = \frac{5S}{108} \text{ h}$$

$$\text{Average speed } V_{av} = \frac{\text{Total distance}}{\text{Total time}}$$

$$V_{av} = \frac{2S}{5S} \times 108$$

$$V_{av} = \frac{216}{5} = 43.2 \text{ km/h}$$

Acceleration

Mostly the velocity of a moving object changes either in magnitude or in direction or in both when the object moves. The body is then said to have acceleration. So it is the rate of change of velocity i.e. change in velocity in unit time is said to be acceleration. It is a vector quantity and

Its S.I unit is m/s² and c.g.s unit is cm/s².

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

$$a = \frac{v - u}{t} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time}}$$

◆ Types of Acceleration

- **Uniform Acceleration (Uniformly Accelerated Motion):** If a body travels in a straight line and its velocity increases in equal amounts in equal intervals of time. Its motion is known as uniformly accelerated motion.

e.g.: Motion of a freely falling body is an example of uniformly accelerated motion (or motion of a body under the gravitational pull of the earth) & motion of a bicycle going down the slope of a road when the rider is not pedaling and wind resistance is negligible.

- **Non-Uniform Acceleration:** If during motion of a body its velocity increases by unequal amounts in equal intervals of time, then its motion is known as non uniform accelerated motion.

e.g.: Car moving in a crowded street & motion of a train leaving or entering the platform.

- **Positive acceleration:** If the velocity of an object increases with respect to time in the same direction, the object has a positive acceleration.
- **Negative acceleration (retardation):** If the velocity of a body decreases with respect to time in the same direction, the body has a negative acceleration or it is said to be retarding.

e.g.: A train slows down, then its acceleration will be negative.

Ex.22 A bus decreases its speed from 80 km h^{-1} to 60 km h^{-1} in 5 s. Find the acceleration of the bus.

Sol. Initial speed of the bus, $u = 80 \text{ km/h} = 80 \times \frac{5}{18} = 22.22 \text{ m/s}$

Final speed of the bus, $v = 60 \text{ km/h} = 60 \times \frac{5}{18} = 16.66 \text{ m/s}$

Time take to decrease the speed, $t = 5 \text{ s}$

$$\text{Acceleration } a = \frac{v - u}{t} = \frac{16.66 - 22.22}{5} = -1.112 \text{ m/s}^2$$

Here, the negative sign of acceleration indicates that the velocity of the car is decreasing.

Ex.23 A train starting from a railway station and moving with uniform acceleration attains a speed 40 km h^{-1} in 10 minutes. Find its acceleration.

Sol. Initial velocity of the train, $u = 0$ (since the train is initially at rest)

Final velocity of the train, $v = 40 \text{ km/h} = 40 \times \frac{5}{18} = 11.11 \text{ m/s}$

Time taken, $t = 10 \text{ min} = 10 \times 60 = 600 \text{ s}$

$$\text{Acceleration } a = \frac{v - u}{t} = \frac{11.11 - 0}{600} = 0.0185 \text{ m/s}^2$$

Hence, the acceleration of the train is 0.0185 m/s^2 .

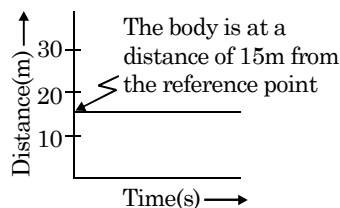
Graphical Representation of Motion

◆ Distance–Time Graph

A moving body changes its position continuously with time. The simplest way to describe the motion of a moving body is to draw its distance–time graph.

The distance–time graphs of a body under the following three conditions are described below:

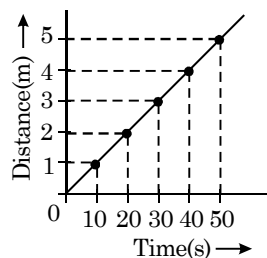
- (i) When the body is at rest.
 - (ii) When the body is moving with a uniform speed
 - (iii) When the body is moving with a non–uniform speed
- Distance–time graph for a body at rest



The distance–time graph for a body at rest is a straight line parallel to the time axis

- Distance–time graph for a body moving with a uniform speed:

When a body covers equal distances in equal intervals of time, it is said to have uniform speed.



Distance–time graph of a body moving with uniform (Constant) speed

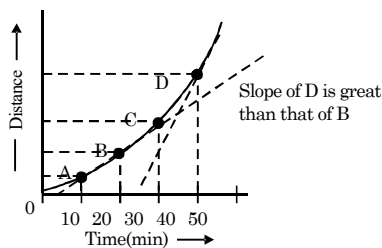
The above graph shows that the distance travelled by a body moving with uniform speed is directly proportional to time.

- Distance–time graph for a body moving with a non–uniform speed:

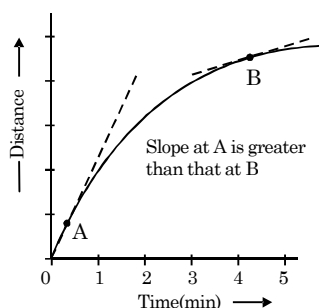
A body moving with a non–uniform speed covers unequal distances in equal intervals of time. Therefore, the distance–time graph of a body moving with a non–uniform speed is a curve.

The shape of the distance–time graph for a body moving with non–uniform speed depends upon the way speed of the body changes with time. Two typical cases are described below:

- (i) **When the speed increases with time:** When the speed of a body increases with time, the distance covered by it in one unit of time also increases with time. Therefore, the distance–time graph for a body moving with an increasing non–uniform speed is a curve whose slope increases with time (Figure).

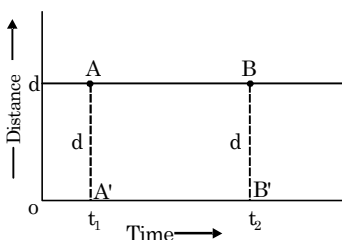


- (ii) **When the speed decreases with time:** When the speed of a body decreases with time, the distance covered by it in one unit of time also decreases with time. Therefore, the distance–time graph for a body moving with a decreasing non–uniform speed is a curve whose slope decreases with time (Figure).



◆ Displacement-Time Graph

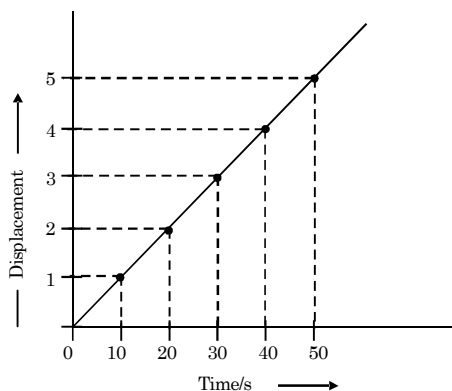
- Displacement – time graph of a body at rest:** The position of a body at rest remains unchanged with time. Let us consider a body at a distance d from a reference point in a particular direction. Then from figure 4.1.



The above graph shows that position of the body does not change w.r.t. time, so that body is said to be at rest.

Thus, the velocity of a body at rest is zero.

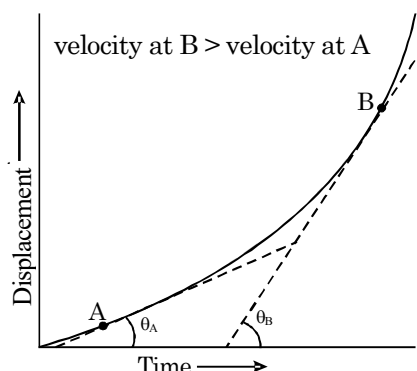
- Displacement – time graph of a body moving with uniform velocity:** The displacement–time graph of a body moving with uniform (constant) velocity is a straight line inclined to the time–axis at certain angle.



The slope of the displacement–time graph for a body moving with uniform velocity is equal to the velocity of the body.

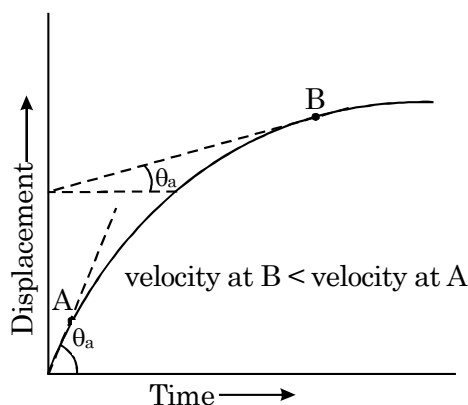
- **Displacement–time graph of a body moving with an increasing non–uniform velocity :** The displacement–time graph of a body moving with an increasing non–uniform velocity is a curve (Figure). Here, the slope of the curve increases with time. So, the velocity of the body increases with time.

i.e., velocity at B > velocity at A



- **Displacement–time graph of a body moving with decreasing non–uniform velocity :** The displacement–time graph of a body moving with a decreasing non–uniform velocity is a curve (Figure). Here, the slope of the curve decreases with time. So, the velocity of the body decreases with time.

i.e., velocity at B < velocity at A



◆ Speed-Time Graph

Some information about the motion of an object can also be obtained from its speed–time graph. Following figures give the speed–time graphs of four different objects in motion. Let us see what information can be obtained from these graphs.

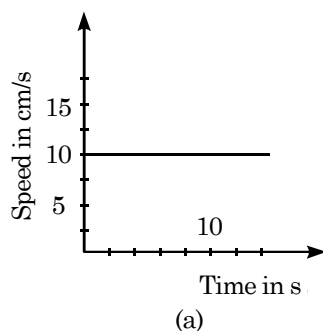


Figure (a) shows that for any time, the speed has the same value (10 cm/s). Thus it represents an object moving with a constant speed.

Whenever an object moves with a constant speed, its speed–time graph is a straight line, parallel to the time–axis.

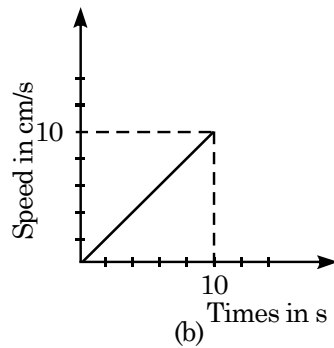


Figure (b) shows that the speed continuously increases with time. At time $t = 0$, the speed is 0. At $t = 10$ s, it becomes 10 cm/s. The straight-line nature of the graph indicates that the speed increases at a constant rate.

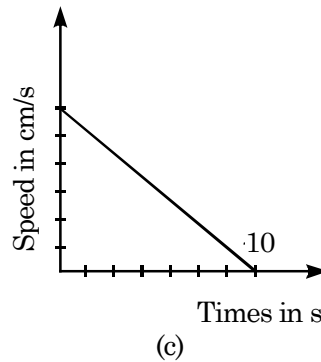


Figure (c) shows that the speed is 10 cm/s at $t = 0$ and gradually decreases as time passes. Thus it represents a decelerating object. Here also the speed changes at constant rate. At $t = 10$ s, the speed becomes zero.

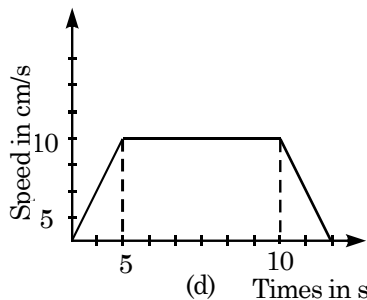
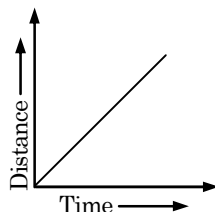


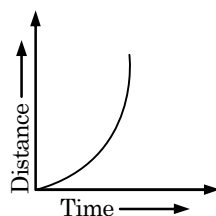
Figure (d) represents the motion of an object which speeds up from $t = 0$ to $t = 5$ s, then moves at a constant speed from $t = 5$ s to 10 s and then decelerates to stop at $t = 15$ s.

Note: Distance covered by the body is shown by the area under speed–time graph.

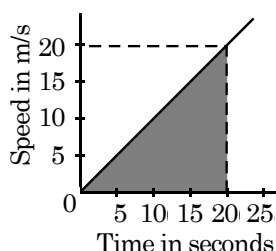
Ex.24 What is the nature of the distance - time graphs for uniform and non-uniform motion of an object?
Sol. The distance–time graph for uniform motion of an object is a straight line (As shown in the following figure).



The distance - time graph for non-uniform motion of an object is a curved line (As shown in the given figure).



Ex.25 Find the distance covered by a particle during the time interval $t = 0$ to $t = 20\text{s}$ for which the speed-time graph is shown in figure.

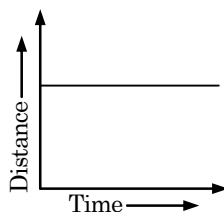


Sol. The distance covered in the time interval 0 to 20s is equal to the area of the shaded triangle. It is

$$\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times (20\text{s}) \times (20 \text{ m/s}) = 200 \text{ m}.$$

Ex.26 What can you say about the motion of an object whose distance-time graph is a straight line parallel to the time axis?

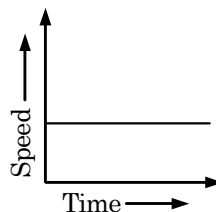
Sol. When an object is at rest, its distance-time graph is a straight line parallel to the time axis.



A straight line parallel to the x-axis in a distance - time graph indicates that with a change in time, there is no change in the position of the object. Thus, the object is at rest.

Ex.27 What can you say about the motion of an object if its speed-time graph is a straight line parallel to the time axis?

Sol. Object is moving uniformly.



A straight line parallel to the time axis in a speed-time graph indicates that with a change in time, there is no change in the speed of the object. This indicates the uniform motion of the object.

◆ Velocity-Time Graph

If a graph is plotted taking the velocity of an object moving along a straight line on the vertical axis and time on the horizontal axis, we get a velocity–time graph.

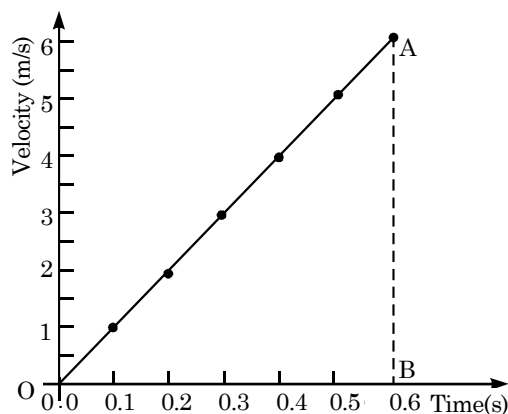
Suppose an object moves along a straight line in a fixed direction. That means the object does not turn around during its motion. Taking the direction of motion as the positive direction, the velocity of the object is given by the same value as its speed. Thus the speed–time graph for such an object is also its velocity–time graph.

The area under the speed–time graph gives the distance covered. But for a particle moving in a fixed direction, the distance covered in a time interval has the same value as its displacement in that time interval. So, the area under the velocity–time graph of an object gives its displacement.

Let us take an example. A ball is dropped from a height. We take the downward direction as positive. As the ball falls, its velocity increases. The velocity of the ball at different instants are given in Table-1. The velocity versus time graph is shown in figure.

Table-1: Velocity of the falling ball at different instants:

Time in s	Velocity in m/s
0	0
0.1	1
0.2	2
0.3	3
0.4	4
0.5	5
0.6	6



What is the displacement of the ball in the time interval 0 to 0.6s? It is equal to the area under the velocity–times graph from $t = 0$ to $t = 0.6$ s. This area is in the shape of a triangle. The area is

$$\frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} (\text{OB}) \times (\text{AB})$$

$$= \frac{1}{2} \times (0.6\text{s}) \times (6 \text{ m/s}) = 1.8 \text{ m.}$$

The ball has fallen through 1.8 m in 0.6 s.

- **Velocity–time graph of an object thrown upwards:** Suppose a ball is thrown upwards. We take the upward direction as the positive direction. The velocity decreases as the ball goes up. Table-2 gives the velocity of the ball at different instants.

Table-2: Velocity of the rising ball at different instants:

Time(s)	Velocity (m/s)
0	10
1	12
2	14
3	16
4	18
5	20

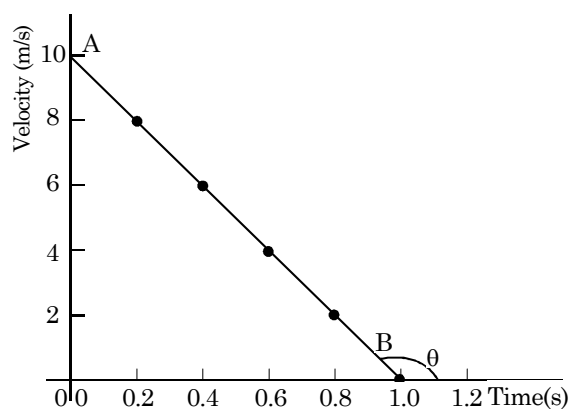
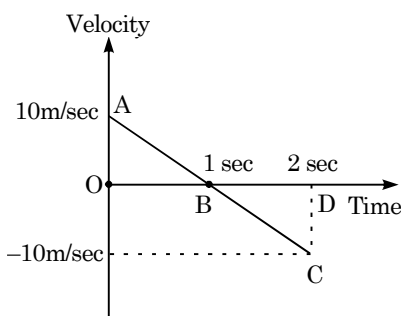


Figure shows the velocity–time graph. The plotted points fall on a straight line, AB. At $t = 1.0$ s, the velocity becomes zero. This means that the ball reaches the highest point at $t = 1.0$ s.

- Ex.28** A ball is thrown vertically upwards with a velocity of 10m/sec. It strikes the ground after 2 sec. Its velocity–time graph is as shown in figure below. Find the displacement travelled by the ball in 2 second.



Sol. Displacement = area under velocity-time curve along time axis
 = area of triangle AOB + area of triangle BDC

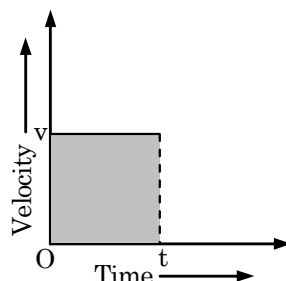
$$= \left(\frac{1}{2} \times OB \times AO \right) + \left(\frac{1}{2} \times BD \times CD \right)$$

$$= \left(\frac{1}{2} \times 1 \text{ sec} \times 10 \text{ m/sec} \right) + \left(\frac{1}{2} \times 1 \text{ sec} \times (-10 \text{ m/sec}) \right)$$

$$= 5 \text{ m} - 5 \text{ m} = 0 \text{ m}$$

Ex.29 What is the quantity which is measured by the area occupied below the velocity - time graph?

Sol. Displacement



The graph shows the velocity-time graph of a uniformly moving body.

Let the velocity of the body at time (t) be v .

Area of the shaded region = length $\times$ breadth

Where, Length = t ; Breadth = v

Area = vt = velocity $\times$ time ... (i)

We know, velocity = displacement / time

$\therefore$ Displacement = Velocity $\times$ Time ... (ii)

From equations (i) and (ii), Area = Displacement

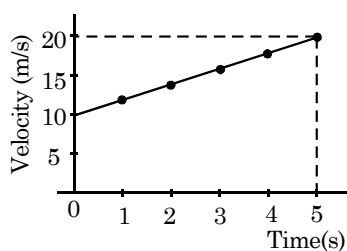
Hence, the area occupied below the velocity-time graph measures the displacement covered by the body.

- **Acceleration from velocity-time graph:** Suppose a particle moves with a uniform acceleration of 2 m/s^2 along a straight line. This means that the velocity increases by 2 m/s in one second. Also suppose its speed at $t = 0$ is 10 m/s .

Let us plot the velocity-time graph for this situation. We first find the values of the velocity at certain instants. At $t = 0$, the velocity is 10 m/s , at $t = 1 \text{ s}$ it will become $10 \text{ m/s} + 2 \text{ m/s} = 12 \text{ m/s}$, at $t = 2 \text{ s}$, it will become 14 m/s and so on. These values are given in Table-3 and the velocity-time graph is shown in Figure.

Table-3: Velocity at different instants

Time(s)	Velocity (m/s)
0	10
1	12
2	14
3	16
4	18
5	20



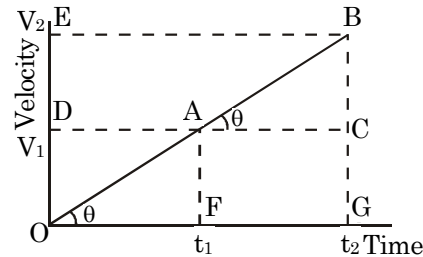
We see that the graph is a straight line. Whenever the acceleration is uniform the velocity-time graph is a straight line. We will now show that the slope of the velocity-time graph gives the acceleration.

Suppose the velocity–time graph of a particle moving along a straight line is as shown in figure. The graph is a straight line. At time t_1 , the velocity is v_1 , and at time t_2 , it is v_2 . These values are represented by the points A and B on the graph.

The acceleration of the object is

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{OE - OD}{OG - OF} = \frac{DE}{FG} = \frac{BC}{AC} = \tan \theta$$

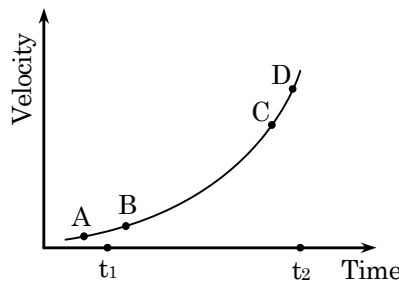
where, θ is the angle made by the graph with the time-axis.



As defined earlier, the ratio $\frac{BC}{AC} = \tan \theta$ is called the slope of the line. Thus, we have the following:

The slope of the velocity–time graphs gives the acceleration for an object moving along a straight line.

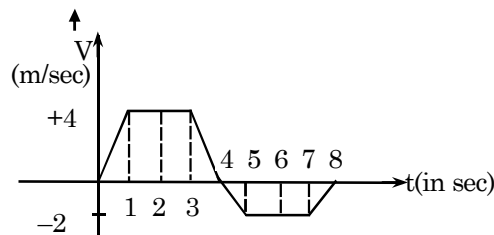
- **Non-uniform acceleration:** If the acceleration of an object moving along a straight line is not constant, the velocity-time graph is not a straight line. Consider the velocity–time graph shown in figure. To find the acceleration at time t_1 we should treat the small part AB as a straight line and find its slope. Similarly, the slope of the small part CD gives the acceleration at time t_2 . It is clear from the figure that the slope of CD is greater than that of AB. Thus, the acceleration at t_2 is greater than that at t_1 . The graph in figure represents a motion in which acceleration increases with time.

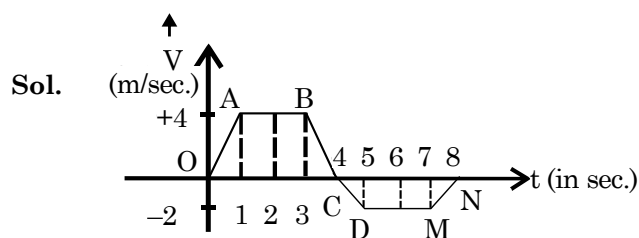


Displacement of particle from velocity time graph.

The area under the velocity–time curve along time axis gives the displacement of the particle.

Ex.30 The velocity versus time graph of a linear motion is shown in figure. Find the distance from the origin in 8 second.





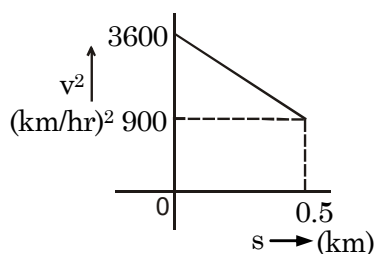
Distance in 8 second

$S = \text{Area of OABC} + \text{Area of CDMN}$

$$S = \frac{(2+4) \times 4}{2} + \frac{(2+4) \times 2}{2}$$

$$S = 18 \text{ m}$$

Ex.31 A graph between the square of the velocity of a particle and the distance moved by the particle is as shown in the figure. Find the acceleration of the particle in km/h^2 .



Sol. Given : $u^2 = 3600 \text{ (km/h)}^2$, $v^2 = 900 \text{ (km/h)}^2$,

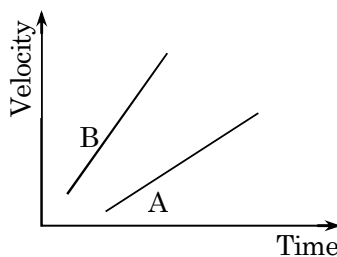
$$s = 0.5 \text{ km}$$

From III equation of motion,

$$v^2 = u^2 + 2as \Rightarrow a = \frac{v^2 - u^2}{2s}$$

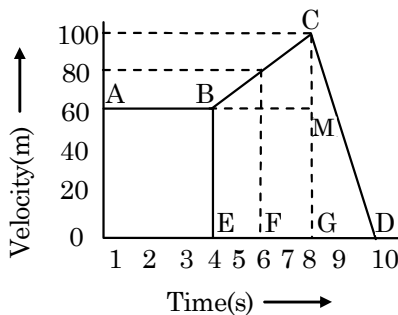
$$\Rightarrow a = \frac{(900) - (3600)}{2 \times 0.5} = \frac{-2700}{2 \times 0.5} = -2700 \text{ km/h}^2$$

Ex.32 Figure shows the velocity–time graphs for two objects, A and B, moving along the same direction. Which object has greater acceleration?



Sol. The slope of the velocity–time graph for B is greater than that for A. Thus, the acceleration of B is greater than that of A.

Ex.33 Time-velocity graph of a particle is shown in figure. Find its instantaneous acceleration at following intervals:



- (i) at $t = 3\text{s}$ (ii) at $t = 6\text{s}$ (iii) at $t = 9\text{s}$

Sol. (i) Instantaneous acceleration at $t = 3\text{s}$, is given by

$$a = \text{slope of line AB} = \text{zero}$$

- (ii) Instantaneous acceleration at $t = 6\text{s}$, is given by $a = \text{slope of line}$

$$BC = \frac{CM}{BM} = \frac{100 - 60}{8 - 4} = 10 \text{ m/s}^2$$

- (iii) Instantaneous acceleration at $t = 9\text{s}$ is given by, $a = \text{slope of line CD} = \frac{0 - 100}{10 - 8} = -50 \text{ m/s}^2$

Ex.34 When will you say a body is in

- (i) uniform acceleration? (ii) non-uniform acceleration?

Sol. (i) A body is said to have uniform acceleration if it travels in a straight path in such a way that its velocity changes at a uniform rate, i.e., the velocity of a body increases or decreases by equal amounts in an equal interval of time.

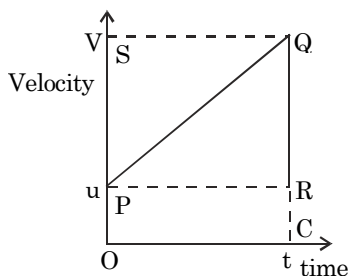
(ii) A body is said to have non-uniform acceleration if it travels in a straight path in such a way that its velocity changes at a non-uniform rate, i.e., the velocity of a body increases or decreases in unequal amounts in an equal interval of time.

Equations of Uniformly Accelerated Motion

◆ Graphical Derivation of Equations of Motion

- **First Equation:** $v = u + at$

It can be derived from $v-t$ graph, as shown in figure: From line PQ, the slope of the line = acceleration



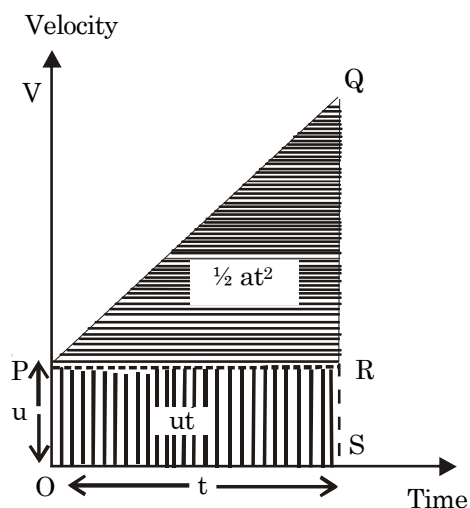
$$a = \frac{QR}{RP} = \frac{SP}{RP}$$

$$\therefore SP = v - u \quad \text{and} \quad RP = t$$

$$\text{So } a = \frac{v - u}{t} \quad \text{or} \quad v = u + at$$

- **Second Equation:** $s = ut + \frac{1}{2} at^2$

It can also be derived from v - t graph as shown in figure. From relation, Distance covered = Area under v - t graphs = Area of trapezium OPQS = Area of rectangle OPRS + Area of triangle PQR



$$= OP \times PR + \frac{RQ \times PR}{2}$$

Putting values,

$$s = u \times t + \frac{1}{2} (v - u) \times t \quad (\because RQ = v - u \text{ \& } PR = OS = t)$$

$$= u \times t + \frac{1}{2} at \times t \quad (\because v - u = at)$$

$$\text{or } s = ut + \frac{1}{2} at^2$$

- **Third Equation :**

$$v^2 = u^2 + 2as$$

From above graph $OP = u$, $SQ = v$, $OP + SQ = u + v$

$$a = \frac{QR}{PR} \text{ Or } PR = \frac{QR}{a} = \frac{v - u}{a}$$

$$s = \text{Area of trapezium OPQS} = \frac{OP + SQ}{2} \times PR$$

On putting the values,

$$s = \frac{u + v}{2} \times \frac{v - u}{a} = \frac{v^2 - u^2}{2a}$$

$$\text{or } v^2 = u^2 + 2as$$

◆ Analytical Derivation for Equations of Uniformly Acceleration Motion

There are three equation of uniformly accelerated motion. They show the relation between initial velocity u , final velocity v , acceleration a , time t and displacement s

- **1st Equation of Motion:** Consider a body moving with initial velocity u and its velocity changes from u to v in time t . Then

$$\text{acceleration} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}}$$

$$\Rightarrow a = \frac{v - u}{t}$$

$$\text{So } at = v - u \text{ and } v = u + at$$

$$1^{\text{st}} \text{ equation of motion: } v = u + at$$

- **2nd Equation of Motion:** We know

$$\text{Distance covered} = (\text{Average velocity}) \times (\text{Time})$$

$$\text{or } s = \frac{u + v}{2} t$$

$$\text{But } v = u + at$$

Substituting the value of v in the equation above, we have

$$s = \frac{u + (u + at)}{2} t$$

$$\text{or } s = \left(\frac{2u + at}{2} \right) t = \left(u + \frac{at}{2} \right) t$$

$$\text{or } s = ut + \frac{1}{2} at^2$$

$$2^{\text{nd}} \text{ equation of motion: } s = ut + \frac{1}{2} at^2$$

- **3rd Equation of Motion:** We know that

$$v = u + at$$

$$\text{or } t = \frac{v - u}{a}$$

$$\text{Distance travelled} = (\text{Average velocity}) \times (\text{time})$$

$$s = \left(\frac{v + u}{2} \right) t = \left(\frac{v + u}{2} \right) \left(\frac{v - u}{a} \right)$$

$$\text{or } s = \frac{v^2 - u^2}{2a} \text{ or } v^2 - u^2 = 2as$$

$$3^{\text{rd}} \text{ equation of motion: } v^2 - u^2 = 2as$$

- **Distance covered in n^{th} second:** Distance travelled in n^{th} second = Distance travelled in n sec – Distance travelled in $(n-1)$ sec.

$$\text{So, } S_{n\text{th}} = S_n - S_{(n-1)}$$

$$\left(un + \frac{1}{2} an^2 \right) - \left[u(n-1) + \frac{1}{2} a(n-1)^2 \right]$$

[Putting $t = n$ and $t = (n-1)$ respectively in equation (ii)]

$$= un + \frac{1}{2} an^2 - un + u - \frac{1}{2} a(n^2 - 2n + 1)$$

$$\text{We have, } S_{n\text{th}} = u + \frac{a}{2} (2n-1)$$

Ex.35 A bus starting from rest moves with a uniform acceleration of 0.1 m s^{-2} for 2 minutes. Find

- (i) the speed acquired, (ii) the distance travelled.

Sol. (i) Initial speed of the bus, $u = 0$ (since the bus is initially at rest)

Acceleration, $a = 0.1 \text{ m/s}^2$

Time taken, $t = 2 \text{ minutes} = 120 \text{ s}$

Let v be the final speed acquired by the bus.

$$\therefore a = \frac{v-u}{t}$$

$$0.1 = \frac{v-0}{120}$$

$$\therefore v = 12 \text{ m/s}$$

(ii) According to the third equation of motion:

$$v^2 - u^2 = 2as$$

where, s is the distance covered by the bus $(12)^2 - (0)^2 = 2(0.1) s$

$$s = 720 \text{ m}$$

Speed acquired by the bus is 12 m/s .

Distance travelled by the bus is 720 m .

Ex.36 A train is travelling at a speed of 90 km h^{-1} . Brakes are applied so as to produce a uniform acceleration of -0.5 m s^{-2} . Find how far the train will go before it is brought to rest.

Sol. Initial speed of the train, $u = 90 \text{ km/h} = 25 \text{ m/s}$

Final speed of the train, $v = 0$ (finally the train comes to rest)

Acceleration = -0.5 m s^{-2}

According to third equation of motion: $v^2 = u^2 + 2as$

$$(0)^2 = (25)^2 + 2(-0.5)s$$

where, s is the distance covered by the train

$$s = \frac{(25)^2}{2(0.5)} = 625 \text{ m}$$

The train will cover a distance of 625 m before it comes to rest.

Ex.37 A trolley, while going down an inclined plane, has an acceleration of 2 cm s^{-2} . What will be its velocity 3 s after the start?

Sol. Initial velocity of the trolley, $u = 0$ (since the trolley was initially at rest)

Acceleration, $a = 2 \text{ cm s}^{-2} = 0.02 \text{ m/s}^2$

Time, $t = 3 \text{ s}$

According to the first equation of motion:

$$v = u + at$$

where, v is the velocity of the trolley after 3 s from start

$$v = 0 + 0.02 \times 3 = 0.06 \text{ m/s}$$

Hence, the velocity of the trolley after 3 s from start is 0.06 m/s.

Ex.38 A racing car has a uniform acceleration of 4 m s^{-2} . What distance will it cover in 10 s after start?

Sol. Initial velocity of the racing car, $u = 0$ (since the racing car is initially at rest)

Acceleration, $a = 4 \text{ m/s}^2$

Time taken, $t = 10 \text{ s}$

According to the second equation of motion:

$$s = ut + \frac{1}{2} at^2$$

where, s is the distance covered by the racing car

$$s = 0 + \frac{1}{2} \times 4 \times (10)^2 = \frac{400}{2} = 200 \text{ m}$$

Hence, the distance covered by the racing car after 10 s from start is 200 m.

◆ Equations of Motion for Freely Falling Object

Since the freely falling bodies fall with uniformly accelerated motion, the three equations of motion derived earlier for bodies under uniform acceleration can be applied to the motion of freely falling bodies. For freely falling bodies, the acceleration due to gravity is 'g', so we replace the acceleration 'a' of the equations by 'g' and since the vertical distance of the freely falling bodies is known as height 'h', we replace the distance 's' in our equations by the height 'h'. This gives us the following modified equations for the motion of freely falling bodies.

General equations of motion

Equations of motion for freely falling bodies

(i) $v = u + at$

changes to

$$v = u + gt$$

(ii) $s = ut + \frac{1}{2} at^2$

changes to

$$h = ut + \frac{1}{2} gt^2$$

(iii) $v^2 = u^2 + 2as$

changes to

$$v^2 = u^2 + 2gh$$

We shall use these modified equations to solve numerical problems. Before we do that, we should remember the following important points for the motion of freely falling bodies.

(i) When a body is dropped freely from a height, its initial velocity 'u' becomes zero.

(ii) When a body is thrown vertically upwards, its final velocity 'v' becomes zero.

(iii) The time taken by a body to rise to the highest point is equal to the time it takes to fall from the same height.

(iv) The distance travelled by a freely falling body is directly proportional to the square of time of fall.

- **Sign conventions:**

- g is taken as positive when it is acting in the same direction as that of motion and g is taken as negative when it is opposing the motion.
- Distance measured upward from the point of projection is taken as positive, while distance measured downward from the point of projection is taken as negative.
- Velocity measured away from the surface of earth (i.e. in upward direction) is taken as positive, while velocity measured towards the surface of the earth is taken as negative.

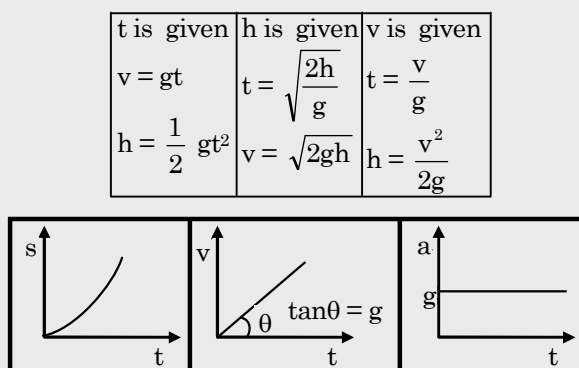
- **To solve numerical problems:**

- If a body is dropped from a height then its initial velocity $u = 0$ but has acceleration (Acting). If a body starts from rest its initial velocity $u = 0$.
- If a body comes to rest its final velocity $v = 0$ or, if a body reaches the highest point after being thrown upwards its final velocity $v = 0$ but has acceleration (acting).
- If a body moves with uniform velocity, its acceleration is zero i.e. $a = 0$.
- Motion of a body is called free fall if only force acting on it is gravity (i.e. earth's attraction).

COMPETITIVE LEVEL

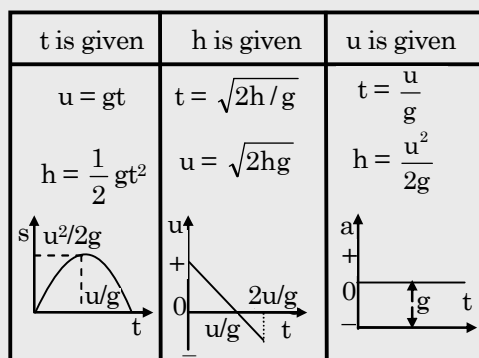
◆ Body Falling Freely Under Gravity

Assuming $u = 0$ for a freely falling body:



- **Body is projected vertically up:** Taking initial position as origin and direction of motion (i.e. vertically up) as positive.

- At the highest point $v = 0$
- $a = -g$



Ex.39 A stone is thrown in a vertically upward direction with a velocity of 5 m s^{-1} . If the acceleration of the stone during its motion is 10 m s^{-2} in the downward direction, what will be the height attained by the stone and how much time will it take to reach there?

Sol. Initially, velocity of the stone, $u = 5 \text{ m/s}$

Final velocity, $v = 0$ (since the stone comes to rest when it reaches its maximum height)

Acceleration of the stone, $a = \text{acceleration due to gravity, } g = 10 \text{ m/s}^2$ (in downward direction) There will be a change in the sign of acceleration because the stone is being thrown upwards.

Acceleration, $a = -10 \text{ m/s}^2$

Let s be the maximum height attained by the stone in time t . According to the first equation of motion:

$$v = u + at$$

$$0 = 5 + (-10) t$$

$$\therefore t = \frac{-5}{-10} = 0.5 \text{ s}$$

According to the third equation of motion: $v^2 = u^2 + 2as$

$$(0)^2 = (5)^2 + 2(-10) s$$

$$s = \frac{5^2}{20} = 1.25 \text{ m}$$

Hence, the stone attains a height of 1.25 m in 0.5 s .

Ex.40 A stone drops from the edge of a roof. It passes a window 2 metre high in 0.1 second. How far is the roof above the top of the window?

Sol. Let the distance between the top of the window and the roof be h . This problem can be solved in two stages.

(A) For the journey across the window i.e., from B to C

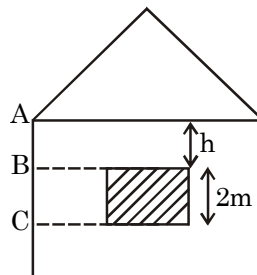
Let, Velocity at B = $u \text{ m/s}$

Distance travelled, $s = 2 \text{ m}$

Time taken, $t = 0.1 \text{ s}$

Acceleration, $a = g = 9.8 \text{ m/s}^2$

Using the relationship,



$$s = ut + \frac{1}{2}gt^2$$

$$2 = u \times 0.1 + \frac{1}{2} \times 9.8 \times (0.1)^2$$

$$2 = 0.1u + 4.9 \times 10^{-2}$$

$$\text{or, } u = \frac{(2 - 0.049)}{0.1} = 19.51 \text{ m/s}$$

The velocity of the stone at the top of the window is 19.51 m/s.

(B) For journey from roof to the top of the window i.e., from A to B

The velocity at the top of the window is the velocity of the stone at the end of falling through 'h'.
So, for this part of the journey,

Initial velocity, $u = 0 \text{ m/s}$

Final velocity, $v = 19.51 \text{ m/s}$

Acceleration due to gravity, $g = 9.8 \text{ m/s}^2$

Then by using the equation, $v^2 - u^2 = 2gh$, one gets

$$(19.51)^2 - 0 = 2 \times 9.8 \times h$$

$$h = \frac{(19.51)^2}{19.6} \text{ m} = 19.42 \text{ m}$$

Thus, the roof is 19.42 m from the upper end (top) of the window.

Ex.41 A car is moving at a speed of 50 km/h after two seconds it is moving at 60 km/h. Calculate the acceleration of the car.

Sol. Here $u = 50 \text{ km/h} = 50 \times \frac{5}{18} \text{ m/s} = \frac{250}{18} \text{ m/s}$

and $v = 60 \text{ km/h} = 60 \times \frac{5}{18} = \frac{300}{18} \text{ m/s}$

Since $a = \frac{v - u}{t} = \frac{\frac{300}{18} - \frac{250}{18}}{2} = \frac{50}{18} = \frac{50}{36} = 1.39 \text{ m/s}^2$

Ex.42 A body starts moving with an initial velocity 50 m/s and acceleration 20 m/s^2 . How much distance it will cover in 4s? Also, calculate its average speed during this time interval.

Sol. Given: $u = 50 \text{ m/s}$, $a = 20 \text{ m/s}^2$,

$t = 4\text{s}$, $s = ?$

$$s = ut + \frac{1}{2}at^2 = 50 \times 4 + \frac{1}{2} \times 20 \times (4)^2$$

$$= 200 + 160 = 360 \text{ m}$$

Average speed during this interval,

$$\bar{V} = \frac{\text{distance travelled}}{\text{time interval}} = \frac{360}{4} = 90 \text{ m/s}$$

Ex.43 A body starts from rest and moves with a constant acceleration. It travels a distance s_1 in first 10 s, and a distance s_2 in next 10 s. Find the relation between s_2 and s_1 .

Sol. Given: $u = 0$, $t_1 = 10$ s

$\therefore$ Distance travelled in first 10 seconds, is given by

$$s_1 = ut + \frac{1}{2} at^2 = 0 + \frac{1}{2} \times a \times (10)^2$$

$$= 50a \quad \text{.....(1)}$$

To calculate the distance travelled in next 10s, we first calculate distance travelled in 20 s and then subtract distance travelled in first 10 s.

$$s = ut + \frac{1}{2} at^2 = 0 + \frac{1}{2} \times a \times (20)^2$$

$$= 200a \quad \text{..... (2)}$$

$\therefore$ Distance travelled in next 10 seconds interval,

$$s_2 = s - s_1 = 200a - 50a \quad \text{..... (3)}$$

$$\text{or } s_2 = 150a$$

$$\text{Now, } \frac{s_2}{s_1} = \frac{150a}{50a} = \frac{3}{1}$$

$$\text{or } s_2 = 3s_1$$

Ex.44 From the top of a tower of height 490 m, a shell is fired horizontally with a velocity 100 m/s. At what distance from the bottom of the tower, the shell will hit the ground?

Sol. We know that the horizontal motion and the vertical motion are independent of each other. Now for vertical motion, we have $u = 0$, $h = 490$ m, $g = 9.8 \text{ m/s}^2$, $t = ?$

Using equation, $h = ut + \frac{1}{2} gt^2$, we get

$$490 = 0 + \frac{1}{2} \times 9.8 \times t^2 \quad \text{or} \quad t^2 = \frac{490}{4.9} = 100$$

$$\text{or } t = 10 \text{ s}$$

$\therefore$ It takes 10 seconds to reach the ground.

Now, horizontal distance

$$= \text{horizontal velocity} \times \text{time}$$

$$= 100 \text{ m/s} \times 10 \text{ s} = 1000 \text{ m}$$

$\therefore$ The shell will strike the ground at a distance of 1000 m from the bottom of the tower.

Circular Motion

◆ Definition

The motion of a body moving around a fixed point in a circular path is known as circular motion.

◆ Uniform Circular motion

If the body covers equal distances along the circumference of the circle in equal intervals of time, the motion is said to be a uniform circular motion. A uniform circular motion is a motion in which speed remains constant but direction of velocity changes.

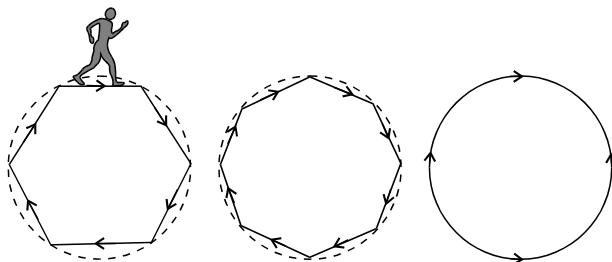
e.g.: Examples of uniform circular motion are:

(i) Motion of moon around the earth.

(ii) Motion of satellite around its planet.

◆ Circular motion is known as accelerated motion

Explanation: Consider a boy running along a regular hexagonal track (path) as shown in figure. As the boy runs along the side of the hexagon at a uniform speed, he has to take a turn at each corner changing direction but keeping the speed same. In one round he has to take six turns at regular intervals. If the same boy runs along the side of a regular octagonal track with same uniform speed, he will have to take eight turns in one round at regular intervals but the interval will become smaller.



By increasing the number of sides of the regular polygon, we find that number of turns per round becomes more and the interval between two turns become still shorter. A circle is a limiting case of a polygon with an infinite number of sides. On the circular track, the turning becomes a continuous process without any gap in between. The boy running along the sides of such a track will be performing a circular motion. Hence, circular motion is the motion of a body along the sides of a polygon of infinite number of sides with uniform speed, the direction changing continuously, it means the body moves with changing velocity in a circular path thus the uniform circular motion is known as accelerated motion.

◆ Difference between a Uniform Linear and Circular Motion

Uniform linear motion	Uniform circular motion
1. The direction of motion does not changes.	1. The direction of motion changes continuously.
2. The motion is non accelerated.	2. The motion is accelerated.

Note: Example of a body performing accelerated motion with uniform speed is circular motion.

COMPETITIVE LEVEL

◆ Radian

It is the unit of plane angle.

- **Definition:** One radian is defined as the angle subtended at the centre of the circle by an arc equal in length to its radius.

e.g.: In figure, the arc AB of the circle has length ℓ and subtends an angle θ at the centre C.

If $\angle ACB = \theta$ radians.

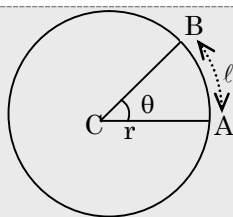
Then, $\theta = \frac{\ell}{r}$ radians.

[For $\ell = r$, $\theta = 1$ radian]

Angle subtended by the circumference at the centre,

$$\theta = \frac{2\pi r}{r} = 2\pi \text{ radians \{or } 2\pi^\circ \}$$

[$^\circ$] is symbol for radian, just as ($^\circ$) is symbol for degree.



Relation between radian and degree: For complete circle at centre

$$2\pi^c = 360^\circ \quad \text{or} \quad 1^c = \left| \frac{360}{2\pi} \right| = 57.3^\circ$$

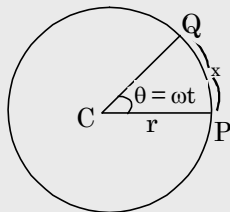
◆ Angular Displacement and Angular Velocity

- **Angular displacement:** In a circular motion, the angular displacement of a body is the angle subtended by the body at the centre in a given interval of time. It is represented by the symbol θ (theta). The unit of angular displacement is radian.
- **Angular velocity:** The angular displacement per unit time is called the angular velocity. It is represented by the symbol ω (omega).
- **Expression for angular displacement and angular velocity:**

- (i) Let a body move along a circle of radius r and perform a uniform circular motion. Let the body be at point P to start with and reach point Q after time t . Then, angular displacement = $\angle PCQ = \theta$

$$\text{angular velocity} = \omega = \frac{\theta}{t} \quad (\text{i.e. } \theta = \omega t)$$

- (ii) In terms of time period and frequency: If the time period of the body is T (time taken in one complete round), the angular displacement = $2\pi^c$



$$\text{Hence } \omega = \frac{2\pi}{T}$$

$$\text{But } \frac{1}{T} = n \text{ (frequency)}$$

$$\text{There } \omega = 2\pi n$$

The unit of angular velocity is rad/s

◆ Angular acceleration

The rate of the change of angular velocity is called angular acceleration. Let angular velocity at time t_1 is ω_1 and at time t_2 is ω_2 change in angular velocity in the time interval $t_2 - t_1$ is $\omega_2 - \omega_1$

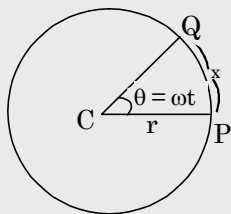
$$\text{thus, rate of change of angular velocity} = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

$$\alpha = \frac{\Delta\omega}{\Delta t} \text{ (here } \alpha \text{ is average angular acceleration)}$$

- The unit of angular acceleration is rad/s^2 .

◆ Relation between Linear and Angular Quantities

(i) Relation between linear displacement and angular displacement.



$$\text{angle} = \frac{\text{arc}}{\text{radius}}$$

$$\theta = \frac{x}{r} \Rightarrow x = r\theta \quad \text{..... (i)}$$

(ii) Relation between linear velocity and angular velocity.

From (i) $x = r\theta$

$$\frac{x}{t} = r \frac{\theta}{t} \Rightarrow v = r\omega \quad \text{..... (ii)}$$

(iii) Relation between linear acceleration and angular acceleration.

From (ii) $v = r\omega$

$$\frac{v}{t} = r \frac{\omega}{t} \Rightarrow a = r\alpha \quad \text{..... (iii)}$$

◆ Centripetal Force

- Always acts towards centre.
- Centripetal force is required to move a particle in a circle.
- Because F_c is always perpendicular to velocity or displacement, hence the work done by this force will always be zero.

Note:

- Circular motion in horizontal plane is usually uniform circular motion.
- Remember that equations of motion are not applicable for circular motion.

◆ Centripetal Acceleration

- In uniform circular motion the particle experiences an acceleration called the centripetal acceleration.
- $a_c = \frac{v^2}{r}$
- The direction of centripetal acceleration is along the radius towards the centre.

Ex.45 A fly wheel making 120 revolutions/minute. Find the angular speed of the wheel:

Sol. $\therefore 120 \text{ revolution/ minute} = 2 \text{ rev/s}$

Angular speed = angle in one revolution $\times$ number of revolution/s

$$= 2\pi \times 2 = 4\pi \text{ rad/s}$$

Ex.46 A stone tied to the end of a string 80 cm is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 sec., what is the magnitude of the angular speed. $\left(\text{use } \pi = \frac{22}{7}\right)$

Sol. angular speed = angle $\times$ number of rev./s

$$\omega = 2\pi \times \frac{14}{25} = 3.52 \text{ rad/s}$$

Ex.47 Earth revolves around the Sun in 365 days. Calculate its angular speed.

Sol. $T = 365 \text{ days} = 365 \times 24 \times 60 \times 60 \text{ s}$

$$\text{So, } \omega = \frac{2\pi}{T} = \frac{2\pi}{365 \times 24 \times 60 \times 60} = 1.99 \times 10^{-7} \text{ rad/s}$$

Ex.48 A particle moves in a circle of radius 2 m and completes 5 revolutions in 10 seconds. Calculate the following:

- (i) Angular velocity (ii) Linear velocity

Sol. Since, it completes 5 revolutions in 10 seconds.

$$\therefore \text{Time period} = \frac{10}{5} = 2 \text{ s}$$

$$(i) \text{ Now angular velocity, } \omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \text{ rad/s}$$

(ii) Linear velocity is given by

$$v = r\omega = 2\pi$$

$$\therefore v = 2\pi \text{ m/s}$$

Ex.49 The length of second's needle in a watch is 1.2 cm. Calculate the following:

- (i) Angular velocity and (ii) Linear velocity of the tip of the needle.

Sol. (i) We know that the second's needle in a watch completes one revolution in 60 seconds.

$$\therefore \text{Time period, } T = 60 \text{ s}$$

$$\text{Angular velocity, } \omega = \frac{2\pi}{T} = \frac{2\pi}{60} = \frac{\pi}{30} \text{ rad/s}$$

(ii) Length of the needle = 1.2 cm = Radius of the circle

Linear velocity of the tip of the needle is given by

$$v = r\omega = 1.2 \times \frac{\pi}{30} = \frac{\pi}{25} \quad \text{or} \quad v = \frac{\pi}{25} = 1.266 \times 10^{-1} \text{ cm/sec.}$$

Ex.50 Earth revolves around the sun in 365 days. Calculate its angular velocity.

Sol. Time period, $T = 365 \text{ days} = 365 \times 24 \times 60 \times 60 \text{ seconds}$

$$\text{Angular velocity, } \omega = \frac{2\pi}{T} = \frac{2\pi}{365 \times 24 \times 60 \times 60} \text{ rad/s} = 1.99 \times 10^{-7} \text{ rad/s.}$$

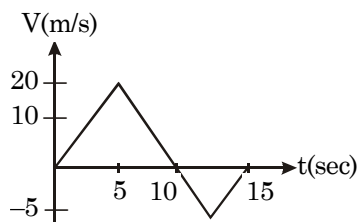
EXERCISE-1

➤ Very Short Answer Type Questions

- Q.1** A ball is thrown up with a certain velocity. It attains a height of 40 m and comes back to the thrower. Find the distance and magnitude of displacement.
- Q.2** What is the S.I. unit of displacement?
- Q.3** A horse runs a distance of 1200 m in 3 min and 20 s. What is the speed of the horse?
- Q.4** What is the S.I. unit of velocity?
- Q.5** What is the S.I. unit of acceleration?

➤ Short Answer Type Questions – Type I

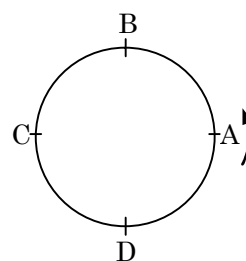
- Q.6** Distance and displacement are equal in some cases. Give reasons.
- Q.7** A stone is thrown upwards, reaches a height h and comes back. What are the distance moved and displacement?
- Q.8** Under what condition(s) is the magnitude of average velocity of an object equal to its average speed?
- Q.9** Define uniform circular motion.
- Q.10** From the following (V-t) graph find:



- (i) Distance and displacement in 10 second.
 (ii) Distance and displacement in 15 second.

➤ Short Answer Type Questions – Type II

- Q.11** Give three examples to explain that motion is relative.
- Q.12** Which of the following is true for displacement?
 (i) It cannot be zero.
 (ii) Its magnitude is greater than the distance travelled by the object.
- Q.13** A particle moves along a circle of radius R as shown in figure. It starts from A and moves in anti-clockwise direction.



Calculate the distance travelled and displacement:

- (i) From A to B
 (ii) From A to C
 (iii) From A to D
- Q.14** A train covers 80 km in 2 hours. Find its average speed in kmh^{-1} , m min^{-1} and ms^{-1} .
- Q.15** An artificial satellite is moving in a circular orbit of radius 42250 km. Calculate its speed, if it takes 24 hours to revolve around the earth.
- Q.16** Which one of the following have maximum and the least average speed?
 (i) Sanjeev moving with 12 kmh^{-1}
 (ii) Rajeev running with 5 ms^{-1}
 (iii) Kabir moving with 150 m min^{-1}
- Q.17** What do you mean by negative and positive acceleration? Explain.



Long Answer Type Questions

Q.18 Why circular motion is accelerated motion?

Q.19 Draw the graph for uniform motion.

(i) Displacement - Time

(ii) Velocity - Time

Q.20 An engine is moving with a velocity 44 m/s. After applying the brakes, it stops after covering a distance of 121 m. Calculate retardation and time taken by the engine to stop.

Q.21 Define rest and motion and give two examples of each.

Q.22 Write five difference between distance and displacement.

Q.23 Write the short notes on:

(i) Uniform motion

(ii) Non uniform motion

(iii) Average speed

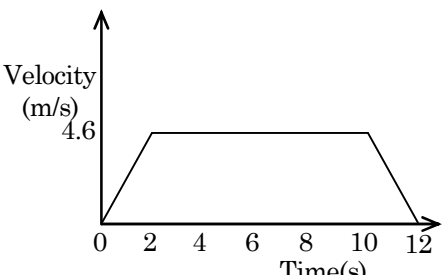
(iv) Velocity

Q.24 The distance between two points A and B is 100 m. A person moves from A to B with a speed of 20 m/s and from B to A with a speed of 25 m/s. Calculate average speed and average velocity.

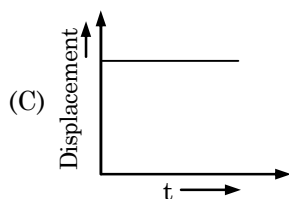
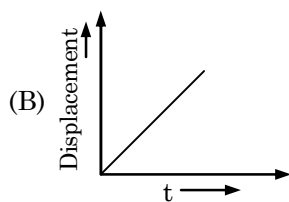
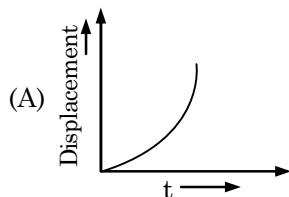
Q.25 A body starting with initial velocity u moves with a constant acceleration a . Find the expression for distance travelled in n th seconds.

EXERCISE-2

- Q.1** ABC is the shortest path length between the two points and ADC is the actual path length. Then which of the two corresponds to displacement?
 (A) ADC (B) ABC
 (C) Can't say (D) None of these
- Q.2** Rest and motion both are
 (A) Relative terms
 (B) Absolute terms
 (C) Can't say
 (D) None of these
- Q.3** An object has travelled 10 km in 15 minutes, its displacement will be
 (A) 10 km
 (B) zero
 (C) More than 10 km
 (D) Cannot be predicted
- Q.4** Which of the following does not need direction to be defined completely
 (A) Speed (B) Velocity
 (C) Force (D) Displacement
- Q.5** Speedometer is a device, which is used to measure
 (A) Distance (B) Displacement
 (C) Speed (D) None of these
- Q.6** A boy travels 50km with 5km/hr and then for next 4hr travels with a uniform speed of 20km/hr. What is the average speed for the whole journey?
 (A) 62/7km/hr (B) 65/7km/hr
 (C) 60/7km/hr (D) 9km/hr
- Q.7** Magnitude of average speed of an object is equal to its average velocity if
 (A) it is moving in a definite direction.
 (B) its initial and final positions are same.
 (C) and only if it is in a uniform motion.
 (D) they travel equal distances.
- Q.8** From the top of a tower, a particle is projected upwards and it reaches the ground after 5 s. The initial velocity of the particle is 12 m/s, the height of the tower is
 (A) 55 m (B) 65 m
 (C) 75 m (D) 85 m
- Q.9** A train passes over a 400 m long bridge. If the speed of the train is 30 m/s and the train takes 20 s to cross the bridge, then the length of the train is
 (A) 400 m (B) 600 m
 (C) 800 m (D) 200 m
- Q.10** If a body covers a distance d with velocity v_1 and another distance d with same velocity v_2 , then average velocity for the whole journey would be equal to
 (A) $\frac{2v_1v_2}{v_1 + v_2}$ (B) $\frac{v_1v_2}{v_1 + v_2}$
 (C) $\frac{v_1v_2}{2v_1 + v_2}$ (D) $\frac{2(v_1v_2)}{v_1v_2}$
- Q.11** If a body covers some distance with speed v_1 for time t_1 and some another distance with speed v_2 for some time t_2 . Then what would be the average velocity for the whole duration?
 (A) $\frac{v_1v_2}{2}$ (B) $\frac{v_1t_1 + v_2t_2}{t_1 + t_2}$
 (C) $\frac{v_1v_2}{v_1 + v_2}$ (D) $\frac{2v_1v_2}{v_1 + v_2}$
- Q.12** A body strikes the floor vertically with a speed 'u' and rebounds at the same speed. The change in velocity would be
 (A) u (B) 3u (C) 2u (D) zero
- Q.13** One car moving on a straight road covers one third of the distance with 20 km/hr and the rest with 60 km/hr. The average speed is
 (A) 40 km/hr (B) 80 km/hr
 (C) $46\frac{2}{3}$ km/hr (D) 36 km/hr

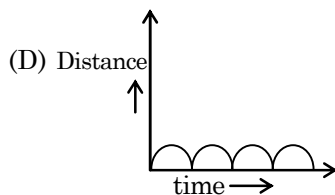
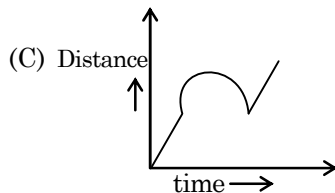
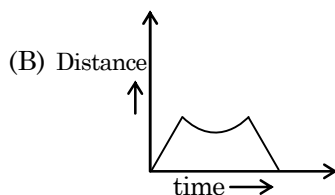
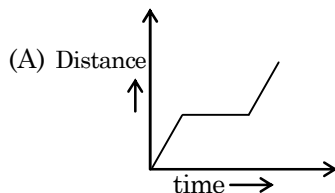
- Q.14** A 150 m long train is moving with a uniform velocity of 45 km/h. The time taken by the train to cross a bridge of length 850 meters is
 (A) 56 sec (B) 68 sec
 (C) 80 sec (D) 92 sec
- Q.15** The ratio of the numerical values of the average velocity and average speed of a body is always
 (A) Unity (B) Unity or less
 (C) Unity or more (D) Less than unity
- Q.16** The formula for average velocity $\frac{u+v}{2}$ is valid for the case when acceleration is
 (A) Variable (B) Zero
 (C) Uniform (D) None of these
- Q.17** If a body travels 20m in 10s starting from rest then the acceleration of the particle is:
 (A) 0.4m/s^2 (B) 0.6m/s^2
 (C) 1 m/s^2 (D) 0.8m/s^2
- Q.18** For the equation $s = ut - \frac{1}{2}at^2$ acceleration is in the:
 (A) Opposite direction of initial velocity
 (B) Opposite direction of displacement
 (C) Same direction of initial velocity
 (D) Both (A) and (B)
- Q.19** An aeroplane lands at 432 km/h and stops after covering a runway of 4 km. The time in which it comes to rest is
 (A) 1.8s (B) 60s
 (C) 66.6s (D) 150s
- Q.20.** The correct statement from the following is
 (A) A body having zero velocity will not necessarily have zero acceleration
 (B) A body having zero velocity will necessarily have zero acceleration
 (C) A body having uniform speed can have only uniform acceleration
 (D) A body having non-uniform velocity will have zero acceleration
- Q.21** The average velocity of a body moving with uniform acceleration travelling a distance of 3.06 m is 0.34 ms^{-1} . If the change in velocity of the body is 0.18ms^{-1} during this time, its uniform acceleration is
 (A) 0.01 ms^{-2} (B) 0.02 ms^{-2}
 (C) 0.03 ms^{-2} (D) 0.04 ms^{-2}
- Q.22** A body under the action of several forces will have zero acceleration
 (A) when the body is very light
 (B) when the body is very heavy
 (C) when the body is a point body
 (D) when the vector sum of all the forces acting on it is zero
- Q.23** The x and y coordinates of a particle at any time t are given by $x = 7t + 4t^2$ and $y = 5t$, where x and y are in metre and t in seconds. The acceleration of particle at $t = 5\text{ s}$ is
 (A) Zero (B) 8 m/s^2
 (C) 20 m/s^2 (D) 40 m/s^2
- Q.24** In the following velocity-time graph of a moving object
- 
- (A) Acceleration in the first 2 seconds is 2.3 ms^{-2}
 (B) Acceleration in the last 2 seconds is 2.3ms^{-2}
 (C) Motion is non-uniform between second and tenth second
 (D) The body comes to rest at $t = 2$ and $t = 10\text{ sec}$
- Q.25** The distance – time graph for an object moving with uniform speed is
 (A) a straight line (B) is curvilinear
 (C) rectilinear (D) parabolic

Q.26 Which of the following is correct for uniformly accelerated motion?



(D) All are correct

Q.27 Which of the following graph is possible?



Q.28 A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further it will penetrate before coming to rest assuming that it faces constant resistance to motion?

- (A) 1.5 cm (B) 1.0 cm
(C) 3.0 cm (D) 2.0 cm

Q.29 A particle experiences a constant acceleration for 20 sec after starting from rest. If it travels a distance S_1 in the first 10 sec and a distance S_2 in the next 10 sec, then

- (A) $S_1 = S_2$ (B) $S_1 = S_2 / 3$
(C) $S_1 = S_2 / 2$ (D) $S_1 = S_2 / 4$

Q.30 A particle starting from rest travels a distance x in first 2 seconds and a distance y in next two seconds with constant acceleration, then

- (A) $y = x$ (B) $y = 2x$
(C) $y = 3x$ (D) $y = 4x$

EXERCISE-3

(Previous Year Questions - NTSE & NSO)

Q.1 Value of one Fermi is

- (A) 10^{-13} metre
- (B) 10^{-14} metre
- (C) 10^{-15} metre
- (D) 10^{-16} metre

Q.2 A student starts with a velocity 40 km/hr for school at 4 km away from his house. Due to closing of school he returns soon to his house with a velocity of 60 km/hr. His average velocity will be

- (A) zero
- (B) 10 km/hr
- (C) 48 km/hr
- (D) 50 km/hr

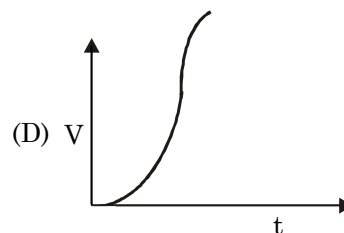
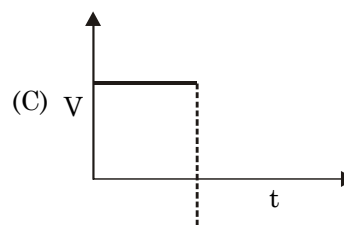
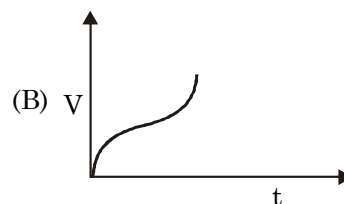
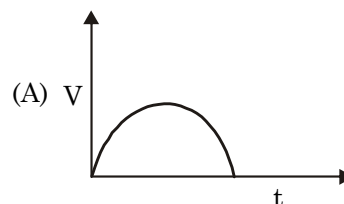
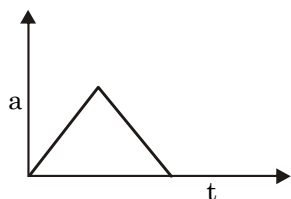
Q.3 A person takes time t to go once around a circular path of diameter $2R$. The speed (v) of this person would be

- (A) $\frac{t}{2\pi R}$
- (B) $\frac{2\pi R}{t}$
- (C) $\frac{\pi R^2}{t}$
- (D) $2\pi R \cdot t$

Q.4 A car travels 40 kms at an average speed of 80 km/h and then travels 40 kms at an average speed of 40 km/h. The average speed of the car for this 80 km trip is

- (A) 40 km/h
- (B) 45 km/h
- (C) 48 km/h
- (D) 53 km/h

Q.5 The acceleration versus time graph of an object is as shown in figure. The corresponding velocity-time graph of the object is



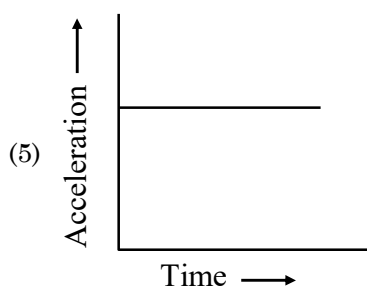
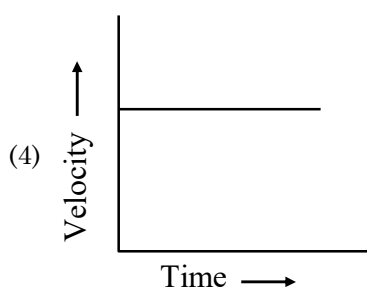
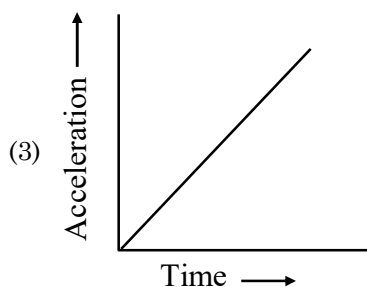
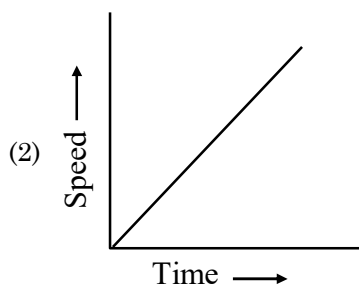
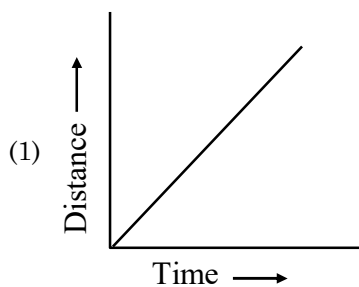
Q.6 A bullet of mass 10 g travelling horizontally with a velocity of 160 ms^{-1} strikes a stationary wooden block and comes to rest in 0.02 s. The distance of penetration of the bullet into the block will be

- (A) 1.20 m
- (B) 1.60 m
- (C) 2.00 m
- (D) 2.40 m

Q.7 The brakes applied to a car produce an acceleration of 8 m/s^2 in the opposite direction to the motion. If the car takes 3 seconds to stop after the application of brakes, the distance it travels during the time will be

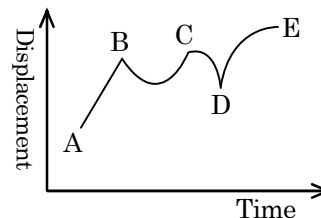
- (A) 30 m
- (B) 36 m
- (C) 25 m
- (D) 40 m

- Q.8** Consider the following five graphs (note the axes carefully). Which of the following represents motion at constant speed?



- (A) 4 only
(B) 4 and 5
(C) 1, 2 and 3
(D) 1 and 4

- Q.9** The figure given below shows the displacement plotted time for a particle. In which regions is the force acting on the particle zero?



- (A) AB
(B) BC
(C) CD
(D) DE

- Q.10** Correct relation is.....

- (A) $v^2 = u^2 + 2a^2s^2$
(B) $v^2 = u^2 - 2a^2s^2$
(C) $v^2 = u^2 + 2as$
(D) $v^2 = u^2 + 2a^2s$

- Q.11** Two cars of unequal masses use similar tyres. If they are moving with same initial speed, the minimum stopping distance (air friction = 0)
- (A) is smaller for the heavier car.
(B) is same for both the cars
(C) is smaller for the lighter car.
(D) depends on the volume of the car

- Q.12** A ball hits a wall horizontally with a velocity of 6.0 ms^{-1} . After hitting wall it rebounds horizontally with a velocity of 4.4 ms^{-1} . If the balls remains in the contact of all for 0.040 sec . the acceleration of ball would be
- (A) -260 m/s^2
(B) $+260 \text{ m/s}^2$
(C) -26 m/s^2
(D) $+26 \text{ m/s}^2$

- Q.13** A man running with a uniform speed 'u' on a straight road observes a stationary bus at a distance 'd' ahead of him. At that instant, the bus starts with an acceleration 'a'. The condition that he would be able to catch the bus is

- (A) $d \leq \frac{u^2}{a}$
(B) $d \leq \frac{u^2}{2a}$
(C) $d \leq \frac{u^2}{3a}$
(D) $d \leq \frac{u^2}{4a}$

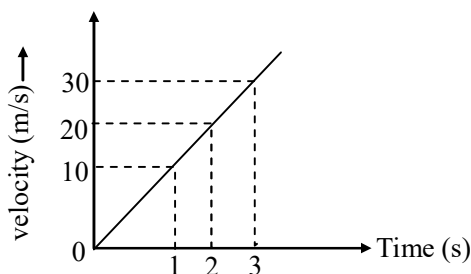
Q.14 A car is moving with a constant speed of 70 km/h. Which of the following statements is correct?

- (A) The acceleration of the car is definitely zero.
- (B) The car has an acceleration only if it is moving along a curved path
- (C) The car may have an acceleration even if it is moving along a straight path
- (D) The car may not have an acceleration even if it is moving along a curved path

Q.15 A body cover half of the distance with a speed of 20m/s and the other half with 30m/s. The average speed of the body during the whole journey is

- (A) Zero
- (B) 24m/s
- (C) 25 m/s
- (D) None of the above

Q.16. Velocity-time graph of a body moving with uniform acceleration is shown in the diagram. The distance travelled by the body in 3 second is

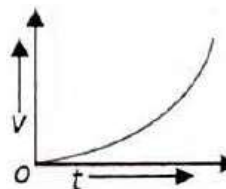


- (A) 90 m
- (B) 45 m
- (C) zero
- (D) 10 m

Q.17 A body starts from rest is accelerated uniformly for 30s. If S_1 , S_2 , S_3 are the distances travelled in first 10s; next 10s and last 10s respectively, then $S_1 : S_2 : S_3$ is

- (A) 1 : 2 : 3
- (B) 1 : 1 : 1
- (C) 1 : 3 : 5
- (D) 1 : 3 : 9

Q.18 The velocity – time graph of a moving body is shown in the figure. Which of the following statements is true?



- (A) The acceleration is constant and positive.
- (B) The acceleration is constant and negative.
- (C) The acceleration is increased and positive.
- (D) The acceleration is decreasing and negative.

Q.19 If the distance travelled by an object is zero, then the displacement of the object is :

- (A) zero
- (B) not zero
- (C) negative
- (D) May or may not be zero

Q.20 How much time the satellite will take to complete one revolution around the earth, if velocity of satellite is 3.14 km/s and its height above earth's surface is 3600 km. (Radius of earth is 6400 km)

- (A) 2000 S
- (B) 20000 S
- (C) 1000 S
- (D) 10000 S

ANSWER KEY

EXERCISE - 1

1. 80 m and zero
3. 6m/s
7. 2h, 0
10. (i) 100 m, 100 m (ii) 112.5 m, 87.5 m
13. (i) $\frac{\pi R}{2}, \sqrt{2}R$ (ii) $\pi R, 2R$ (iii) $\frac{3\pi R}{2}, \sqrt{2}R$
14. 40km/hr, 666.67m/min, 11.11m/sec
15. 3.069 km/s
20. 8 m/sec², 5.5s
24. 22.2 m/s

EXERCISE - 2

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	A	D	A	C	B	A	B	D	A	B	C	D	C	B
Ques.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	C	A	D	C	A	B	D	B	A	A	A	A	B	B	C

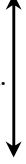
EXERCISE - 3

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	A	B	D	D	B	B	D	A	C	D	A	B	B	B
Ques.	16	17	18	19	20										
Ans.	B	C	A	A	B										

SOLUTIONS

EXERCISE-1

➤ Very Short Answer Type Questions

Sol.1 
 Displacement = 0
 Distance = 40m + 40m = 80m

Sol.2 Metre

Sol.3 $\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{1200}{200} = 6 \text{ m/sec.}$

Sol.4 m/sec

Sol.5 m/sec²

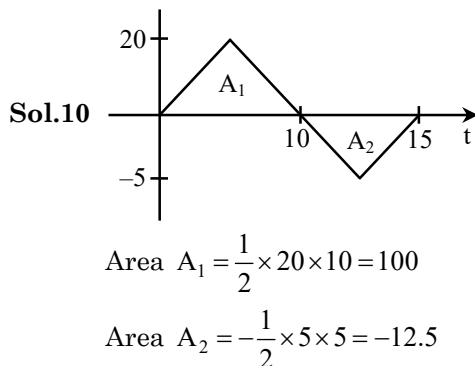
➤ Short Answer Type Questions – Type I

Sol.6 When body moving in straight line in certain direction.

Sol.7 Displacement = 0
 Distance = 2h
 Travelled

Sol.8 When body moving in straight line in certain direction.

Sol.9 If a body covers equal distance in equal interval along the circumference of the circle in equal interval of time.



(i) Distance = Displacement = Area A_1
 = 100 m.

(ii) Displacement = Area A_1 + Area A_2
 = 100 – 12.5 = 87.5 m

Distance = (Area A_1) + (Area A_2)
 = 100 + 12.5 = 112.5 m.

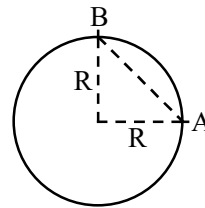
➤ Short Answer Type Questions – Type II

Sol.11 A train is moving on the track, the passenger are seated, will be stationary w.r.t. to each other but in moving condition w.r.t. to station.

Sol.12 Both are incorrect.

Sol.13 (i) $A \rightarrow B$

$$\text{Distance} = \frac{1}{4}(2\pi R) = \frac{\pi}{2}R$$



$$\text{Displacement} = \sqrt{2} R$$

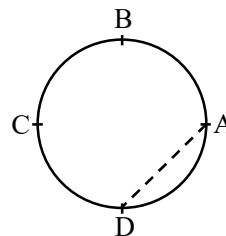
(ii) $A \rightarrow C$

$$\text{Distance} = \frac{1}{2}(2\pi R) = \pi R$$

$$\text{Displacement} = 2R$$

(iii) $A \rightarrow D$

$$\text{Distance} = \frac{3}{4}(2\pi R) = \frac{3\pi R}{2}$$



$$\therefore \text{Displacement} = \sqrt{2} R$$

Sol.14 Avg. Speed = $\frac{\text{total distance}}{\text{total time}}$

$$= \frac{80 \text{ km}}{2 \text{ hr}} = 40 \text{ Km/hr.}$$

or $= \frac{80 \times 10^3 \text{ m}}{2 \times 60 \text{ min}} = 666.67 \text{ m/min.}$

or $= \frac{80 \times 10^3 \text{ m}}{2 \times 60 \times 60 \text{ sec}} = 11.12 \text{ m/min.}$

Sol.15 Speed = $\frac{\text{Distance}}{\text{Time}} = \frac{2\pi R}{t} = \frac{2\pi R(42250)}{24}$
 $= 11,055.42 \text{ Km/hr}$

Sol.16 Convert all the units in SI (m/sec)

(i) $12 \text{ km/hr} = 3.33 \text{ m/sec}$

(ii) 5 m/sec

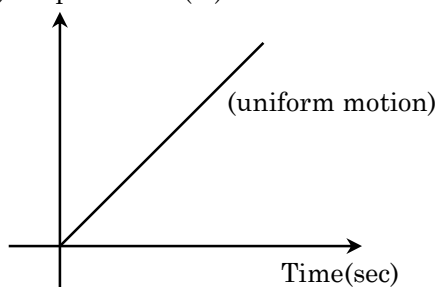
(iii) $150 \text{ m/min} = 2.5 \text{ m/sec}$

So Rajeev has maximum speed & kabir has minimum speed.

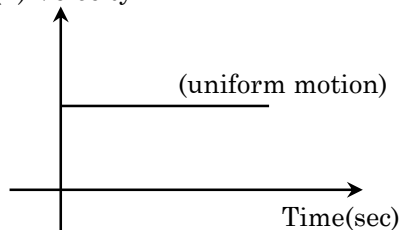
Sol.17 If direction of acceleration is in the direction of motion of body then it be positive acceleration and if the acceleration is in opposite direction of motion of body then it be negative acceleration.

Sol.18 In circular motion velocity is changing at every instant that's why circular motion is called as accelerated motion.

Sol.19 (i) Displacement(m)



(ii) Velocity



Sol.20 $u = 44 \text{ m/sec}$

$s = 121 \text{ M}$

$v = 0$

By using. Equation of motion.

$v^2 = u^2 + 2as$

$0 = (44)^2 + 2(a)(121)$

$a = -8 \text{ m/sec}^2$

$v = u + at$

$0 = 44 + (-8)(t)$

$t = 5.5 \text{ sec}$

➤ Long Answer Type Questions

Sol.21 An object is said to be at rest if it does not change its position w.r.t. its surrounding with the passage of time.

A body is said to be in motion if its position change continuously with the surroundings with the passage of time.

Although rest it motion are relative terms.

Sol.22

Distance	Displacement
1. Distance is the length of the path actually traveled by a body in an direction.	1. Displacement is the short distance between the initial and the final position.
2. Distance b/w two given points depends upon the path chosen	2. Displacement b/w two points is measured by the straight path b/w the points.
3. Distance is always positive.	3. Displacement may be positive negative or zero.
4. Distance is a scalar quantity.	4. Displacement is vector quantity.
5. Distance will never decreases	5. Displacement may decreases.

Sol.23 (i) Uniform motion :

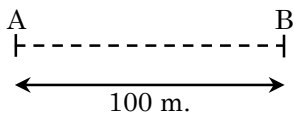
If an object moves equal distance in equal interval of time in a certain direction.

(ii) Non uniform motion :

If an object moves unequal distance in equal interval of time.

(iii) Average speed :

Total distance traveled by body divided by total time taken.

Sol.24 

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$= \frac{100 + 100}{\frac{100}{20} + \frac{100}{25}} = \frac{200}{9} \text{ m/sec}$$

$$\text{Average velocity} = \frac{\text{displacement}}{\text{total time}} = 0$$

Sol.25 $S_{n^{\text{th}}} = S_n - S_{n-1}$

$$= \left(ut + \frac{1}{2} at^2 \right) - \left(u(n-1) + \frac{1}{2} a(n-1)^2 \right)$$

$$= un + \frac{1}{2} an^2 - un + u - \frac{1}{2} an^2 - \frac{a}{2} + an$$

$$S_{n^{\text{th}}} = u - \frac{a}{2} + \frac{an}{2}$$

EXERCISE-2

Sol.1 Shortest path between two points is shown as displacement.

Sol.2 Rest and motion both are relative terms.

Sol.3 Displacement $\leq$ distance traveled.
So cannot be predicted by this information only.

Sol.4 Speed is a scalar quantity.

Sol.5 odometer is used to measure speed.

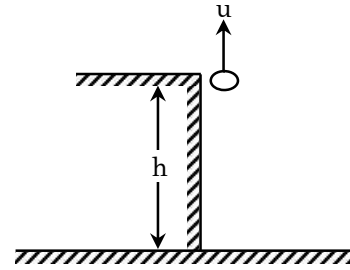
Sol.6 $\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$$= \frac{S_1 + S_2}{t_1 + t_2}$$

$$= \frac{50 + (20 \times 4)}{\left(\frac{50}{5}\right) + 4} = \frac{65}{7} \text{ km/hr.}$$

Sol.7 $|\text{avg speed}| = |\text{avg velocity}|$
 $\Rightarrow$ distance = displacement
 It is only possible when body moving in definite direction.

Sol.8 $U = + 12 \text{ m/sec}$
 $t = 5 \text{ sec}$
 $s = -h$
 $a = -g = - 10 \text{ m/sec}^2$



$$S = ut + \frac{1}{2} at^2$$

$$-h = 12 \times 5 + \frac{1}{2} (-10) (25)$$

$$h = 65 \text{ m}$$

Sol.9 Let the length of the train is x.
 Distance = Speed $\times$ Time
 $(400 + x) = 30 \times 20$
 $x = 200 \text{ m}$

Sol.10 $\text{Average velocity} = \frac{\text{total displacement}}{\text{time taken}}$

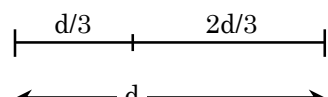
$$= \frac{2d}{t_1 + t_2}$$

$$= \frac{2d}{\frac{d}{v_1} + \frac{d}{v_2}} = \frac{2v_1 v_2}{v_1 + v_2}$$

Sol.11 $\text{Average velocity} = \frac{\text{total displacement}}{\text{total time taken}}$

$$= \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2}$$

Sol.12 Change in velocity
 = final velocity – initial velocity
 $= u \hat{j} - (-u \hat{j}) = 2u \hat{j}$
 (as velocity is a vector quantity)

Sol.13 

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$\begin{aligned}
&= \frac{d}{t_1 + t_2} \\
&= \frac{d}{\frac{d/3}{20} + \frac{2d/3}{60}} \\
&= \frac{d}{\frac{d}{60} + \frac{d}{90}} = 36 \text{ km/hr}
\end{aligned}$$

Sol.14 Total distance travelled by train
 $= 850 \text{ m} + 190 \text{ m}$
Speed $= 45 \text{ km/hr}$
time $= \frac{\text{distance}}{\text{speed}} = \frac{1}{43} \text{ hr} = 80 \text{ sec.}$

Sol.15 $|\text{avg velocity}| \leq |\text{avg speed}|$

Sol.16 If uniform acceleration.
 $v^2 = u^2 + 2as$
 $v^2 - u^2 = 2\left(\frac{v-u}{t}\right)s$
 $s = \frac{(v+u)}{2}t$
Average velocity $= \frac{\text{total displacement}}{\text{total time taken}}$
 $= \frac{\frac{(v+u)}{2}t}{t} = \frac{v+u}{2}$

Sol.17 $S = ut + \frac{1}{2}at^2$
 $20 = 0 + \frac{1}{2} \times a \times 100 \Rightarrow a = 0.4 \text{ m/sec}^2$

Sol.18 acceleration is negative
initial velocity is positive.
displacement is positive.
So acceleration is in opposite direction &
both displacement in velocity.

Sol.19 $u = 432 \text{ Km/hr} = 120 \text{ m/sec}$
 $s = 4 \text{ km} = 400 \text{ m}$
 $v^2 = u^2 + 2as$
 $0 = (120)^2 + 2(a)(400)$
 $a = -1.8 \text{ m/sec}^2$
 $v = u + at$
 $0 = 120 - 1.8 \pi t$
 $t = \frac{120}{1.8} = 66.66 \text{ sec}$

Sol.20 A body having zero velocity will not necessarily have zero acceleration

Sol.21 avg velocity $= \frac{\text{total displacement}}{\text{total time taken}}$

$$0.34 = \frac{3.06}{t}$$

$$t = \frac{3.06}{0.34} = 9$$

$$\text{acceleration} = \frac{v-4}{t} = \frac{0.18}{9} = 0.02 \text{ m/sec}^2$$

Sol.22 $\vec{F}_{\text{net}} = m\vec{a} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \dots$

If vector sum of all forces is zero.
acceleration of body will be zero.

Sol.23 $x = 7t + 4t^2 \Rightarrow v_x = \frac{dx}{dt} = 7 + 8t \Rightarrow a_x = 8$

$$y = 5t \Rightarrow v_y = \frac{dy}{dt} = 5 \Rightarrow a_y = 0$$

$$\vec{a}_{\text{net}} = a_x \hat{i} + a_y \hat{j} = 8 \hat{i}$$

Sol.24 Slope of velocity time graph provide the value of acceleration.

time $0 \rightarrow 2$ $a = 2.3 \text{ m/sec}^2$

time $2 \rightarrow 10$ $a = 0$ (uniform motion)

time $10 \rightarrow 12$ $a = -2.3 \text{ m/sec}^2$

Sol.25 Slope of distance time graph provides the value of speed.

If uniform speed slope is constant.
 $\Rightarrow$ Straight line.

Sol.26 Uniformly accelerated motion means velocity is increasing.

Slope of displacement v/s time graph provide velocity

Sol.27 Distance always increase with time it never decrease with time.

Sol.28 Let is assume initial speed of bullet be U after a displacement of 3cm its velocity becomes u/2.

$$v^2 = u^2 + 2as$$

$$\left(\frac{u}{2}\right)^2 = u^2 + 2(a)(3) \Rightarrow a = -\frac{u}{8}$$

as the resistance is constant, finally velocity becomes zero.

$$v^2 = u^2 + 2as$$

$$0 = \left(\frac{u}{2}\right)^2 + 2\left(-\frac{u}{8}\right)s$$

$$s = 1\text{cm}$$

Sol.29 $S = ut + \frac{1}{2}at^2$

$$S_{20} = 0 + \frac{1}{2} \times a \times (20)^2$$

$$S_{10} = 0 + \frac{1}{2} \times a \times (10)^2$$

$$S_1 = S_{10} - 0 = 50a$$

$$S_2 = S_{20} - S_{10} = 200a - 50a = 150a$$

$$\frac{S_2}{3} = S_1$$

Sol.30 $S = 4t + \frac{1}{2}at^2$

$$S_4 = 0 + \frac{1}{2}a \times (4)^2 = 8a$$

$$S_2 = 0 + \frac{1}{2}a \times (2)^2 = 2a$$

$$x = S_2 - 0 = 2a$$

$$y = S_4 - S_2 = 6a$$

$$y = 3x$$

EXERCISE-3

Sol.1 1 fermi = 10^{-15} m.

Sol.2 avg velocity = $\frac{\text{displacement}}{\text{total time}}$

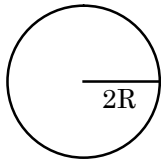
in this case displacement = 0

$\Rightarrow$ avg velocity = 0

Sol.3 circumference = $2\pi(R)$

time = t

speed = $\frac{\text{distance travelled}}{\text{time}} = \frac{2\pi R}{t}$



Sol.4 average speed = $\frac{\text{total distance travelled}}{\text{total time taken}}$

$$= \frac{40 + 40}{t_1 + t_2} = \frac{80}{\frac{40}{80} + \frac{40}{40}} = \frac{160}{3} \approx 53.33 \text{ km/hr}$$

Sol.5 Acceleration is increasing and then decreasing but acceleration is positive.

Velocity time graph first increases parabolically and then decreases in same manner.

Sol.6 $u = 160 \text{ m/sec}$

$t = 0.02 \text{ sec}$

$v = u + at$

$0 = 160 + a(0.02)$

$a = -8000 \text{ m/sec}^2$

$v^2 = u^2 + 2as$

$0 = (160)^2 + 2(-8000)(s)$

$s = 1.6\text{m}$

Sol.7 $a = -8 \text{ m/sec}^2$

$t = 3\text{sec}$

$v = 0$

$v = u + at$

$0 = u + (-8)(3)$

$u = 24 \text{ m/sec}$

$S = ut + \frac{1}{2}at^2$

$= 24 \times 3 + \frac{1}{2} \times (-8)(3)^2 = 36\text{m}.$

Sol.8 Slope of distance v/s time graph provide speed.

Sol.9 A $\rightarrow$ B velocity is constant

$\Rightarrow$ Acceleration is zero.

$\Rightarrow$ Force is zero

Sol.10 $v^2 = u^2 + 2as$

Sol.11 $a = \frac{F}{m}$

Heavy car will have low retardation.

$\therefore v^2 = u^2 - 2as$

$0 = u^2 - 2as$

$s = \frac{u^2}{2a}$

As retardation is low for heavy car distance cover will be more for it.

Sol.12 $\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t} = \frac{-4.4\hat{i} - (6\hat{i})}{(0.04)} = -260 \text{ m/sec}^2$

Sol.13 Let after time t man catches bus
For bus distance travel in Time t

$$s = \frac{1}{2}at^2$$

Man has to cover distance $(d + s)$ in time t ,

So

$d + s = \text{speed} \times \text{time}$

$$d + \frac{1}{2}at^2 = ut$$

$$\text{or } \frac{2d}{a} + t^2 = \frac{2u}{a} \cdot t$$

$$\text{or } t^2 - \frac{2u}{a}t + \frac{2d}{a} = 0$$

for positive roots.

$$b^2 - 4ac \geq 0$$

$$\frac{4u^2}{a^2} - 4(1)\left(\frac{2d}{a}\right) \geq 0$$

$$d \leq \frac{u^2}{2a}$$

Sol.14 In uniform circular motion speed is constant
and it is an accelerated motion.

Sol.15 Let the total distance be d .

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$= \frac{d}{t_1 + t_2} = \frac{d}{\frac{d}{20} + \frac{d}{30}} = 24 \text{ m/sec.}$$

Sol.16 Slope of the line $= (30-0)/3 = 10 \text{ M/S}^2$

$$a = 10 \text{ m/s}^2$$

$$s = \frac{1}{2} at^2$$

$$s = 45 \text{ m}$$

$$\text{Sol.17 } S_1 = \frac{1}{2}a(10)^2 = 100\left(\frac{a}{2}\right)$$

$$S_2 - S_1 = \frac{1}{2}a(20)^2$$

$$S_3 - S_2 - S_1 = \frac{1}{2}a(30)^2$$

$$S_3 = 500\left(\frac{a}{2}\right)$$

$$S_1 : S_2 : S_3 = 100 : 300 : 500 = 1 : 3 : 5$$

Sol.18 Velocity of object is continue increasing
parabolically hence acceleration is also
increasing in positive manner.

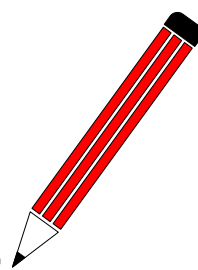
Sol.19 If distance travel by object is zero its mean
it is in the rest.

Sol.20 Radius of circle $= 6400 + 3600 = 10,000 \text{ km}$

$$\text{time} = \frac{\text{Distance}}{\text{Speed}} = \frac{2 \times 3.14 \times 10,000}{3.14}$$

$$= 20,000 \text{ sec.}$$

NOTES



CHEMISTRY

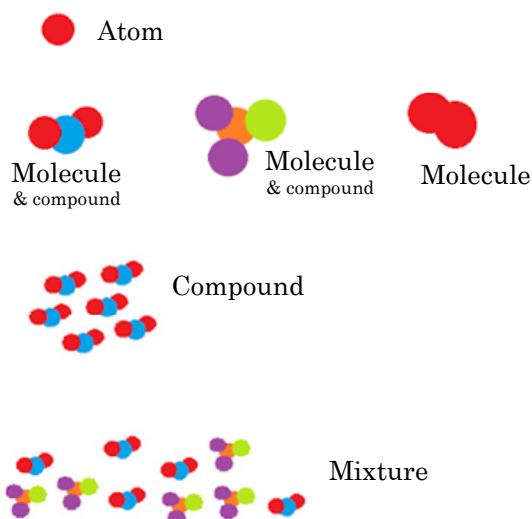
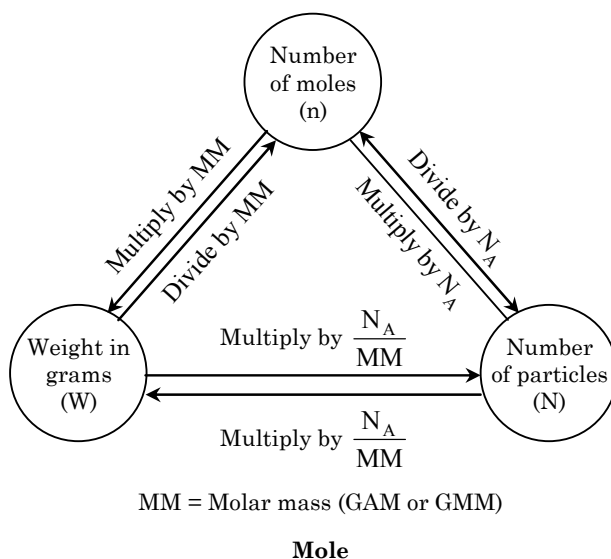
ATOMIC FOUNDATIONS OF MATTER

Chapter Outline

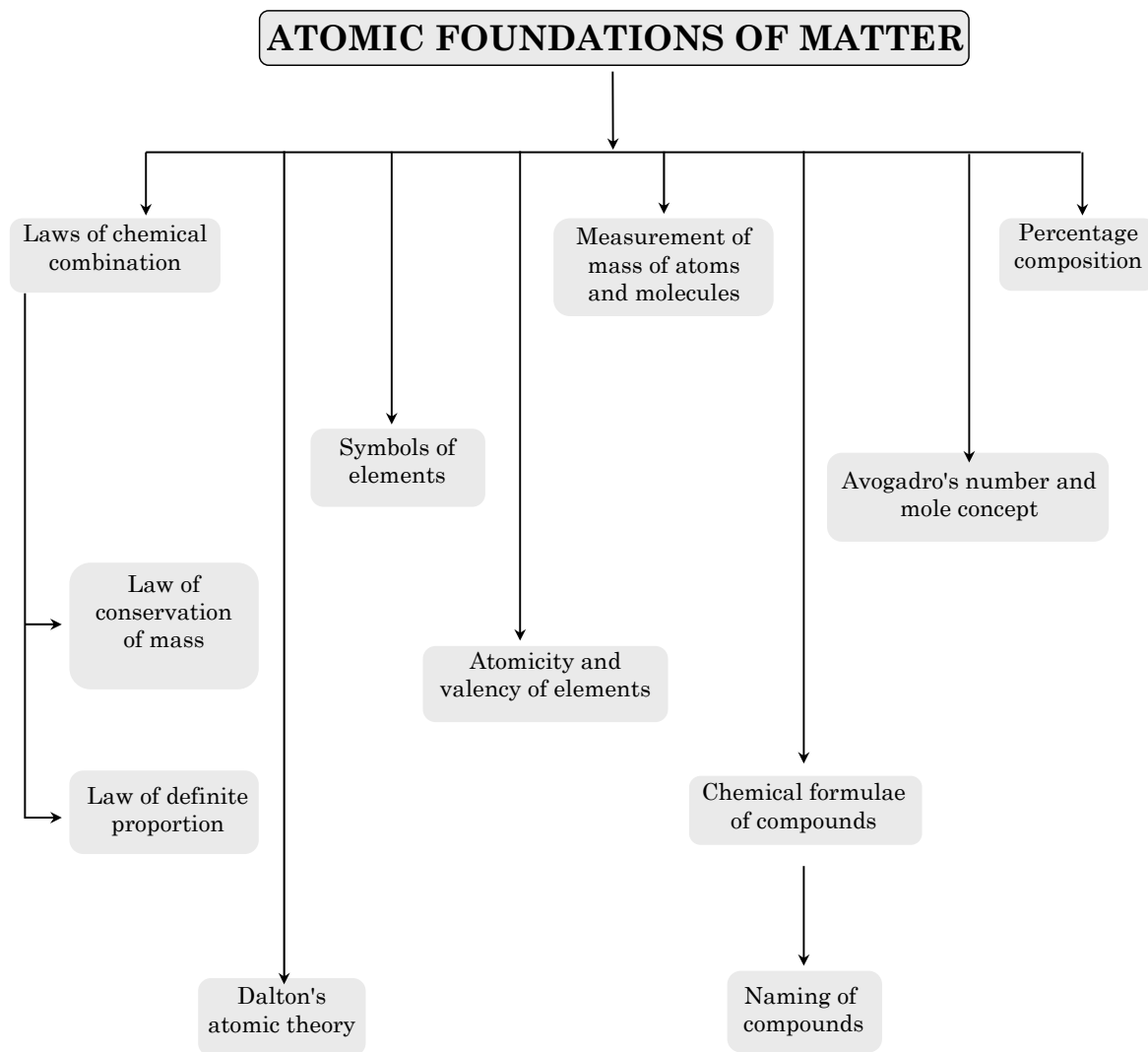
- ✧ Laws of Chemical Combination
- ✧ Dalton's Atomic Theory
- ✧ Atoms
- ✧ Symbols of an Element
- ✧ Molecules
- ✧ Concept of Mass
- ✧ Ions
- ✧ Chemical Formula
- ✧ Naming of Compounds
- ✧ Mole Concept
- ✧ Percentage Composition
- ✧ Empirical Formula
- ✧ Limiting Reagent

	Hydrogen		Carbon		Oxygen
	Phosphorus		Sulphur		Iron
	Copper		Lead		Silver
	Gold		Platina		Mercury

Symbols of an elements



MIND MAP



ATOMIC FOUNDATIONS OF MATTER

Introduction

An atom is the smallest part of an element that retains the properties of that element.

It was first suggested by an ancient Greek, named Democritus (the Greek word “atomos” means indivisible).

Democritus theorized that if you take an object and cut it in half again and again you would eventually end up with some particle which could not be further divided.

Note: In the neutral atom the total number of protons in the nucleus is equal to the total number of electrons revolving around the nucleus.

Laws of Chemical Combination

Chemistry is an experimental science and deals mainly with the reactions carried out time to time. In eighteenth century, chemists made several experiments and observed that all the chemical reactions always take place in accordance to certain laws. The experimental studies revealed the following laws.

♦ Law of Conservation of Mass (Lavoisier, 1798)

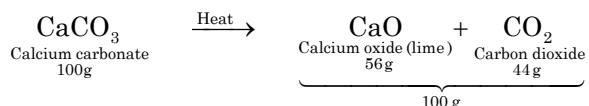
A french chemist *Antoine L. Lavoisier* (the father of chemistry) proposed a very significant law known as law of conservation of mass.

The law can be stated as follows: "The total mass of the products obtained during any physical or chemical change, is always equal to the total mass of reactants taken at the beginning of the change".

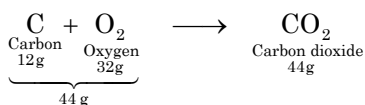
The matter is neither created nor destroyed during any physical or chemical change. However, it may change from one form to another.

♦ Illustration of law of conservation of mass:

- (i) If 100 g of calcium carbonate (CaCO_3) is strongly heated, we get 56 g of calcium oxide (lime) and 44 g of carbon dioxide. The total mass of products ($56 + 44 = 100$ g) is exactly the same as the mass of calcium carbonate taken initially to carry out the reaction.



- (ii) 12 g of carbon (C) consumes exactly 32 g of oxygen (O_2) during combustion and forms exactly 44 g of carbon dioxide. The total mass of reactants in this case ($12 + 32 = 44$ g) is exactly the same as the mass of the product i.e., (CO_2) obtained.



Ex.1 1.7 g of silver nitrate dissolved in 100 g of water is taken. 0.585 g of sodium chloride dissolved in 100 g of water is added to it. In chemical reaction 1.435 g of silver chloride and 0.85 g of sodium nitrate are formed. Justify that the data obey law of conservation of mass.

Sol. Total mass of reactants (initially) = mass of AgNO_3 + mass of NaCl
$$= 1.7 + 0.585$$
$$= 2.285 \text{ g}$$

Total mass of products (finally) = mass of AgCl + mass of NaNO_3
$$= 1.435 + 0.85$$
$$= 2.285 \text{ g}$$

Thus total mass before reaction = total mass after reaction = 2.285 g.

This confirms the law of conservation of mass.

Ex.2 8.4 g of sodium bicarbonate was added to 6.0 g acetic acid solution. Carbon dioxide gas was produced which was allowed to escape while the residue (of sodium acetate and water) was found to weigh 10.0 g. What is the mass of carbon dioxide which escaped out into the atmosphere, keeping in view the law of conservation of mass?

Sol. $\text{NaHCO}_3 + \text{CH}_3\text{COOH} \rightarrow \text{CH}_3\text{COONa} + \text{H}_2\text{O} + \text{CO}_2$

According to the law of conservation of mass.

Total mass before reaction = mass of NaHCO_3 + mass of CH_3COOH
$$\qquad\qquad\qquad 8.4 \text{ g} \qquad\qquad\qquad 6.0 \text{ g}$$

Total mass after reaction = $\underbrace{\text{mass of } \text{CH}_3\text{COONa} + \text{mass of } \text{H}_2\text{O}}_{10 \text{ g}} + \text{mass of } \text{CO}_2$

$\therefore$ Mass of $\text{CO}_2 = 14.4 - 10.0 = 4.4 \text{ g}$

◆ **Law of constant proportion or Law of Definite Proportion (Proust 1799)**

A French chemist, *Proust* in 1799 proposed the law of constant proportion or Law of definite proportion. According to this law,

"A chemical compound, no matter from which source it is obtained, always contains the same elements combined together in the definite proportion by mass".

For example: water (H_2O) is comprised of two elements hydrogen and oxygen in the ratio 1: 8 by mass. Pure water obtained from any source, always contain hydrogen and oxygen in the same fixed ratio 1: 8 by mass.

Note: To solve numerical problems of law of constant proportion, calculate the percentage of constituents in each case and prove that they are the same.

Ex.3 The common salt can be obtained from two different sources. In one sample, the percentage of chlorine was found to be 60.75 %. In the second sample, 3.888 g of chlorine were present in 6.4 g of the salt. Show that these data are in accordance to the law of constant proportion.

Sol. The first sample of common salt contains 60.75 % chlorine.

$\therefore$ 6.4 g of salt contains chlorine = 3.888 g

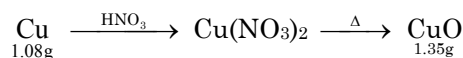
$\therefore$ 100 g of salt will contain chlorine = $\frac{3.888 \times 100}{6.4} = 60.75 \text{ g}$

Hence, the percentage of chlorine in second sample = 60.75 %

Since, both the samples of common salt contain the same percentage of chlorine, the given data are in accordance to the law of constant proportion.

Ex.4 1.08 g of copper wire was allowed to react with nitric acid. The resulting solution when dried and ignited gave 1.35 g of copper oxide. In another experiment 2.30 g of copper oxide was heated in presence of hydrogen yielding 1.84 g of copper. Show that the above data are in accordance with the law of constant proportion.

Sol. Case I:

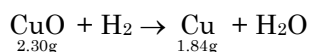


$\therefore$ 1.35 g CuO contains 1.08 g Cu

$\therefore$ 100 g CuO contains $(1.08 \times 100) / 1.35$ g Cu = 80 g Cu

$\therefore$ % of Cu in CuO = 80 ; % of O in CuO = 20

Case II:



$\therefore$ 2.30 g CuO contains 1.84 g Cu

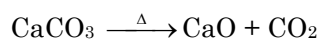
$\therefore$ 100 g CuO contains $(1.84 \times 100) / 2.30$ = 80 g Cu

$\therefore$ % of Cu in CuO = 80 ; % of O in CuO = 20

Since the percentage of Cu and O₂ in CuO in both the cases is same, the data are in accordance with the law of definite proportion.

Ex.5 1.4 g of calcium oxide obtained by heating limestone was found to contain 0.4 g of oxygen. Another sample of 3.5 g of calcium oxide obtained by direct combination of calcium and oxygen was found to contain 2.5 g of calcium. Show that the data are in accordance with the law of definite proportion.

Sol. Case I:



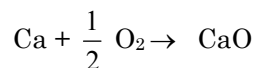
$\therefore$ 1.4 g CaO contains 0.4 g O₂

$\therefore$ 100 g CaO contains $(0.4 \times 100) / 1.4$ = 28.57 g O₂

$\therefore$ % of O in CaO = 28.57

$\therefore$ % of Ca in CaO = 100 – 28.57 = 71.43

Case II:



$\therefore$ 3.5 g CaO contains 2.5 g Ca

$\therefore$ 100 g CaO contains $(2.5 \times 100) / 3.5$ = 71.43

$\therefore$ % of Ca in CaO = 71.43

and % of O in CaO = 100 – 71.43 = 28.57

Thus, percentage of Ca and oxygen in CaO samples are same. This confirms the law of definite proportion.

COMPETITIVE LEVEL

(C) Law of Multiple Proportion: This law is given by Johan Dalton. "When two elements combine to form more than one compound then the different mass of one element which combine with a fixed mass of the other element bear a simple whole number ratio to one another".

e.g.:

- (i) Carbon and oxygen when combine, can form two oxides that are CO (carbon monoxide). CO₂ (carbon dioxide). In CO, 12 g carbon combined with 16 g of oxygen. In CO₂, 12 g carbon combined with 32 g of oxygen. Thus, we can see the mass of oxygen which combine with a constant mass of carbon (12 g) bear simple ratio of 16: 32 or 1: 2.
- (ii) Nitrogen and oxygen combine to form five oxides, which are: Nitrous oxides (N₂O), nitric oxide (NO), nitrogen trioxide (N₂O₃), nitrogen tetraoxide (N₂O₄) and nitrogen pentoxide (N₂O₅).

Weights of oxygen which combine with the fixed weight of nitrogen in these oxides are calculated as under:

Oxides	Ratio of weights of nitrogen and oxygen in each compound
N ₂ O	28 : 16 = 14 : 8
NO	14 : 16 = 14 : 16
N ₂ O ₃	28 : 48 = 14 : 24
N ₂ O ₄	28 : 64 = 14 : 32
N ₂ O ₅	28 : 80 = 14 : 40

Number of parts by weight of oxygen which combine with 14 parts by weight of nitrogen from the above are 8, 16, 24, 32 and 40 respectively. Their ratio is 1: 2: 3: 4: 5, which is a simple ratio. Hence the law illustrated.

Ex.6 Carbon is found to form two oxides, which contain 42.8% and 27.27% of carbon respectively. Show that these figures illustrate the law of multiple proportions.

Sol. % of carbon in first oxide = 42.8

$$\therefore \text{ \% of oxygen in first oxide} = 100 - 42.8 = 57.2$$

$$\text{ \% of carbon in second oxide} = 27.27$$

$$\therefore \text{ \% of oxygen in second oxide}$$

$$= 100 - 27.27 = 72.73$$

For the first oxide -

Mass of oxygen in grams that combines with 42.8 g of carbon = 57.2

$$\therefore \text{ Mass of oxygen that combines with 1 g of carbon} = \frac{57.2}{42.8} = 1.34 \text{ g}$$

For the second oxide -

Mass of oxygen in grams that combines with 27.27 g of carbon = 72.73

$$\therefore \text{ Mass of oxygen that combines with 1 g of carbon} = \frac{72.73}{27.27} = 2.68 \text{ g}$$

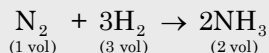
Ratio between the masses of oxygen that combine with a fixed mass (1 g) of carbon in the two oxides = 1.34: 2.68 or 1: 2 which is a simple ratio. Hence, this illustrates the law of multiple proportion.

◆ Gay Lussac's Law of Gaseous Volumes

According to Gay Lussac's Law of combining volume.

"Whenever gases react chemically, they do so in volumes, which bear a simple ratio to each other and also to the products if they are also gaseous, provided the volumes are measured at same temperature and pressure for all the gases".

e.g.: Consider the reaction between nitrogen and hydrogen. The balanced equation is written as



The ratio of nitrogen gas that reacts with hydrogen gas forming ammonia gas is 1: 3: 2.

To illustrate 100 ml of nitrogen gas will react with 300 ml of hydrogen gas to give 200 ml of ammonia gas.

◆ Avogadro's Law

Avogadro's law states that. "Equal volumes of all gases contain the same number of moles under same conditions of temperature and pressure".

Illustration

If 1ℓ of hydrogen gas at STP contains say, 'n' molecules, then according to Avogadro law.

1ℓ of oxygen gas at STP has the same number of molecules. i.e., 'n' molecules.

1ℓ of nitrogen gas at STP also has the same number of molecules, i.e., 'n' molecules and

1ℓ of CO₂ at STP will also have the same number of molecules, i.e., 'n' molecules.

◆ Applications of Avogadro's Law:

- In the calculation of atomicity of elementary gases.

e.g.: 3 volumes of hydrogen combine with 1 volume of nitrogen to form two volumes of ammonia.

Hydrogen + Nitrogen → Ammonia

3 vol. 1 vol. 2 vol.

Applying Avogadro's hypothesis

Hydrogen + Nitrogen → Ammonia

3n molecules n molecules 2 n molecules

or

3 molecule 1 molecule 2 molecule

$\frac{3}{2}$ molecules $\frac{1}{2}$ molecules 1 molecules

Thus 1 molecule of ammonia contains $\frac{1}{2}$ molecule of nitrogen. But 1 molecule of ammonia contains 1 atom of nitrogen. Hence $\frac{1}{2}$ molecule of nitrogen = 1 atom of nitrogen or 1 molecules of nitrogen = 2 atoms of nitrogen i.e. atomicity of nitrogen = 2.

Dalton's Atomic Theory (1808)

Keeping in view the laws of chemical combinations and the work of Greek philosophers, a meaningful atomic theory was finally proposed by John Dalton in 1808. The basic postulates of Dalton's theory are as follows:

- (i) Each element is composed of extremely small particles called atoms.
- (ii) All atoms of a given element are identical i.e., atoms of a particular element are alike in all aspects.
- (iii) Atoms of different elements possess completely different properties and differ in all aspects.
- (iv) Atoms are indestructible i.e., atoms are neither created nor destroyed in chemical reactions.
- (v) Atoms of element take part in chemical reaction to form molecules.
- (vi) In a given compound, the relative number and kind of atoms are constant.
- (vii) Atom of same type of elements combine in different ratio to form more than one type of compounds.

◆ Merits of Dalton's Atomic Theory

- (i) Dalton's theory provides us a conceptual picture of matter. Atoms are the basic building blocks of matter. They are the smallest units of an element that can combine with other elements in a chemical reaction. In compounds, the atoms of two or more elements combine in definite arrangement.
- (ii) Dalton's theory embodies several simple laws of chemical combination that were known at that time. Postulate (iv) indicates for the law of conservation of mass. Postulate (vi) indicates for the law of definite proportion.
- (iii) Dalton's theory also explains the law of multiple proportion. Postulate (vii) indicates for the law of multiple proportion.
- (iv) It explains that what makes an element to differ from others.

◆ Limitations of Dalton's Atomic Theory

- (i) Distinction between atoms and molecules: According to Dalton, the smallest particle of an element as well as of compound was atom, however, he called 'compound atoms' to be smallest particle of compound. Later on Amedeo Avogadro used the term molecule for the 'compound atom' i.e., molecule is the smallest particle of compound.
- (ii) It could not explain why do atoms combine to form a molecule.
- (iii) It could not explain the nature of forces which hold the atoms and molecules in solid, liquid and gaseous state.
- (iv) The theory fails to explain the existence of isotopes and isobars.

Atoms

- ◆ All the matter is made up of atoms. An atom is the smallest particle of an element that can take part in a chemical reaction. Atoms of most of the elements are very reactive and do not exist in the free state. They exist in combination with the atoms of the same element or another element.
- ◆ Atoms are very, very small in size. The size of an atom is indicated by its radius which is called 'atomic radius'. Atomic radius is measured in 'nanometres'. The symbol of a nanometre is nm.

$$1 \text{ nanometre} = \frac{1}{10^9} \text{ metre} \quad \text{or} \quad 1 \text{ nm} = \frac{1}{10^9} \text{ m}$$

$$\text{or } 1 \text{ nm} = 10^{-9} \text{ m}$$

Hydrogen atom is the smallest atom of all.

Note: *Atoms cannot be viewed by simple optical microscopes. However, through modern techniques such as scanning tunneling microscope it is possible to produce magnified images of surfaces of elements showing atoms.*

Ex.7 How do atoms exist?

Sol. The atoms of only a few elements called noble gases (such as Helium, Neon, Argon, Krypton, Xenon and Radon) are chemically unreactive and exist in the free state (as single atoms). Atoms of most of the elements are chemically very reactive and do not exist in the free state (as single atoms).

Atoms usually exist in two ways.

(i) In the form of molecules

(ii) In the form of ions.

When atoms form molecules or ions, they become stable (because while doing so they acquire the stable electronic configuration like noble gases).

Molecules

According to Avogadro, molecule is the smallest unit of a substance (element or compound) which can exist independently.

It is important to know that the atoms of the same element or different elements can join together to form molecules. On this basis, molecules are of two types:

(1) Molecules of elements

(2) Molecules of compounds

◆ Molecules of Elements

The molecule which is made up of two or more atoms of the same element that are chemically bonded together is called molecule of an element.

e.g.: H_2 , N_2 , O_2 , Cl_2 , O_3 , P_4 , S_8 etc. are the molecules of elements. Molecules of helium (He), neon (Ne), argon (Ar) etc. are made up of single atoms. These are chemically unreactive elements.

The number of atoms present in a single molecule of a substance is called atomicity of that molecule or substance. The molecules of elements composed of the same type of atoms are called *homoatomic* molecules. They may be monoatomic, diatomic, triatomic or polyatomic.

(i) **Monoatomic molecules:** The molecules of noble gases like helium (He), neon (Ne), argon (Ar), krypton (Kr), xenon (Xe) and radon (Rn) are composed of one atom only. Hence, they are monoatomic in nature and exist independently in monoatomic state.

(ii) **Diatomic molecules:** Diatomic molecules of most of the gases formed by non-metal atoms consists of two atoms of same element held strongly together by attractive forces

e.g.: a molecule of oxygen consists of two oxygen atoms and is represented as O_2 .

Examples of other diatomic molecules are nitrogen molecule (N_2), hydrogen molecule (H_2), chlorine molecule (Cl_2), fluorine molecule (F_2) etc. In each of the above examples subscript 2 indicates that there are two atoms.

- (iii) **Triatomic molecules:** Triatomic molecules consist of three atoms of same element held tightly together by attractive forces

e.g.: a molecule of ozone consists of three atoms of oxygen and is represented as O_3 .

- (iv) **Polyatomic molecules:** The polyatomic molecules of some non-metals consist of more than three atoms and are referred as polyatomic molecules

e.g.: phosphorus molecules (P_4), sulphur molecule (S_8) etc.

The molecules of metals and some other elements, such as carbon (e.g. C-60) consist of a very large and indefinite number of atoms bonded together.

◆ Molecules of Compounds

Molecules in which atoms of different elements are chemically bonded together are called molecules of compounds. Compounds in which atoms of different elements are chemically bonded together are called molecular or covalent compounds.

Water (H_2O), ammonia (NH_3), carbon dioxide (CO_2), methane (CH_4), hydrogen chloride (HCl), sulphuric acid (H_2SO_4) etc. are all molecules of compounds.

In all these compounds, atoms of different elements are chemically bonded together in the constant ratio by mass of elements.

- ◆ **Heteroatomic molecules:** The molecules of compounds composed of two or more atoms of different elements linked together in definite proportions are called *heteroatomic* molecules.

For example, a molecule of water is composed of two atoms of hydrogen (H) and one atom of oxygen (O) therefore, a molecule of water is represented as H_2O .

The heteroatomic molecules of compounds can also be classified as diatomic (NO), triatomic (H_2O) and polyatomic (H_2O_2) depending upon the total number of atoms present in them.

Note: Every compound is a molecule, but every molecule is not a compound.

Molecules of Some Compounds

Compound	Constituent Elements	Ratio by Mass	Formula of the Molecule	Atomicity
Hydrogen chloride	Hydrogen (H), Chlorine (Cl)	1 : 35.5	HCl	2
Water	Hydrogen (H), Oxygen (O)	1 : 8	H_2O	3
Ammonia	Nitrogen (N), Hydrogen (H)	14 : 3	NH_3	4
Carbon monoxide	Carbon (C), Oxygen (O)	3 : 4	CO	2
Sulphur dioxide	Sulphur (S), Oxygen (O)	1 : 1	SO_2	3
Nitrous oxide	Nitrogen (N), Oxygen (O)	7 : 4	N_2O	3
Nitric oxide	Nitrogen (N), Oxygen (O)	7 : 8	NO	2

◆ Properties of Covalent Compounds

- Covalent compounds have weak intermolecular forces.
- Therefore, they melt and boil easily.
- Most covalent compounds are soft in nature.
- They do not conduct electricity because they do not contain free ions.
- Exception: Graphite conducts electricity.
- Most covalent compounds do not dissolve in water.
- They dissolve in solvents like benzene, ether and petrol.
- Covalent compounds may exist as solids, liquids or gases.
- Atoms share electrons to complete their outermost shell.

Ions

An ion is a positively or negatively charged atom (or group of atoms). An ion is formed by the loss or gain of electrons by an atom, so it contains an unequal number of electrons and protons.

e.g.: Sodium ion (Na^+), magnesium ion (Mg^{2+}), chloride ion (Cl^-), and oxide ion (O^{2-}) etc.

◆ Type of Ions

1. **On the basis of nature of charge:** There are two types of ions: cations and anions.

- ◆ **Cation:** A positively charged ion is known as cation. A cation is formed by the loss of one or more electrons by an atom.

e.g.: Sodium atom loses 1 electron to form a sodium ion, Na^+ , which is cation:

Na	$\xrightarrow{-1 \text{ electron}}$	Na^+
Sodium atom		Sodium ion (cation)
Protons = 11 (+ charge)		Protons = 11 (+charge)
Electrons = 11 (– charge)		Electrons = 10 (–charge)
Overall charge = 0		Overall charge = 1+

e.g.: A magnesium atom loses 2 electrons to form magnesium ion, so magnesium ion bears 2 units of positive charge and it is represented as Mg^{2+}



e.g.: An aluminium atom loses 3 electrons to form aluminium ion, so aluminium ion bears 3 units of positive charge and it is represented as Al^{3+}

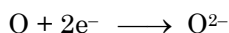


- ◆ **Anion:** A negatively charged ion is known as anion. An anion is formed by the gain of one or more electrons by an atom.

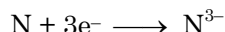
e.g.:A chlorine atom gains 1 electron to form a chloride ion, Cl^- , which is an anion.

Cl	$\xrightarrow{+1 \text{ electron}}$	Cl^-
Chlorine atom		Chlorine ion (anion)
Protons = 17 (+ charge)		Protons = 17 (+charge)
Electrons = 17 (– charge)		Electrons = 18 (–charge)
Overall charge = 0		Overall charge = 1–

e.g.: A oxygen atom gains 2 electrons to form oxide ion, so oxide ion bears 2 units of negative charge and it is represented as O^{2-}



e.g.: A nitrogen atom gains 3 electrons to form nitride ion, so nitride ion bears 3 units of negative charge and it is represented as N^{3-}



Note:

- *The ions of all the non metal elements are anions.*
- *The ions of all the metal elements are cations.*
- *Size of a cation is always smaller and anion is always greater than that of the corresponding neutral atom.*

2. On the basis of number of atoms: There are two types of ions: monoatomic ions and polyatomic ions.

♦ **Monoatomic ions:** These ion which are formed from single atom are called simple ions.

e.g.: Na^{+} , K^{+} , Ca^{2+} , O^{2-} , F^{-} etc.

♦ **Polyatomic ions:** Those ions which are formed from groups of joined atoms are called compound ions

e.g.: NH_4^{+} , H_3O^{+} , OH^{-} etc.

Chemical Formulae

The chemical formula of a compound or an element represents the composition of a molecule of the compound or an element, in terms of the symbols of elements and the number of atoms of each element present in one molecule of the compound or an element.

◆ Formulae of Elements

The chemical formula of an element is a representation of the composition of its molecule in which symbol represents the element and subscript represents, how many atoms are present in one molecule.

e.g.: One molecule of hydrogen element contains two atoms of hydrogen, therefore, the formula of hydrogen is H_2 . It should, however, be noted that 2H represents two separate atoms of hydrogen, while H_2 represents one molecule of hydrogen.

◆ Formulae of Compounds

The chemical formula of a compound is a representation of the composition of its molecule in which symbol represents, which elements are present and the subscript shows us how many atoms of each element are present in one molecule of a compound.

e.g.: One molecule of carbon dioxide contains 1 atom of carbon and 2 atoms of oxygen. Hence, the formula of carbon dioxide is CO_2 .

(i) In the chemical formula of a compound, the elements present are denoted by their symbols and the number of atoms of each element are denoted by writing their number as subscripts to the symbols of the respective elements.

e.g.: Water is a compound whose one molecule is made up of 2 atoms of hydrogen and 1 atom of oxygen and hence, its chemical formula is H_2O .

(ii) While writing the formula of an ionic compound, the metal is written on the left hand side, while the non metal is written on the right hand side.

e.g.: Magnesium oxide is written as MgO , Sodium chloride is written as NaCl etc.

The name of the metal remains as such, but that of the non-metal is changed to have the ending 'ide'.

- (iii) Molecular compounds, formed by the combination between two different non-metals, are written in such a way that the less electronegative element is written on the left hand side, while the more electronegative element is written on the right hand side. In naming molecular compounds, the name of the less electronegative non-metal is written as such but the name of the more electronegative element is changed to have the ending 'ide'.

e.g.: H_2S is named as hydrogen sulphide and HCl is named as hydrogen chloride .

- (iv) When there are more than one atoms of an element present in the formula of the compound, then the number of atoms are indicated by the use of appropriate prefixes (mono for 1, di for 2, tri for 3, tetra for 4 atoms etc. respectively) in the name of the compound.

e.g.: CO is named as carbon monoxide, CO_2 is named as carbon dioxide and CCl_4 is named as carbon tetrachloride.

- (v) The prefixes are needed in naming those binary compounds in which the two non-metals form more than one compounds (by having different number of atoms).

e.g.: Two non-metals, nitrogen and oxygen, combine to form different compounds like nitrogen monoxide (NO), nitrogen dioxide (NO_2), dinitrogen trioxide (N_2O_3) etc.

- (vi) If two non-metals form only one compound, then prefixes are not used in naming such compounds.

e.g.: Hydrogen and sulphur combine to form only one compound H_2S . So, H_2S is named as hydrogen sulphide and not as hydrogen monosulphide or dihydrogen monosulphide.

The number ratio of atoms present in a molecule can be determined by dividing the ratio by mass with the respective atomic masses of the corresponding elements and finding the simplest ratio.

LIST OF COMMON ELECTROVALENT POSITIVE RADICALS

Monovalent Electropositive		Bivalent Electropositive		Trivalent Electropositive		Tetravalent Electropositive	
1	Hydrogen H^+	1	Magnesium Mg^{2+}	1	Aluminium Al^{3+}	1	Stannic [Tin(IV)] Sn^{4+}
2	Ammonium NH_4^+	2	Calcium Ca^{2+}	2	Ferric [Iron(III)] Fe^{3+}	2	Plumbic [lead(IV)] Pb^{4+}
3	Sodium Na^+	3	Zinc Zn^{2+}	3	Chromium Cr^{3+}		
4	Potassium K^+	4	Plumbous [Lead(II)] Pb^{2+}				
5	Cuprous [Copper(I)] Cu^+	5	Cupric [Copper(II)] Cu^{2+}				
6	Argentous [Silver(I)] Ag^+	6	Argentia [Silver(II)] Ag^{2+}				
7	Mercurous [Mercury(I)] Hg^{1+}	7	Stannous [Tin(II)] Sn^{2+}				
		8	Ferrous [Iron(II)] Fe^{2+}				
		9	Mercuric [Mercury(II)] Hg^{2+}				
		10	Barium Ba^{2+}				

LIST OF COMMON ELECTROVALENT NEGATIVE RADICALS

Monovalent Electronegative		Bivalent Electronegative		Trivalent Electronegative		Tetravalent Electronegative	
1	Fluoride F ⁻	1	Sulphate SO ₄ ²⁻	1	Nitride N ³⁻	1	Carbon C ⁴⁻
2	Chloride Cl ⁻	2	Sulphite SO ₃ ²⁻	2	Phosphide P ³⁻		
3	Bromide Br ⁻	3	Sulphide S ²⁻	3	Phosphite PO ₃ ³⁻		
4	Iodide I ⁻	4	Thiosulphate S ₂ O ₃ ²⁻	4	Phosphate PO ₄ ³⁻		
5	Hydride H ⁻	5	Zincate ZnO ₂ ²⁻				
6	Hydroxide OH ⁻	6	Oxide O ²⁻				
7	Nitrite NO ₂ ⁻	7	Peroxide O ₂ ²⁻				
8	Nitrate NO ₃ ⁻	8	Dichromate Cr ₂ O ₇ ²⁻				
9	Bicarbonate or Hydrogen carbonate HCO ₃ ⁻	9	Carbonate CO ₃ ²⁻				
10	Bisulphite or Hydreogen sulphite HSO ₃ ⁻	10	Silicate SiO ₃ ²⁻				
11	Bisulphide or Hydrogen sulphide HS ⁻						
12	Bisulphate or Hydrogen sulphate HSO ₄ ⁻						
13	Acetate CH ₃ COO ⁻						

◆ Ion and method of writing the chemical formula of an ionic compound

An ion is an atom or a group of atoms (of the same or different elements) that carries a positive or a negative charge and behaves like a single unit.

The species containing a single atom and carrying a positive or negative charge are called simple ions i.e., Na^+ , Ca^{2+} , H^+ , Cl^- , Br^- etc. The species containing a group of atoms and having a net charge on them are called polyatomic ions, i.e; NH_4^+ , SO_4^{2-} , PO_4^{3-} , NO_3^- etc.

Positively charged ions are termed as cations, whereas the negatively charged ions are termed as anions.

The valency of an ion is usually equal to the number of charges present on it. Depending upon their valencies, ions are also referred to as monovalent (valency = 1), bivalent (valency = 2), trivalent (valency = 3) and tetravalent (valency = 4).

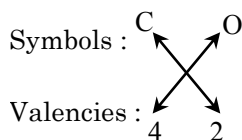
Note: 1. 2^+ means two positive charges, i.e., $++$ e.g. Ca^{2+} ; 3^+ means three positive charges, i.e., $+++$ e.g. Fe^{3+} ; whereas 4^+ indicates the presence of four positive charges ($++++$) e.g. Similarly 1^- means one negative charge i.e. -1 e.g. Cl^- & so on.

2. Roman number written in brackets represent the valency of the ion i.e. Fe^{2+} as $Fe(II)$ in compounds.

Ex.8 Writing the formula of carbon dioxide.

Sol. Following steps are used to write the formula of carbon dioxide -

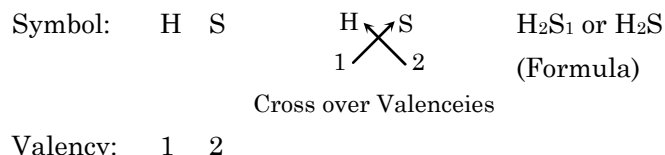
- Carbon dioxide is a compound composed of two elements, carbon and oxygen. So, we first write their symbols C and O respectively.
- The valency of carbon is 4 and the valency of oxygen is 2. Now, these valencies are to be written under the corresponding symbols of elements.



- Now, the valencies of carbon and oxygen are to be exchanged. So, the subscript corresponding to C is 2 and that corresponding to O is 4. Hence, the formula of the compound becomes C_2O_4 .
- But, the valencies 2 and 4 have a common factor 2. So, on dividing the whole formula by 2, we get the simplest formula CO_2 . Thus, the formula of carbon dioxide is CO_2 .

Ex.9 Write the formula of hydrogen sulphide.

Sol. Elements present in hydrogen sulphide are hydrogen and sulphur. Valency of hydrogen is 1 and that of sulphur is 2.



In the formula H_2S :

Valency of H is 1, atoms of H are 2.

Valency of S is 2, atom of S is 1.

Total valencies of H-atom = $1 \times 2 = 2$

Total valency of S-atom = $2 \times 1 = 2$

Since total valency of atoms of metals and non-metals comes out to be equal, the formula (H_2S) is correct of hydrogen sulphide.

◆ Chemical Formula of Ionic Compound

The compounds which are made up of ions are known as ionic compounds. In an ionic compound, the positively charged ions (cations) and negatively charged ions (anions) are held together by the strong electrostatic forces of attraction. The forces which hold the ions together in an ionic compound are known as ionic bonds or electrovalent bonds. Since an ionic compound consists of an equal number of positive ions and negative ions, so the overall charge on an ionic compound is zero.

e.g.: Sodium chloride ($NaCl$) is an ionic compound which is made up of equal number of positively charged sodium ions (Na^+) and negatively charged chloride ions (Cl^-).

Some ionic compound and their ions

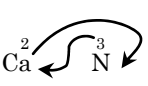
S. No.	Name	Formula	Ions present
1	Sodium chloride	NaCl	Na^+ and Cl^-
2	Potassium chloride	KCl	K^+ and Cl^-
3	Ammonium chloride	NH_4Cl	NH_4^+ and Cl^-
4	Magnesium chloride	MgCl_2	Mg^{2+} and Cl^-
5	Calcium chloride	CaCl_2	Ca^{2+} and Cl^-
6	Magnesium oxide	MgO	Mg^{2+} and O^{2-}
7	Calcium oxide	CaO	Ca^{2+} and O^{2-}
8	Aluminium oxide	Al_2O_3	Al^{3+} and O^{2-}
9	Sodium hydroxide	NaOH	Na^+ and OH^-
10	Copper sulphate	CuSO_4	Cu^{2+} and SO_4^{2-}
11	Calcium nitrate	$\text{Ca}(\text{NO}_3)_2$	Ca^{2+} and NO_3^-

An ionic compound is electrically neutral and is usually formed by the combination of positive and negative ions in such a way that the positive charge of the cation is completely neutralized by the negative charge present on the anion. This fact should be kept in mind while writing the formula of a compound.

The method used to write the formula of a compound is called *Criss-cross method*. It is carried out in the following steps:

- Write the symbols of cation and anion side by side. Say for example magnesium oxide. The cation Mg^{2+} i.e., the ion having positive charge is written first i.e., O^{2-} . The anion (O^{2-}) is written after it.
- Write the charge of each ion on the top of its symbol. While doing so, ignore the + and – sign i.e.
 $\overset{2}{\text{Mg}} \quad \overset{2}{\text{O}}$
- Divide the charges by their highest common factor (if any) to get a simple ratio (in this case by 2).
- Interchange the charge and write them on the lower right side of the symbols i.e. $\text{MgO}(\text{Mg}_1\text{O}_1)$. The number 1 does not need to be written. This is known as (cris cross method).
- The formula of magnesium oxide MgO .

Let us consider another example of calcium nitride.

- Writing the symbols of cation and anion $\text{Ca}^{2+} \text{N}^{3-}$
- Writing the charge of ions: $\overset{2}{\text{Ca}} \quad \overset{3}{\text{N}}$
- Interchange their charge: $\overset{2}{\text{Ca}} \quad \overset{3}{\text{N}}$

- Thus, the formula of calcium nitride is Ca_3N_2 .

◆ Properties of Ionic Compounds

- Ionic compounds are generally hard solids.
- They have a crystalline structure.
- Strong electrostatic force of attraction exists between ions.
- Therefore, a large amount of heat is required to break the bond.
- Ionic compounds conduct electricity only when dissolved in water or in molten form because ions become free to move.
- They do not conduct electricity in solid state.
- Most ionic compounds dissolve easily in water. Ex: Sodium chloride (NaCl).
- They are generally insoluble in petrol, kerosene and benzene.
- Positive and negative ions are strongly attracted to each other.
- Ionic compounds break easily when pressure is applied.

Naming Compounds from Their Formulae

◆ Naming Simple Compounds Formed by a Metal and a Non-metal

The name of a simple compound formed by a metal and a non-metal (or group of non-metals) can easily be written by identifying cation and anion in its formula. The name of the compound is obtained by first writing the name of the metal cation followed by the name of the non metal anion.

Formula	Name
NaCl	Sodium chloride
KCl	Potassium chloride
MgCl ₂	Magnesium chloride
CaCl ₂	Calcium chloride
MgO	Magnesium oxide
CaO	Calcium oxide
Al ₂ O ₃	Aluminium oxide

Note: The valency of an element is always a whole number. Some elements exhibit more than one valency, i.e., they have variable valency.

Generally, the Latin / Greek name for the element is modified to end in 'ous' for the lower valency and to end in 'ic' for the higher valency.

The table below lists some elements that exhibit variable valencies.

Element	Symbol	Valency	Ions	Nomenclature
Copper	Cu	1	Cu ¹⁺	Copper [I] or Cuprous
		2	Cu ²⁺	Copper [II] or Cupric
Silver	Ag	1	Ag ¹⁺	Silver [I] or Argentous
		2	Ag ²⁺	Silver [II] or Argentic
Mercury	Hg	1	Hg ¹⁺	Mercury [I] or Mercurous
		2	Hg ²⁺	Mercury [II] or Mercuric
Iron	Fe	2	Fe ²⁺	Iron [II] or Ferrous
		3	Fe ³⁺	Iron [III] or Ferric
Tin	Sn	2	Sn ²⁺	Tin [II] or Stannous
		4	Sn ⁴⁺	Tin [IV] or Stannic
Lead	Pb	2	Pb ²⁺	Lead [II] or Plumbous
		4	Pb ⁴⁺	Lead [IV] or Plumbic

In case of the metal forming the compound is capable of showing variable valencies, the valency of the metal is written in roman numerals in paranethessis after the name of the metal. This representation is called stock notation. For example:

Naming Simple Compounds from their formulae

Formula	Name
CuCl	Copper (I) chloride or Cuprous chloride
CuCl ₂	Copper (II) chloride or Cupric chloride
FeCl ₂	Ferrum (II) chloride or Ferrous chloride
FeCl ₃	Ferrum (III) chloride or Ferric chloride

◆ Naming Compounds formed by Two non-metals

Compounds formed by two non-metals are named by writing first non metal and then other using the suffixes di, tri, tetra, penta etc. before the name of the non-metals if they appear with more than one atom. The name of more electronegative non metal comes in last. For example.

Formula	Name
CO ₂	Carbon dioxide
NO ₂	Nitrogen dioxide
OF ₂	Oxygen difluoride
PCl ₃	Phosphorus trichloride
CCl ₄	Carbon tetrachloride
PCl ₅	Phosphorus pentachloride
N ₂ O ₅	Dinitrogen pentoxide

Concept of Mass

Atoms of all the elements (except noble gases) are not able to exist independently. They combine with similar or different atomic species to form molecules or ions. These molecules or ions aggregate together in large numbers to form the matter which is around us.

Only a few elements, (noble gases) e.g.: helium, neon, argon, krypton, xenon and radon are able to exist freely in atomic state i.e., these show independent existence and exist as mono atomic gases in nature.

The absolute atomic mass of an atom of hydrogen is 1 u or 1.67×10^{-24} g as well as absolute atomic masses of other atoms are also of the same order. Since the absolute atomic masses of all atoms are extremely small and are in the order of 10^{-24} g and therefore, it is very inconvenient to use mass of one atom in routine calculations. A new term *relative atomic mass* was therefore used. The relative atomic mass is its absolute mass compared to the absolute mass of an atom of a standard substance i.e. carbon, Thus,

$$\text{Relative atomic mass} = \frac{\text{Absolute mass of atom of the given substance}}{\text{Absolute mass of an atom of a standard substance}}$$

Direct determination of relative mass as defined above is again impossible as it will again require absolute masses of the atoms.

The relative atomic mass of an element is the number which tells how many times an atom of that element is heavier than an atom of hydrogen.

◆ Definition of Mass

Mass is the measure of quantity of matter within a sample.

◆ Atomic Mass Unit

The atomic mass unit (amu) is equal to one-twelfth ($1/12$) of the mass of an atom of carbon-12. The mass of an atom of carbon-12 isotope was given the atomic mass of 12 units, i.e. 12 amu or 12 u.

The atomic masses of all other elements are now expressed in atomic mass units.

◆ Atomic Mass

- ◆ **Relative Atomic Mass:** The atomic mass of an element is a relative quantity and it is the mass of one atom of the element relative to one-twelfth ($1/12$) of the mass of one carbon-12 atom. Thus,

$$\text{Relative atomic mass} = \frac{\text{Mass of one atom of the element}}{\frac{1}{12} \times \text{mass of one C-12 atom}}$$

$$[1/12 \text{ the mass of one C-12 atom} = 1 \text{ amu},$$

$$1 \text{ amu} = 1.66 \times 10^{-24} \text{ g} = 1.66 \times 10^{-27} \text{ kg.}]$$

- ◆ **Gram Atomic Mass:** The atomic mass of an element expressed in grams is called the Gram Atomic Mass of the element.

$$\text{The number of gram-atoms} = \frac{\text{Mass of the element in grams}}{\text{Gram Atomic mass of the element}}$$

Note: Most of the atomic masses determined with hydrogen as the standard were not whole numbers. Later on, O^{16} isotope of oxygen was taken as the reference substance and the relative atomic masses of most of the elements were calculated on its basis and found very close to whole numbers.

In 1961, a new standard carbon-12, (C^{12} , the most stable isotope of carbon) was adopted by International Union of Chemistry to calculate the relative atomic masses of elements. The new scale adopted is called atomic mass unit scale.

◆ Molecular Mass

As the term itself explains, molecular mass is the mass of one molecule of a substance. But like an atom, a molecule is so small that it is impossible to determine its absolute mass. Therefore its better to refer the relative molecular mass.

- ◆ **Relative molecular mass:** The relative molecular mass of a substance is the mass of a molecule of the substance as compared to one-twelfth of the mass of one carbon -12 atom i.e.,

$$\text{Relative molecular mass} = \frac{\text{Mass of one molecule of the substance}}{\frac{1}{12} \times \text{mass of one C-12 atom}}$$

The molecular mass of a molecule, thus, represents the number of times it is heavier than 1/12 of the mass of an atom of carbon-12 isotope.

- ◆ **Gram Molecular Mass:** The molecular mass of a substance expressed in grams is called the Gram Molecular Mass of the substance.

$$\text{The number of gram molecules} = \frac{\text{Mass of the substance in grams}}{\text{Gram molecular mass of the substance}}$$

e.g.:

(i) Molecular mass of hydrogen (H_2) = 2u.

∴ Gram Molecular Mass of hydrogen (H_2) = 2 g.

(ii) Molecular mass of methane (CH_4) = 16u

∴ Gram Molecular Mass of methane (CH_4) = 16 g.

Note: The molecular mass of a substance is the sum of the atomic masses of its constituent atoms present in a molecule.

Ex.10 Calculate the molecular mass of water.

(Atomic masses: H = 1u, O = 16u).

Sol. The molecular formula of water is H_2O .

∴ Molecular mass of water = $(2 \times \text{atomic mass of H}) + (1 \times \text{atomic mass of O}) = 2 \times 1 + 1 \times 16 = 18$

i.e., molecular mass of water = 18 amu.

Ex.11 Find out the molecular mass of sulphuric acid. (Atomic mass: H = 1u, O = 16u, S = 32u).

Sol. The molecular formula of sulphuric acid is H_2SO_4

∴ Molecular mass of H_2SO_4 = $(2 \times \text{atomic mass of H}) + (1 \times \text{atomic mass of S}) + (4 \times \text{atomic mass of O})$

$$= (2 \times 1) + (1 \times 32) + (4 \times 16) = 2 + 32 + 64 = 98$$

i.e., Molecular mass of H_2SO_4 = 98 amu.

◆ Formula Mass

The term 'formula mass' is used for ionic compounds and others where discrete molecules do not exist, e.g.: sodium chloride, which is best represented as $(\text{Na}^+\text{Cl}^-)_n$, but for reasons of simplicity as NaCl or Na^+Cl^- . Here, formula mass means the sum of the masses of all the species in the formula.

Thus, the formula mass of sodium chloride = (atomic mass of sodium) + (atomic mass of chlorine)

$$= 23 + 35.5$$

$$= 58.5 \text{ amu}$$

Mole Concept

Mole in Latin means heap or collection or pile. A mole of atoms is a collection of atoms whose total mass is the number of grams equal to the atomic mass in magnitude. Since an equal number of moles of different elements contain an equal number of atoms, it becomes convenient to express the amounts of the elements in terms of moles. A mole represents a definite number of particles, viz, atoms, molecules, ions or electrons. This definite number is called the Avogadro Number (now called the Avogadro constant) which is equal to 6.023×10^{23} .

A mole is defined as the amount of a substance that contains as many atoms, molecules, ions, electrons or other elementary particles as there are atoms in exactly 12 g of carbon -12 (^{12}C).

♦ Moles of Atoms:

- (i) 1 mole atoms of any element occupy a mass which is equal to the Gram Atomic Mass of that element.
e.g.: 1 Mole of oxygen atoms weigh equal to Gram Atomic Mass of oxygen, i.e. 16 grams.
- (ii) The symbol of an element represents 6.023×10^{23} atoms (1 mole of atoms) of that element.
e.g.: Symbol N represents 1 mole of nitrogen atoms and 2N represents 2 moles of nitrogen atoms.

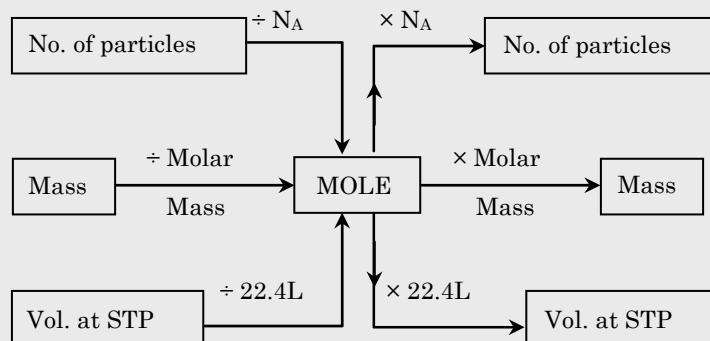
♦ Moles of Molecules:

- (i) 1 mole molecules of any substance occupy a mass which is equal to the Gram Molecular Mass of that substance.
e.g.: 1 mole of water (H_2O) molecules weigh equal to Gram Molecular Mass of water (H_2O), i.e. 18 grams.
- (ii) The symbol of a compound represents 6.023×10^{23} molecules (1 mole of molecules) of that compound.
e.g.: Symbol H_2O represents 1 mole of water molecules and 2 H_2O represents 2 moles of water molecules.

◆ Some important relations & Formulae

- 1 mole of atoms = Gram Atomic mass = mass of 6.023×10^{23} atoms
- 1 mole of molecules = Gram Molecular Mass = 6.023×10^{23} molecules
- Number of moles of atoms = $\frac{\text{Mass of element in grams}}{\text{Gram Atomic Mass of element}}$
- Number of moles of molecules = $\frac{\text{Mass of substance in grams}}{\text{Gram Molecular Mass of substance}}$
- Number of moles of molecules = $\frac{\text{No. of molecules of element}}{\text{Avogadro number}} = \frac{N}{N_A}$

- Number of moles of atoms = $\frac{\text{No. of atoms of an element}}{\text{Avogadro number}} = \frac{N}{N_A}$
- Number of atoms = No. of molecules $\times$ atomicity



Ex.12 To calculate the number of moles in 16 grams of Sulphur (Atomic mass of Sulphur = 32 u).

Sol. 1 mole of atoms = Gram Atomic Mass.

So, 1 mole of Sulphur atoms = Gram Atomic Mass of Sulphur = 32 grams.

Now, 32 grams of Sulphur = 1 mole of Sulphur

So, 16 grams of Sulphur = $(1/32) \times 16 = 0.5$ moles

Thus, 16 grams of Sulphur constitute 0.5 mole of Sulphur.

Ex.13 Which has more number of atoms, 100 grams of sodium or 100 grams of iron. Given that atomic mass of Na = 23 u, Fe = 56 u.

Sol. In case of atoms of elements, number of atoms depend upon the number of moles of the elements. Greater the number of moles, greater is the number of atoms.

$$\text{Moles of sodium} = \frac{\text{Mass in gram}}{\text{Atomic mass}} = \frac{100}{23} = 4.35$$

$$\text{Moles of iron} = \frac{\text{Mass in gram}}{\text{Atomic mass}} = \frac{100}{56} = 1.78$$

Therefore, number of particles in 100 gram sodium are more than 100 gram iron.

If we want to calculate the exact number of atoms, then multiply the number of moles by 6.022×10^{23} .

$$\begin{aligned} \text{Number of sodium atoms} &= 4.35 \times 6.022 \times 10^{23} \\ &= 26.196 \times 10^{23} \end{aligned}$$

$$\begin{aligned} \text{Number of iron atoms} &= 1.78 \times 6.022 \times 10^{23} \\ &= 10.72 \times 10^{23} \end{aligned}$$

Therefore, number of atoms of sodium are more than those in iron.

Ex.14 Calculate the number of moles, molecules, number of H-atoms, number of S-atoms and number of O-atoms in 49 gram H_2SO_4 .

Sol. In such questions, number of moles are calculated first. Moles on multiplication with 6.022×10^{23} give the number of molecules. Then see how many atoms of the given element are present in one molecule of the compound? Then calculate the number of atoms.

Mass of H_2SO_4 (m) = 49 gram,

Molar mass of H_2SO_4 (M) = 98 gram.

$$\text{Moles of } \text{H}_2\text{SO}_4 = \frac{\text{Mass in gram (m)}}{\text{Molar mass (M)}} = \frac{49}{98} = 0.5$$

$$\begin{aligned}\text{Number of molecules of } \text{H}_2\text{SO}_4 &= \text{Moles} \times 6.022 \times 10^{23} \\ &= 0.5 \times 6.022 \times 10^{23} \\ &= 3.011 \times 10^{23}\end{aligned}$$

Now one molecules of H_2SO_4 contains: Two H-atom, One S-atom, Four O-atoms

$$\begin{aligned}\text{Number of H-atoms} &= 2 \times \text{number of molecules of } \text{H}_2\text{SO}_4 \\ &= 2 \times 3.011 \times 10^{23} \\ &= 6.022 \times 10^{23}\end{aligned}$$

$$\begin{aligned}\text{Number of S-atoms} &= 1 \times \text{Number of molecules of } \text{H}_2\text{SO}_4 \\ &= 1 \times 3.011 \times 10^{23} \\ &= 3.011 \times 10^{23}\end{aligned}$$

$$\begin{aligned}\text{Number of O-atoms} &= 4 \times \text{Number of molecules of } \text{H}_2\text{SO}_4 \\ &= 4 \times 3.011 \times 10^{23} \\ &= 12.044 \times 10^{23}\end{aligned}$$

Ex.15 What is the mass of

- (a) One mole of nitrogen atoms.
 - (b) 4 moles of aluminium atoms.
 - (c) 10 moles of sodium sulphite (Na_2SO_3).
- (Given that N = 14, Al = 27, Na = 23, S = 32, O = 16)

Sol. (a) Mass (in gram) of N-atoms = Moles $\times$ At. mass

$$= 1 \times 14 = 14 \text{ g}$$

(b) Mass (in gram) of Al-atoms = Moles $\times$ At.mass

$$= 4 \times 27 = 108 \text{ g}$$

(c) Molar mass of $\text{Na}_2\text{SO}_3 = (2 \times 23) + (1 \times 32) + (3 \times 16)$

$$= 126 \text{ g}$$

Mass of Na_2SO_3 = Moles $\times$ Molar mass

$$= 10 \times 126 = 1260 \text{ g}$$

Ex.16 An element contains 9.033×10^{24} atoms. How many moles does it contain?

Sol. Number of moles = $\frac{\text{Number of particles}}{6.022 \times 10^{23}}$

$$= \frac{9.033 \times 10^{24}}{6.022 \times 10^{23}} = 15 \text{ mole}$$

Ex.17 Calculate the mass of the following:

- (i) 1 atom of sodium
- (ii) 10 molecules of argon
- (iii) 1 molecule of CO_2
- (iv) 1 molecule of H_2SO_4

Sol. (i) The Gram-atomic mass of sodium = 23 g = 6.022×10^{23} sodium atoms.

$$\therefore \text{Mass of one atom of sodium} = \frac{23}{6.022 \times 10^{23}} = 3.819 \times 10^{-23} \text{ g}$$

(ii) Argon is monoatomic. Therefore, its one molecule contains only one atom.

Hence,

Molar mass of argon = 40 g mol^{-1} = 6.022×10^{23} molecules of argon.

$\therefore$ Mass of 6.022×10^{23} molecules of argon = 40 g.

$$\therefore \text{Mass of 10 molecules of argon} = \frac{10 \times 40}{6.022 \times 10^{23}} = 6.642 \times 10^{-22} \text{ g}$$

(iii) Molar mass of CO_2 = 44 g = 6.022×10^{23} molecules CO_2 .

$$\therefore \text{Mass of one molecule of } \text{CO}_2 = \frac{44}{6.022 \times 10^{23}} = 7.307 \times 10^{-23} \text{ g.}$$

(iv) Molar mass of H_2SO_4 = 98 g = 6.022×10^{23} molecules of H_2SO_4 .

$$\therefore \text{mass of 1 molecule of } \text{H}_2\text{SO}_4 = \frac{98}{6.022 \times 10^{23}} = 16.27 \times 10^{-23} \text{ g}$$

Ex.18 Calculate the number of mole and number of molecules in 4.4 g of CO_2 .

Sol. The molecular mass of CO_2 = $12 + (2 \times 16) = 44\text{u}$. Hence, the molar mass of CO_2 = 44 g

$$\therefore \text{Number of mole} = \frac{\text{Mass in grams}}{\text{molar mass}}$$

$$\therefore \text{Number of mole contained in 4.4 g of } \text{CO}_2 = \frac{4.4}{44} = 0.10$$

$\therefore$ One mole of CO_2 has 6.022×10^{23} molecules.

$\therefore$ Number of molecules in 0.10 mole of CO_2 = $0.10 \times 6.022 \times 10^{23} = 6.022 \times 10^{22}$ g

$$\bullet \text{ Number of moles} = \frac{\text{Given volume}}{\text{molar volume}}$$

(Molar volume = 22.4L,)

Ex.19 Calculate the mass of 60 mL of chlorine at S.T.P. (Atomic mass of Cl = 35.5 u)

Sol. Molecular mass of chlorine (Cl_2) = $35.5 \times 2 = 71 \text{ u}$

Molar mass of Cl_2 = 71 g mol^{-1}

1 mole of chlorine = 71 g = 22.4 L at S.T.P.

Therefore,

$\therefore$ Mass of 22400 mL Cl_2 at S.T.P. = 71 g

$$\therefore \text{Mass of 60 mL of Cl}_2 \text{ at S.T.P.} = \frac{71}{22400} \times 60 = 0.19 \text{ g}$$

Ex.20 What volume will be occupied by 4.4 g of CO₂ gas at S.T.P.? (Given: C = 12, O = 16)

Sol. Molecular mass of CO₂ = 12 + (16 × 2) = 44 u.

1 mole of CO₂ = 44 g mol⁻¹ = 22.4 L at S.T.P.

$\therefore$ 44 g of CO₂ at S.T.P. occupy a volume = 22.4 L

$$\therefore 4.4 \text{ g of CO}_2 \text{ at S.T.P. will occupy a volume} = \frac{22.4}{44} \times 4.4 = 2.24 \text{ L}$$

The various relationships involving mole are shown in figure.

Ex.21 Calculate the number of molecules present in

(i) 100g of methane (CH₄), and

(ii) 10 L of hydrogen at S.T.P.

(Given: Atomic mass of C = 12, H = 1)

Sol. (i) Molecular mass of CH₄ = 12 + (1 × 4) = 16 u

1 mole of CH₄ = 16 g CH₄ = 6.022 × 10²³ molecules of CH₄

Therefore,

$\therefore$ Number of molecules in 16 g of CH₄ = 6.022 × 10²³

$$\therefore \text{Number of molecules in 100 g of CH}_4 = \frac{6.022 \times 10^{23}}{16} \times 100 = 3.76 \times 10^{24}$$

$$(ii) \therefore \text{Number of molecules present in 10 L of H}_2 \text{ at S.T.P.} = \frac{6.022 \times 10^{23}}{22.4} \times 10 = 2.69 \times 10^{23}$$

Percentage Composition

The percentage composition of elements in a compound is calculated from the molecular formula of the compound. The molecular mass of the compound is calculated from the atomic masses of the various elements present in the compound. The percentage by mass of each element is then computed with the help of the following relations.

$$\text{Percentage mass of the element in the compound} = \frac{\text{Total mass of the element}}{\text{Molecular mass}} \times 100$$

Ex.22 What is the percentage of calcium in calcium carbonate (CaCO₃)?

Sol. Molecular mass of CaCO₃ = 40 + 12 + 3 × 16 = 100 amu.

Mass of calcium in 1 mol of CaCO₃ = 40g.

$$\therefore \text{Percentage of calcium} = \frac{40 \times 100}{100} = 40 \%$$

Determination of Empirical & Molecular Formula

The chemical formula of the compound that can be calculated from the element analysis data (percentage by mass of each element in the compound) is called the empirical formula.

Empirical formula represents the simplest whole number ratio of the atoms of different constituent elements present in one molecule of the compound.

◆ Steps taken to arrive at the empirical formula

- Determine the percentage of each element: Percentage of each element is determined quantitatively.
- Divide the percentage of each element by its atomic mass to obtain the relative numbers of atoms (or atomic ratio).

$$\text{Atomic ratio} = \frac{\text{Percentage of an element}}{\text{Atomic mass of the same element}}$$

- Divide the atomic ratio by smallest quotient to get the simplest ratio of various elements.
- Convert the simplest ratio to the whole number ratio either by,
 - Rounding them off to whole number, if the value is quite close to whole number e.g.: 1.98, 2.99, 3.95 are rounded off as 2, 3 and 4 respectively.
 - Multiply the figures by suitable integer (2, 3 or 4 etc.)
- Write the empirical formula of the compound by writing the symbols of the various elements side by side. Now insert the whole number ratio of each element as the subscripts to the lower right hand corner of each symbol.

Ex.23 An inorganic salt gave the following percentage composition Na = 29.11, S = 40.51 and O = 30.38. Calculate the empirical formula of the salt.

Sol. Calculation of empirical formula:

Element	Symbol	Percentage of element	At. mass of elements	Moles of the element = Percentage At. mass (Relative no. of moles)	Simplest molal ratio	Simplest whole no. molar ratio
Sodium	Na	29.11	23	$\frac{29.11}{23} = 1.266$	$\frac{1.266}{1.266} = 1$	2
Sulphur	S	40.51	32	$\frac{40.51}{32} = 1.266$	$\frac{1.266}{1.266} = 1$	2
Oxygen	O	30.38	16	$\frac{30.38}{16} = 1.897$	$\frac{1.89}{1.266} = 1$	3

Thus, the Empirical formula is $Na_2S_2O_3$.

◆ Molecular Formula

It is that formula of the compound which gives the actual number of atoms of various elements in a molecules of that compound.

For example, molecular formula of glucose is $C_6H_{12}O_6$. Molecular formula of a compound is related to empirical formula as being a simple whole number multiple of empirical formula. The molecular and empirical formula of any chemical compound are related as a $n \times \text{Empirical formula} = \text{Molecular formula}$, where 'n' is positive integer.

$$n = \frac{\text{Molecular mass of compound}}{\text{Empirical formula mass}}$$

e.g.: Let us consider hydrogen peroxide whose empirical formula is HO and its molecular formula mass is 34.

$$n = \frac{\text{Molecular formula mass of hydrogen peroxide}}{\text{Empirical formula mass of hydrogen peroxide}} = \frac{34}{1+16} = \frac{34}{17} = 2$$

Thus, the molecular formula of hydrogen peroxide = (empirical formula)_n = (HO)₂ = H₂O₂.

When n = 1.

Molecular formula = Empirical formula

The molecular mass of a volatile compound can be determined by Victor Meyer's method (based on the principle that 22.4 L of vapours of a substance at STP have mass equal to gram molecular mass) or by employing the relation.

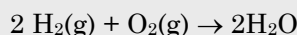
Molecular mass = 2 × Vapour density.

Limiting Reagent

In many situations, an excess of one or more substance is available for chemical reaction. Some of these excess substances will therefore be left over when the reaction is complete; the reaction stops immediately as soon as one of the reactant is totally consumed.

The substance that is totally consumed in a reaction is called limiting reagent because it determines or limits the amount of product. The other reactant present in excess are called as excess reagents.

Let us consider a chemical reaction which is initiated by passing a spark through a reaction vessel containing 10 mole of H₂ and 8 mole of O₂.



The reaction stops only after consumption of 5 moles of O₂ as no further amount of H₂ is left to react with unreacted O₂. Thus H₂ is a limiting reagent in this reaction.

Ex.24 8 litre of H₂ and 6 litre of Cl₂ are allowed to react to maximum possible extent. Find out the final volume of reaction mixture. Suppose P and T remains constant throughout the course of reaction.

Sol.

H ₂	+	Cl ₂	→	2HCl
Volume before		8 litre		6 litre
reaction				
Volume after		2		0
reaction				12 litre

here, Cl₂ is a limiting reagent

hence

Volume after reaction = Volume of H₂ left + volume of HCl formed = 2 + 12 = 14 litre.

Ex.25 10 g N₂ is reacting with 20 g of H₂ to form ammonia then find limiting reagent in this reaction is also find the maximum amount of NH₃ formed.

Sol. N₂ + 3H₂ → 2NH₃

$$\text{no. of moles of N}_2 (n_{\text{N}_2}) = \frac{10}{28} = 0.357 \text{ mole}$$

$$\text{no. of moles of H}_2 (n_{\text{H}_2}) = \frac{20}{2} = 10 \text{ mole}$$

∴ 1 mole if N₂ is reacting with 3 mole of H₂.

$$\begin{aligned} \therefore 0.357 \text{ mole of N}_2 \text{ would react} &= 3 \times 0.357 \\ &= 1.078 \text{ mole of H}_2 \end{aligned}$$

Hence N₂ is L.R.

Formation of ammonia will depend on L.R.

$\therefore$ 1 mole of N_2 forming 2 mole of NH_3

$\therefore$ 0.357 mole of N_2 would form $= 2 \times 0.357$
 $= .714$ mole of NH_3

Wt. of $\text{NH}_3 = 0.714 \times 17 = 12.138 \text{ g}$

EXERCISE-1

➤ Very Short Answer Type Questions

- Q.1** Name of the building block of all matter.
- Q.2** What is the molecular mass of H_2SO_4 ?
- Q.3** Which of the following is tetraatomic molecule CH_3OH , CH_4 , H_2O_2 ?
- Q.4** "If 100 grams of calcium carbonate are decomposed completely, then 56 grams of calcium oxide and 44 grams of carbon dioxide are obtained" Which law of chemical combination is illustrated by this statement?
- Q.5** An element X of valency 3 combines with another element Y of valency 2. What will be the formula of the compound formed?

➤ Short Answer Type Questions – Type I

- Q.6** Name two elements whose symbols are derived from Latin names. Give their symbols.
- Q.7** Calculate the number of moles of phosphorus molecule in 12.044×10^{25} atoms of phosphorus.
- Q.8** An element B shows valencies of 4 and 6. Write the formulae of its two oxides.
- Q.9** What do the symbols, H_2 , S and O_4 mean in the formula H_2SO_4 ?
- Q.10** Give two examples of trivalent metal ions.

➤ Short Answer Type Questions – Type II

- Q.11** What is the ratio by mass of nitrogen and hydrogen in ammonia?
- Q.12** Define 'formula unit mass' of an ionic compound. What is the formula unit of
(i) sodium chloride
(ii) magnesium chloride
- Q.13** The mass of one atom of an element X is 2.0×10^{-23} g.
(i) Calculate the atomic mass of element X.
(ii) What could element X be?
- Q.14** The mass of one molecule of a substance is 4.65×10^{-23} g.
(i) What is its molecular mass?
(ii) What could this substance be?

- Q.15** What weight of oxygen gas will contain the same number of molecules as 56 g of nitrogen gas? (O = 16 u ; N = 14 u)
- Q.16** Define mole with example. What is meant by 1 mole of carbon atoms?
- Q.17** (i) What is an ion? How is an ion formed?
(ii) What is the difference between a cation and an anion? Explain with examples.
- Q.18** Explain the difference between 2N and N_2 .
- Q.19** Which of the following has more number of atoms and by how much?
(i) A mole of ammonia (NH_3)
(ii) A mole of methane (CH_4)
- Q.20** Write the chemical formulae of the following
(i) Aluminium hydroxide
(ii) Ammonium sulphate
(iii) Potassium carbonate

➤ Long Answer Type Questions

- Q.21** Give the main postulates of Dalton's atomic theory. Which postulate of Dalton's atomic theory explain the law of conservation of mass.
- Q.22** Calculate the following:
(i) Number of S atoms in 3.2 g of S.
(ii) Number of molecules of CH_4 in 80.0 g of it
(iii) The mass of 1 molecule of NH_3 .
(iv) The mass of 0.25 moles of calcium
(v) Number of bromide ion in 0.2 mole of MgBr_2 .
- Q.23** Calculate the mass of
(a) 0.2 mol of O_2
(b) One atom of aluminium
(c) 3.0 mole of chloride ion
(d) 2.5 mol of NaCl
- Q.24** Write the formula and names of compounds formed by
(a) Na^+ and HCO_3^- (b) K^+ and CO_3^{2-}
(c) Cu^{2+} and SO_4^{2-} (d) Cu^{2+} and O^{2-}
(e) Na^+ and SO_4^{2-} (f) NH_4^+ and CO_3^{2-}
- Q.25** Calculate the number of moles of phosphorus (P) atoms in 155 g of phosphorus. If phosphorus is considered to contain P_4 molecules, then how many moles of P_4 molecules are there?

EXERCISE-2

- Q.1** Atomic radius of atoms is measured in nanometers (nm). One nanometer is equal to -
 (A) 10^{-10} metre (B) 10^{-9} metre
 (C) 10^{-10} cm (D) 10^{-9} cm
- Q.2** Symbol of which element has been derived from its Latin name?
 (A) Sodium (B) Iron
 (C) Copper (D) All
- Q.3** Symbol of which element has been derived from its English name?
 (A) Hydrogen (B) Carbon
 (C) Aluminium (D) All
- Q.4** One unified mass (u) is equal to -
 (A) 1.66×10^{-24} g (B) 1.66×10^{-23} g
 (C) 1.66×10^{-24} kg (D) 1.66×10^{-23} kg
- Q.5** Polyatomic ion has -
 (A) More than one unit negative charge
 (B) More than one unit positive charge
 (C) Either more than one unit negative charge or positive charge
 (D) Two or more atoms having charge
- Q.6** Which of the following are monovalent ions?
 (A) Chloride and sulphate
 (B) Nitrite and nitrate
 (C) Carbonate and sulphide
 (D) Oxide and bromide
- Q.7** Which of the following are divalent ions?
 (A) Carbonate and sulphide
 (B) Sulphide and oxide
 (C) Sulphate and cupric
 (D) All
- Q.8** Which of the following are trivalent ions?
 (A) Cupric and ferric
 (B) Mercuric and phosphate
 (C) Ferric and phosphate
 (D) Aluminium and sulphate
- Q.9** Correct name is -
 (A) CO is named as carbon monoxide
 (B) PCl_3 is named as phosphorus chloride
 (C) N_2O_5 is named as dinitrogen oxide
 (D) HCl is named as hydrogen monochloride
- Q.10** Incorrect name is -
 (A) PCl_5 is named as phosphorus pentachloride
 (B) N_2O_4 is named as dinitrogen tetraoxide
 (C) H_2S is named as dihydrogen sulphide
 (D) HCl is named as hydrogen chloride
- Q.11** Correct formula is -
 (A) $\text{Al}_2(\text{SO}_4)_3$ and CaCO_3
 (B) Ca_3PO_4 and NaCl
 (C) $\text{Ca}(\text{HCO}_3)_2$ and KSO_4
 (D) CaCl_2 and NaCO_3
- Q.12** Incorrect formula is -
 (A) $\text{Al}_2(\text{CO}_3)_3$ and CaCl_2
 (B) Na_2NO_3 and FeCl
 (C) FeSO_4 and CuCl
 (D) HgCl_2 and FeCl_3
- Q.13** Incorrect statement is -
 (A) One mole = 6.022×10^{23} particles
 (B) Gram atomic mass = mass of 6.022×10^{23} particles in gram
 (C) One mole of H_2SO_4 has 6.022×10^{23} atoms
 (D) Mass of 6.022×10^{23} molecules of ammonia is 17 gram
- Q.14** Oxide and sulphide ions are respectively -
 (A) Both are divalent
 (B) Univalent and divalent
 (C) Divalent and univalent
 (D) Both are trivalent
- Q.15** True statements is -
 (A) Cation has more electrons than its neutral atom.
 (B) Anion has less electrons than its neutral atom
 (C) Cation and anion of the same element contain equal number of electrons
 (D) Cation has less electrons while anion has more electrons than their neutral atoms
- Q.16** Which of the following would weigh the highest?
 (A) 0.2 mole of sucrose ($\text{C}_{12}\text{H}_{22}\text{O}_{11}$)
 (B) 2 moles of CO_2
 (C) 2 moles of CaCO_3
 (D) 10 moles of H_2O

- Q.17** Which of the following has maximum number of atoms?
 (A) 18 g of H_2O (B) 18 g of O_2
 (C) 18 g of CO_2 (D) 18 g of CH_4
- Q.18** Which of the following contains maximum number of molecules?
 (A) 1 g CO_2 (B) 1 g N_2
 (C) 1 g H_2 (D) 1 g CH_4
- Q.19** Mass of one atom of oxygen is –
 (A) $\frac{16}{6.023 \times 10^{23}}$ g (B) $\frac{32}{6.023 \times 10^{23}}$ g
 (C) $\frac{1}{6.023 \times 10^{23}}$ g (D) 8 u
- Q.20** A change in the physical state can be brought about –
 (A) only when energy is given to the system
 (B) only when energy is taken out from the system
 (C) when energy is either given to or taken out from the system
 (D) without any energy change
- Q.21** The law of chemical combination that is applied when two elements combine to form two or more compounds –
 (A) Law of conservation of mass
 (B) Law of definite proportion
 (C) Law of multiple proportion
 (D) all the three laws (a, b and c)
- Q.22** The law of conservation of mass is seen in –
 (A) an unbalanced chemical equation
 (B) a balanced chemical equation
 (C) both (A) and (B)
 (D) none of these
- Q.23** The law of multiple proportion can be applied to which compounds out of NO_2 , CO_2 , SO_2 , SO_3 , N_2O
 (i) CO_2 , SO_2 (ii) CO_2 , SO_3
 (iii) SO_3 , H_2O (iv) SO_2 , SO_3
 (v) NO_2 , N_2O
 (A) i and ii (B) ii and iii
 (C) iii and iv (D) iv and v
- Q.24** Matter can neither be created nor destroyed in a chemical reaction. The statement is in accordance with the –
 (A) Law of Constant Proportions
 (B) Law of Conservation of Mass
 (C) Law of Multiple Proportions
 (D) none of these
- Q.25** If the ratio of weights of oxygen in two oxides of an element, that combines with a fixed weight of the element is 1: 2, then the ratio of weights of the element that combines with a fixed weight of oxygen is –
 (A) 1: 2 (B) 2: 1 (C) 4: 1 (D) 1: 4
- Q.26** The molecular formula of ethanoic acid is $\text{C}_2\text{H}_4\text{O}_2$. Its empirical formula is –
 (A) $\text{C}_4\text{H}_8\text{O}_2$ (B) CH_2O
 (C) CHO (D) CHO_2
- Q.27** Chemical analysis of a carbon compound gave the following percentage composition by weight of the elements present in it. Carbon = 10.06 %, hydrogen = 0.84 %, chlorine = 89.10 %. Calculate the empirical formula of the compound –
 (A) $\text{C}_2\text{H}_2\text{Cl}_2$ (B) CHCl_2
 (C) CHCl_3 (D) $\text{C}_4\text{H}_4\text{Cl}_4$
- Q.28** Percentage of carbon is highest in –
 (A) CH_4 (B) C_3H_4 (C) C_6H_6 (D) C_3H_8
- Q.29** 1 mole of SO_2 and 1 mole of H_2S react completely to form
 $\text{SO}_2 + 2\text{H}_2\text{S} \rightarrow 2\text{H}_2\text{O} + 3\text{S}$ ($\text{S} = 32$, $\text{O} = 16$)
 (A) 96 g S (B) 48 g S
 (C) 24 g S (D) 64 g S
- Q.30** $2\text{H}_2 + \text{O}_2 \rightarrow 2\text{H}_2\text{O}$
 2g H_2 and 1g O_2 reacts to form H_2O :
 (A) 3.0 g (B) 1.125 g
 (C) 4.5 g (D) 2.50 g

EXERCISE-3

(Previous Year Questions - NTSE & NSO)

- Q.1** In a chemical reaction one molecule of hydrogen sulphide gas reacts with two molecules of nitric acid. The molecules of nitrogen dioxide and water and atoms of sulphur formed has a ratio of -
(A) 1: 2: 2 (B) 2: 1: 2
(C) 2: 2: 1 (D) 1: 1: 1
- Q.2** Molecular formula of a compound of metal M (equivalent weight 8) is M_2CO_3 . Atomic weight of the metal will be -
(A) 24 (B) 16 (C) 8 (D) 4
- Q.3** The quantity of potassium chlorate (molecular weight 122.5) required in grams to produce 33.6 litre of oxygen at normal temperature and pressure will be -
(A) 122.5 (B) 132.6
(C) 245.0 (D) 254.0
- Q.4** The group of symbols of elements based on Latin names out of the following is -
(A) Ca, Ag, Cu
(B) Ag, Au, K
(C) Pb, Cl, Cd
(D) Cs, Fe, Na
- Q.5** The molecular formula of the compound formed by the combination of elements ${}_{13}A^{27}$ and ${}_{8}B^{16}$ is:
(A) AB (B) A_2B (C) AB_2 (D) A_2B_3
- Q.6** 1.204×10^{24} molecules are present in a sample of methane. What will be its weight (in grams)?
(A) 32 (B) 24 (C) 16 (D) 12
- Q.7** $Na_2CO_3 + 2HCl \longrightarrow 2NaCl + CO_2 + H_2O$
According to the above reaction, how many grams of Na_2CO_3 will be needed to obtain 11.2 litre CO_2 ?
(A) 51 (B) 53 (C) 83 (D) 106
- Q.8** The empirical formula of a compound is CH_2 . If its vapour density be 21, the molecular formula of this compound will be -
(A) C_2H_4 (B) C_3H_6
(C) C_4H_8 (D) C_5H_{10}
- Q.9** A gas X is five times heavier than hydrogen gas while the other gas Y is two times heavier than X. The molecular weight of Y will be -
(A) 2.5 (B) 5
(C) 20 (D) 10
- Q.10** Which one of the following is cation ?
(A) Na^+
(B) Cl^-
(C) H_2
(D) None of the above
- Q.11** One Litre of a gas at STP has mass equal to 1.25 g. The gas is -
(A) H_2 (B) O_2
(C) N_2 (D) CO_2
- Q.12** 2 litre of hydrogen and 1.2 litre of chlorine are mixed and exploded. The composition by volume of the resultant mixture will be -
(A) 0.8 litre of hydrogen and 2.4 litre of chlorine
(B) 0.8 litre of hydrogen and 2.4 litre of hydrogen chloride
(C) 2.4 litre of hydrogen chloride
(D) 0.8 litre of chlorine and 2.4 litre of hydrogen chloride
- Q.13** When 2 moles of N_2 gas and 9 moles of H_2 gas are mixed and reaction is completed to form NH_3 gas then reaction mixture will contain:
(A) 11 moles of NH_3
(B) 4 moles of NH_3 + 3 moles of H_2
(C) 6 moles of NH_3 + 3 moles of H_2
(D) 2 moles of NH_3 + 1 mole of N_2

- Q.14** What mass of hydrogen and oxygen will be produced on complete electrolysis of 18g of water?
 (A) 2g hydrogen and 32g oxygen
 (B) 2g hydrogen and 16g oxygen
 (C) 4g hydrogen and 32g oxygen
 (D) 4g hydrogen and 14g oxygen
- Q.15** Number of molecules in 14 g of carbon monoxide is –
 (A) 12.044×10^{23}
 (B) 6.022×10^{23}
 (C) 3.011×10^{23}
 (D) 1.5050×10^{23}
- Q.16** Which of the following molecule has an atomicity of four:
 (A) H_2O (B) NH_3 (C) CH_4 (D) CO_2
- Q.17** One mole of CO_2 means:
 (A) 4.4 gm CO_2
 (B) 2.24 litres gas at STP
 (C) 6.022×10^{23} molecules of CO_2
 (D) 22 gm CO_2
- Q.18** Total number of atoms in 4 gms of oxygen molecule is -
 (A) 6.022×10^{23} (B) 7.52×10^{22}
 (C) 1.5055×10^{23} (D) 0.0752×10^{23}
- Q.19** Number of molecules present in 14 gm of N_2 molecule is:
 (A) 6.022×10^{23} (B) 3.011×10^{23}
 (C) 1.51×10^{23} (D) 6.022×10^{22}
- Q.20** Formula of aluminium carbonate is:
 (A) $\text{Al}_2(\text{CO}_3)_3$ (B) Al_2CO_3
 (C) Al_2HCO_3 (D) AlCO_3
- Q.21** Number of molecules present in 32g of O_2 is
 (A) 6.022×10^{23} (B) 3.011×10^{23}
 (C) 1.51×10^{23} (D) 6.022×10^{22}
- Q.22** The formula of chloride of an element X is XCl_3 . The formula of its oxide will be -
 (A) XO_2 (B) XO_3
 (C) X_2O_3 (D) X_3O_2
- Q.23** Write chemical formula of Magnesium Chloride
 (A) MgCl_2 (B) CaCl_2
 (C) $\text{Cu}(\text{NO}_3)_2$ (D) CaCO_3

ANSWER KEY

EXERCISE - 1

2. 98 g
7. 50 moles
11. 14 : 3
13. (i) 12.044 g
14. (i) 28 g
15. 64 g
19. (i) 24.088×10^{23} (ii) 30.11×10^{23}
 CH_4 has 6.022×10^{23} more atoms than NH_3
22. (i) 0.075×10^{23} (ii) 30.11×10^{23}
 (iii) 2.82×10^{-23} g (iv) 10 g
 (v) 2.40×10^{23}
23. (i) 6.4 g (ii) 4.48×10^{-23} g
 (iii) 106.5 g (iv) 146.25 g
25. Number of moles of phosphorus atom = 5
 Number of moles of phosphorus molecule = 1.25

EXERCISE - 2

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	D	D	A	D	B	D	C	A	C	A	B	C	A	D
Ques.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	C	D	C	A	C	C	B	D	B	B	B	C	C	B	B

EXERCISE - 3

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	C	A	B	D	A	B	B	C	A	C	B	B	B	C
Ques.	16	17	18	19	20	21	22	23							
Ans.	B	C	C	B	A	A	C	A							

SOLUTIONS

EXERCISE-1

➤ Very Short Answer Type Questions

Sol.1 Atoms.

Sol.2 H_2SO_4
Molar mass = $(2 \times 1) + (1 \times 32) + (4 \times 16)$
= 98 g

Sol.3 H_2O_2
Atomicity is 4.

Sol.4 $\text{CaCO}_3 \longrightarrow \text{CaO} + \text{CO}_2$
100g 56g 44g
Law of conservation of mass.

Sol.5 Low of mass conservation is obeyed
Element X has valency 3
Element Y has valency 2
Formula X_2Y_3 (Criss cross method)

➤ Short Answer Type Questions – Type I

Sol.6 Latin names :
Sodium – Natrium - Na
Potassium – Kalium - K

Sol.7 1 mole of phosphorus molecule,
 $\text{P}_4 = N_A$ molecules
= $4N_A$ atoms = $4 \times 6.022 \times 10^{23}$
Moles of phosphorus molecule in 12.044×10^{25} atoms
= $\frac{12.044 \times 10^{25}}{4 \times 6.022 \times 10^{23}}$
= $\frac{12.044 \times 10^{25}}{24.088 \times 10^{23}} = \frac{1}{2} \times 10^2 = 50$ moles

Sol.8 Valency 4 = BO_2
Valency 6 = BO_3

Sol.9 Two hydrogen atoms (H_2), 1 sulphur atom and 4 oxygen atoms. Combine to form 1 molecule of H_2SO_4 .

Sol.10 Al^{+3} , Fe^{+3} , Mn^{3+} , CO^{3+} , Cr^{3+} .

➤ Short Answer Type Questions – Type II

Sol.11 Ammonia, NH_3
Mass of Nitrogen = 14g
Mass of Hydrogen = 3g.
Mass of nitrogen and hydrogen in ammonia is in the ratio of 14 : 3.

Sol.12 Formula unit mass : Formula mass means the sum of the masses of all the species in the formula.
(i) NaCl $23 + 35.5 = 58.5$ g
(ii) MgCl_2 $24 + 2(35.5) = 24 + 71 = 95$ g

Sol.13 Mass of one atoms $\text{X} = 2 \times 10^{-23}$ g
Mass of 6.023×10^{23} atom = ?
= $\frac{2 \times 10^{-23} \text{ g}}{1 \text{ atom}} \times 6.023 \times 10^{23} \text{ atom}$
= 12.046 g
(i) Atomic mass of X = 12 g
(ii) Element is carbon.

Sol.14 Mass of 1 molecule = 4.65×10^{-23} g
Mass of 6.023×10^{23} molecule
= $\frac{4.65 \times 10^{-23} \text{ g}}{1 \text{ molecule}} \times 6.023 \times 10^{23} = 28$ g
(i) Molar mass of molecule = 28 g
(ii) Nitrogen

Sol.15 Molecules in 56g of $\text{N}_2 = \frac{6.023 \times 10^{23}}{28} \times 56$
= 12.046×10^{23}
 N_A molecule in 32 g O_2
Mass of 12.046×10^{23} molecule in 32 g O_2
= $\frac{12.046 \times 10^{23}}{6.023 \times 10^{23}} \times 32 = 64$ g

Sol.16 Mole : Amount of substance present in 6.023×10^{23} particles of atom or molecule or ion.
1 mole of carbon atom means it contains 6.023×10^{23} atoms of carbon.

Sol.17 (i) An ion is a positively or negatively charged atom (or group of atoms). An ion is formed by the loss or gain of electrons by an atom.

(ii)

Cation	Anion
Positively charged ion where no. of protons are greater than electrons. Eg. Na^+ , Al^{3+}	Negatively charged ion where no. of electrons are greater than no. of protons. Eg. Cl^- , SO_4^{2-}

Sol.18 2N is for 2 nitrogen atom and it contains $2 \times N_A$ no. of atoms.
 N_2 is molecule that contains N_A of molecules of nitrogen.

Sol.19 1 mole of $\text{NH}_3 = 4N_A$ atoms
Atomicity of $\text{NH}_3 \rightarrow 4$
1 mole of $\text{CH}_4 = 5 N_A$ atoms
Atomicity of $\text{CH}_4 \rightarrow 5$
A mole of methane (CH_4) has $N_A (6.022 \times 10^{23})$ more atoms than a mole of ammonia (NH_3).

Sol.20 Aluminium hydronide – $\text{Al}(\text{OH})_3$
Ammonium sulphate – $(\text{NH}_4)_2\text{SO}_4$
Potassium carbonate – K_2CO_3 .

➤ Long Answer Type Questions

Sol.21 The basic postulates of Dalton's theory are as follows:

- Each element is composed of extremely small particles called atoms.
- All atoms of a given element are identical i.e., atoms of a particular element are alike in all aspects.
- Atoms of different elements possess completely different properties and differ in all aspects.
- Atoms are indestructible i.e., atoms are neither created nor destroyed in chemical reactions.
- Atoms of element take part in chemical reaction to form molecules.
- In a given compound, the relative number and kind of atoms are constant.
- Atom of same type of elements combine in different ratio to form more than one type of compounds.

Atoms are indestructible i.e., atoms are neither created nor destroyed in chemical reactions. This postulate of Dalton's atomic theory explain the law of conservation of mass.

Sol.22 (i) Number of S atoms in 3.2 g of S_8
Molar mass = $8 \times 32 = 256 \text{ g}$
256 g contain $8 \times N_A$ atoms
 $3.2 \text{ g contain} = \frac{8 \times 6.023 \times 10^{23}}{256 \text{ g}} \times 3.2 \text{ g}$
 $= 6.023 \times 10^{22}$

(ii) Number of molecules of CH_4 in 80 g of it
16 g contain N_A molecule
 $\therefore 80 \text{ g contain} = \frac{6.023 \times 10^{23}}{16} \times 80$
 $= 30.11 \times 10^{23}$

(iii) The mass of 1 molecule of NH_3
17g contains N_A molecule
 $\therefore 1 \text{ molecule is} = \frac{17}{6.023 \times 10^{23}} \times 1$
 $= 2.8 \times 10^{-23}$

(iv) 1 mole of Ca = 40g
 $\therefore 0.25 \text{ mole of Ca} = 40 \times 0.25 = 10 \text{ g}$

(v) $\text{MgBr}_2 \rightleftharpoons \text{Mg}^{2+} + 2\text{Br}^-$
1 mole of MgBr_2 contain 2 mole of bromide ions
1 mole of Bromide ions = 6.022×10^{23} bromide ion
0.2 mole of MgBr_2 will have 0.4 mole of bromide ion
Bromide ions in 0.2 mole of MgBr_2
 $= 0.4 \times 6.022 \times 10^{23}$
 $= 2.40 \times 10^{23}$ bromide ions

Sol.23 (a) 1 mole of $\text{O}_2 = 32 \text{ g}$
 $0.2 \text{ mol of } \text{O}_2 = \frac{32 \text{ g}}{1 \text{ mole}} \times 0.2 \text{ mole} = 6.4 \text{ g}$
(b) One atom of aluminium
 N_A of Al contain = 27 g
1 atom contain = $\frac{27}{6.023 \times 10^{23}} = 4.5 \times 10^{-23}$
(c) 1 mole of $\text{Cl}^- = 35.5 \text{ g}$
3 mole of $\text{Cl}^- = 3 \times 35.5 = 106.5 \text{ g}$
(d) 1 mole of NaCl weighs 58.5 g
2.5 mole of NaCl = $58.5 \times 2.5 = 146.25 \text{ g}$

Sol.24 (a) NaHCO_3 (b) K_2CO_3
(c) CuSO_4 (d) CuO
(e) Na_2SO_4 (f) $(\text{NH}_4)_2\text{CO}_3$

Sol.25 Molar mass of phosphorus atom, P = 31
Molar mass of phosphorus molecule,
 $\text{P}_4 = 4 \times 31 = 124$
Given, mass of phosphorus = 155 g
 $\therefore$ No of moles phosphorus atom,
 $\text{P} = 155/31 = 5 \text{ moles.}$
 $\therefore$ No of moles phosphorus molecule,
 $\text{P}_4 = 155/124 = 1.25 \text{ moles.}$

EXERCISE-2

Sol.1 [B]

One nm = 10^{-9} m.

Sol.2 [D]

Latin name – Sodium-Natrium,
Iron-Ferrum, Copper-Cuprum.

Sol.3 [D]

English name – Carbon, Hydrogen, Aluminium.

Sol.4 [A]

One unified mass = 1.66×10^{-24} g.

Sol.5 [D]

Polyatomic ion has two or more atoms
having charge.

Sol.6 [B]

NO_2^- & NO_3^- monovalent ions.

Sol.7 [D]

CO_3^{2-} , S^{2-} , O^{2-} , SO_3^{2-} , Cu^{2+} all divalent ions.

Sol.8 [C]

Fe^{3+} & PO_4^{3-} – Trivalent ions.

Sol.9 [A]

$\text{CO} \Rightarrow$ Carbon monoxide.

Sol.10 [C]

$\text{H}_2\text{S} \Rightarrow$ Hydrogen sulphide.

Sol.11 [A]

$\text{Al}_2(\text{SO}_4)_3$ Aluminium sulphate
 CaCO_3 Calcium carbonate.

Sol.12 [B]

Incorrect formulae are Na_2NO_3 & FeCl .
Correct formula NaNO_3 & FeCl_3 .

Sol.13 [C]

One mole of H_2SO_4 contains
 $7 \times 6.022 \times 10^{23}$ atoms.

Sol.14 [A]

Oxide (O^{2-}) & S^{2-} sulphide $\Rightarrow$ divalent.

Sol.15 [D]

True statements is –
Cation has less electrons
Anion has more electrons.

Sol.16 [C]

2 moles of CaCO_3
Molar mass = $40 + 12 + 48 = 100$ g
Mass of 2 moles = $2 \times 100 = 200$ g

Sol.17 [D]

Maximum no. of atoms
18 g of CH_4
16 g contain $5 \times 6.023 \times 10^{23}$ atoms
18 g contain $= \frac{5 \times 6.023 \times 10^{23}}{16} \times 18$
 $= 33.75 \times 10^{23}$ atoms

Sol.18 [C]

Maximum no. of molecules
1 g H_2 2g $\text{H}_2 = N_A$ molecule
 $\therefore 1\text{g H}_2 = \frac{6.022}{2} \times 10^{23}$ molecules
 $= 3.011 \times 10^{23}$ molecules

Sol.19 [A]

Mass of 1 atom of oxygen
 $\frac{16}{6.023 \times 10^{23}} \text{ g} = 2.66 \times 10^{-23}$.

Sol.20 [C]

Energy given or taken causes for change in
physical state of the system.

Sol.21 [C]

According to law of multiple proportion when
two or more elements combines to form more
than two compounds, the various masses of
one element that combine with fixed mass of
other element can bear a simple ratio.

Sol.22 [B]

The law of conservation of mass is seen in
balanced chemical equation, $M_P = M_R$.

Sol.23 [D]

The law of multiple proportion
 SO_2 & SO_3 NO_2 & N_2O
 $\text{S} + \text{O}_2 \rightarrow \text{SO}_2$ $\text{N}_2 + 2\text{O}_2 \rightarrow 2\text{NO}_2$
 $2\text{S} + 3\text{O}_2 \rightarrow 2\text{SO}_3$ $\text{N}_2 + \frac{1}{2}\text{O}_2 \rightarrow \text{N}_2\text{O}$

Sol.24 [B]

The law of conservation of mass matter is neither created nor destroyed.

Sol.26 [B]

Molecular formula of ethanoic acid = $\text{C}_2\text{H}_4\text{O}_2$
Empirical formula = $\text{C}_2\text{H}_4\text{O}_2 = \text{CH}_2\text{O}$

Sol.27 [C]

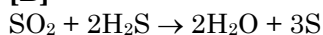
Elements	%	Mole	Mole (simple ratio)	Wholes (ratio mole)
C	10.06	$\frac{10.06}{12}$	0.84	$\frac{0.84}{0.84} = 1$
H	0.84	$\frac{0.84}{1}$	0.84	$\frac{0.84}{0.84} = 1$
Cl	89.10	$\frac{89.1}{35.5}$	2.5	$\frac{2.5}{0.84} = 3$

Empirical formula of compound = CHCl_3 .

Sol.28 [C]

Benzene, C_6H_6

$$\% \text{ carbon} = \frac{72}{78} \times 100 = 92.3\%$$

Sol.29 [B]

1 mole of SO_2 gives 3 moles of sulphur

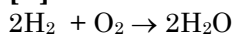
2 mole of H_2S gives 3 mole of sulphur

H_2S is limiting reagent

2 mole $\text{H}_2\text{S} \rightarrow 3$ moles 'S'

$$\therefore 1 \text{ mole } \text{H}_2\text{S} \rightarrow \frac{3}{2} \times 1 = 1.5 \text{ mole}$$

$$1.5 \times 32 = 48 \text{ g}$$

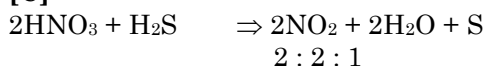
Sol.30 [B]

2g $\text{H}_2 \rightarrow 18\text{g water}$

32g $\text{O}_2 \rightarrow 36\text{g water}$

$$1 \text{ g } \text{O}_2 \rightarrow \frac{36}{32} = \frac{9}{8} = 1.125 \text{ g}$$

EXERCISE-3

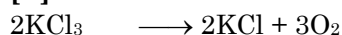
Sol.1 [C]**Sol.2 [C]**

Molecular formula of a compound = M_2CO_3

Equivalent weight = 8

Valency = 1

Atomic weight = 8

Sol.3 [A]

$$122.5 \times 2 \longrightarrow 3 \times 22.4 \text{ litre}$$

$$? \longrightarrow 33.6 \text{ litre}$$

$$\begin{aligned} \text{Mass of } \text{KClO}_3 &= \frac{122.5 \times 2}{3 \times 22.4} \times 33.6 \\ &= \frac{245.0 \times 33.6}{67.2} = \frac{8232}{67.2} \\ &= 122.5 \end{aligned}$$

Sol.4 [B]

Latin name

Ag – Argentum

Au – Aurum

K – Kalium

Sol.5 [D]

$_{13}\text{A}^{27}$ and $_8\text{B}^{16}$

A = Aluminium

B = Oxygen

Valency = 3

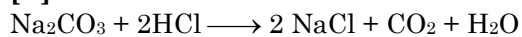
Valency = 2

$$\therefore \text{MF} = \text{Al}_2\text{O}_3 = \text{A}_2\text{B}_3$$

Sol.6 [A]

16 g in 6.023×10^{23} molecules

$$\begin{aligned} 1.204 \times 10^{24} \text{ molecule} &= \frac{1.204 \times 10^{24}}{6.023 \times 10^{23}} \times 16 \\ &= 32 \text{ g} \end{aligned}$$

Sol.7 [B]

106 g of $\text{Na}_2\text{CO}_3 \longrightarrow 44 \text{ g } \text{CO}_2 = 22.4 \text{ litre}$

11.2 litre $\text{CO}_2 \longrightarrow 53 \text{ g}$.

Sol.8 [B]

Molecular mass = $2 \times \text{V.D.}$

$$= 2 \times 21 = 42$$

EF = CH_2

EF mass = 14

n = MF mass/ EF mass

$$= 42/14 = 3$$

$$\text{MF} = 3(\text{EF}) = \text{C}_3\text{H}_6.$$

Sol.9 [C]

Weight of X = $5 \times 2 = 10$

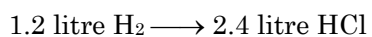
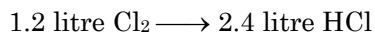
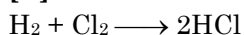
Weight of Y = $2 \times 10 = 20$.

Sol.11 [C]

1 litre $\longrightarrow 1.25 \text{ g}$

$$22.4 \text{ litre} \longrightarrow \frac{1.25}{1} \times 22.4 = 28 \text{ molar mass}$$

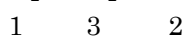
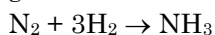
Nitrogen (N_2) has molar mass = 28.

Sol.12 [B]

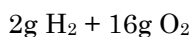
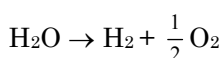
Remaining mixture = 0.8 lit. H_2 & 2.4 lit. HCl .

Sol.13 [B]

When 2 moles of N_2 gas and 9 moles of H_2 gas are mixed and



Remains : 3 mole H_2 + 4 mole NH_3 .

Sol.14 [B]**Sol.15 [C]**

Molar mass of $\text{CO} = 28$

28 g contains 6.023×10^{23} molecules

$\therefore$ 14 g contains 3.011×10^{23} molecules

Sol.16 [B]

NH_3 atoms = 4.

Sol.17 [C]

One mole $\rightarrow$ Avogadro no. of molecules
 6.023×10^{23} molecule.

Sol.18 [C]

Molar mass of $\text{O}_2 = 32$

1 mole of $\text{O}_2 = 6.022 \times 10^{23} \times 2$ atoms

Given 4 g oxygen means $\frac{1}{8}$ mole

$$\begin{aligned} \text{Number of atoms} &= 6.022 \times 10^{23} \times 2 \times \frac{1}{8} \\ &= 1.5055 \times 10^{23} \end{aligned}$$

Sol.19 [B]

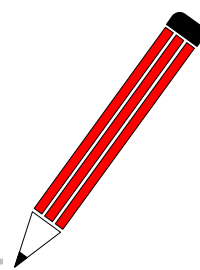
Molar mass of $\text{N}_2 = 28$

14 g contains 3.011×10^{23} molecule.

Sol.20 [A]

Aluminium carbonate, $\text{Al}_2(\text{CO}_3)_3$.

NOTES

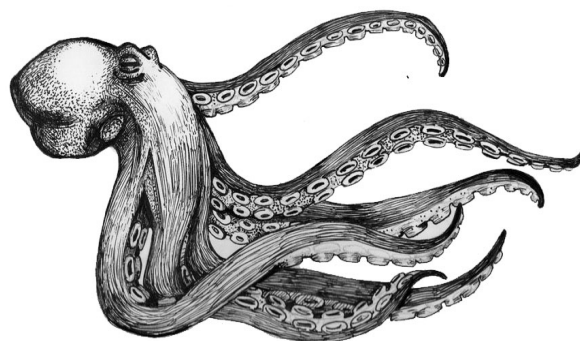


BIOLOGY

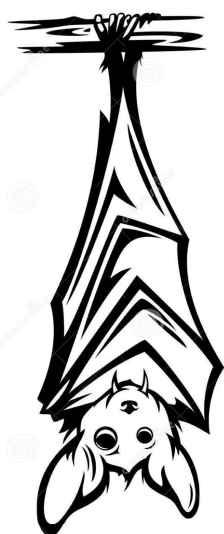
PATTERNS IN LIFE : DIVERSITY AND CLASSIFICATION

Chapter Outline

- ✧ Taxonomy
- ✧ Hierarchy
- ✧ Nomenclature
- ✧ Classification
- ✧ Kingdom Monera
- ✧ Kingdom Protista
- ✧ Kingdom Plantae
- ✧ Kingdom Animalia
- ✧ Viruses
- ✧ Discovery of Viruses
- ✧ Structural Organization of Viruses



Octopus



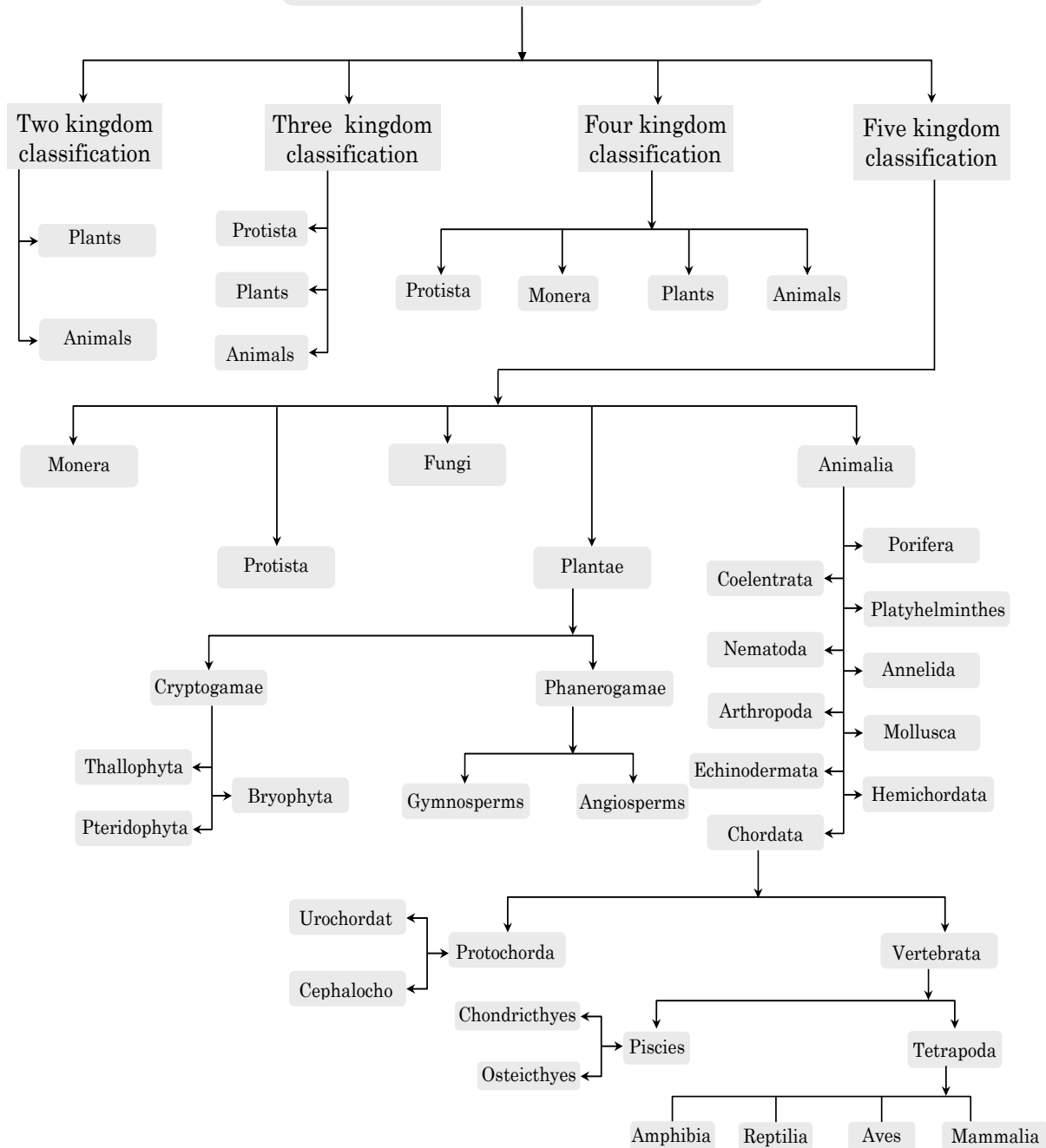
Bat



Panthera Tigris

MIND MAP

PATTERNS IN LIFE DIVERSITY AND CLASSIFICATION



PATTERNS IN LIFE : DIVERSITY AND CLASSIFICATION

Introduction to Biodiversity

The Earth is home to an enormous variety of living organisms. From microscopic bacteria to giant trees and animals, life exists in countless forms and habitats. This great variety of life is known as **biodiversity**.

Biodiversity includes all plants, animals and microorganisms found on Earth along with the ecosystems in which they live. It is essential for maintaining ecological balance and supporting life.

- Every organism performs a specific role in nature.
- Green plants prepare food through photosynthesis and release oxygen into the atmosphere.
- Fungi and bacteria decompose dead plants and animals, converting them into nutrients that enrich the soil.
- Birds, bees and bats help in pollination, while animals help in seed dispersal.

Humans also depend greatly on biodiversity for food, medicines, clothing, shelter and livelihood. Farmers preserve different crop varieties because biodiversity increases resistance against diseases, pests and changing environmental conditions. A rich biodiversity strengthens food security and ecosystem stability.

Thus, all living organisms are interconnected through food chains and ecological relationships.

- ◆ **Biodiversity** : The term “**biodiversity**” is a concise form of “**biological diversity**” and was coined by **Walter G. Rosen** in 1986.



India as a Biodiversity Hotspot

India is one of the richest countries in biodiversity because of its varied geographical features and climatic conditions. The country has mountains in the north, deserts in the west, plateaus in the south, dense forests in the north-east and long coastal regions. These diverse habitats support a large number of plants and animals.

Some species are found naturally only in a particular region and nowhere else in the world. Such species are known as **endemic species**. Examples include:

- Nilgiri tahr
- Lion-tailed macaque
- *Nepenthes khasiana* (pitcher plant)
- Neelakurinji



(a)



(b)



(c)



(d)

Fig. (a) Nilgiri tahr, (b) Lion-tailed macaque,
(c) Nepenthes Khasiana, and (d) Neelakurinji

Regions that contain a large number of endemic species and face serious habitat loss are called **biodiversity hotspots**. These regions are important for conservation because they contain unique organisms that may become extinct if not protected.

Major biodiversity hotspots of India include:

1. Western Ghats
2. Himalayas
3. Indo-Burma region (North-East India)
4. Sundaland (Nicobar Islands)

◆ How has Biodiversity Evolved?

The biodiversity seen today did not appear suddenly. It developed gradually over millions of years through continuous changes in organisms and their surroundings.

Small variations occur naturally among organisms. Some variations help organisms survive better under environmental changes. Organisms possessing favourable characteristics survive and reproduce successfully. These useful variations accumulate over generations and eventually give rise to new forms of life.

Thus, biodiversity is the result of:

- Continuous changes over time
- Adaptation to environmental conditions
- Evolution from common ancestors

Taxonomy

- The method of arranging organisms into groups on the basis of similarities and differences is called classification.
 - (Gr. Taxis-arrangement, nomos-law) The functional branch of biology dealing with the identification, classification and nomenclature of living organisms is called taxonomy. “Carolus Linnaeus” (1707-1778) is known as “Father of taxonomy”.
- (a) **Identification** : It is defined as the determination that a particular organism is similar to some other known individual.
- (b) **Nomenclature** : It is the system of naming of any living creature whether it is from a lower phylum or from higher phylum.
- (c) **Classification** : It is the categorization of living creatures according to their peculiar features.

Hierarchy

“Hierarchy” means ordering of living creatures according to their growing similarities. It includes the following pattern of classifying any living one.

The hierarchical order of classifying organisms is :

Kingdom → Phylum/ Division → Class → Order → Family → Genus → Species

1. **Kingdom** : It is the highest category of taxonomic studies. All animals are included in kingdom Animalia and all plants are included in kingdom Plantae.
2. **Phylum /Division** : Many Classes with some common characters are included in Phylum.
3. **Class** : Similar orders are placed together in a class.
4. **Order** : A number of families having common characters are placed in order.
5. **Family** : A number of genera having several common characters form a family.
6. **Genus** : It is a group of closely related species with common ancestry.
7. **Species** : A group of organisms capable of interbreeding to produce offspring.

Ex.1 In the hierarchy of classification, which grouping will have the smallest number of organisms with a maximum of characteristics in common and which will have the largest number of organisms ?

Ans. Species which lie at the bottom of the taxonomic ladder bear the smallest number of organisms and since the members of a species can interbreed among themselves, they possess maximum characteristics in common. In the hierarchy of classification, kingdom is at the top. It bears the largest number of organisms because a kingdom is split into several phyla, classes, orders, families, genera and species, each having their specific characteristics.

COMPETITIVE LEVEL

- Biodiversity “**hotspots**” are areas of the world with a high number of endemic species (that is, species found only in that place).
- **Taxonomists** : Biologists who are specialized in identifying and classifying life on the planet, have named approximately 1.7 million species. About 13,000 more species are added to the list of known organisms. (National Wildlife Magazine)

Nomenclature

- It is the system of naming an individual.
- Nomenclature is done on the basis of a set of rules stated in the ICN i.e. International code of nomenclature.
- **Binomial nomenclature** : Proposed by Carolus Linnaeus in his book *Systema Naturae* who is also called father of taxonomy.
- According to this system of nomenclature, each animal or plant is given two names, the first one is the generic name and the second one is the name of the species.
- Scientific names are always in *Latin*.
- The first letter of the generic name is always capitalized and that of the species name is written in small letter. For example, the scientific name of frog is *Rana tigrina*, in which *Rana* is the generic name and *tigrina* is the name of the species.

Ex.2 Why is binomial nomenclature called so ?

Ans. The binomial system of nomenclature gives the scientific name of an organism which consists of two separate words-first one designates generic. Name and the second one designates specific name. Since this system of naming organisms gives two words name to an organism, it is known as a binomial nomenclature.

Ex.3 Define advantage of using scientific name instead of common name ?

Ans. Common names of plants and animals differ from one geographical area to another area but the scientific name is same all over the world, e.g., in Hindi, mango is called as **aam** but in Tamil, it is called as **maangai**. But the scientific name is *Mangifera indica*, which is same all over the world.

COMPETITIVE LEVEL

- In 1980, in the tropical rainforests of Panama, scientists discovered 1,200 species of beetles living in and around just 19 trees and fully 80% of these species were previously unknown to science.
- Approximately half of all synthetic drugs have a natural origin, including 10 of the 25 highest-selling drugs in the US.
- Based on data of recorded extinctions of known species over the past century, scientists estimate that current rates of species extinction are about 100 times higher than long-term average rates based on fossil data.

❖ Classification of Organisms :

The method of arranging organisms into groups on the basis of similarities and differences is called classification.

(a) Significance of Classification :

- It establishes hierarchy of groups of organisms on the basis of their common features.
- It makes the systematic study easier.
- It is essential to understand the inter-relationship amongst different groups of organisms.
- It serves as a base for the development of other biological sciences as well as different fields of applied biology like public health, environment etc.

(b) System of Classification :

(i) **Artificial system** : Biological classification in early times was based upon single arbitrarily chosen character suiting the convenience of taxonomist.

e.g. On the basis of habitat and ability to fly.

(ii) **Natural system** : It was based on morphological and anatomical similarities and differences.

(iii) **Phylogenetic system** : It was based on evolutionary sequence as well as genetic relationships amongst the organisms. Charles Darwin showed that living organisms evolved by the process of descent with modifications.

❖ Different Classification Systems :

(a) **Two kingdom system** : It was given by **Carolus Linnaeus** in 1758. Organisms were divided into Plantae kingdom and Animalia kingdom. Fungi, bacteria and *Euglena* could not find an appropriate position.

(b) **Three kingdom system** : It was given by **Ernst Haeckel** (1894). In this kingdom Protista was also included along with plant kingdom and animal kingdom.

(c) **Four kingdom system** : It was given by **Copeland**. kingdom Monera was also included in this system of classification.

(d) **Five kingdom classification** : This scheme was proposed by **R.H. Whittaker** in 1969. Fungi could not find a suitable place in earlier system of classifications and thus Whittaker's five kingdom theory was found to be most favoured amongst most biologists.

This classification includes :

(i) Monera (ii) Protista (iii) Fungi (iv) Plantae (v) Animalia

Ex.4 Why do we classify organisms ?

Ans. There is vast number of living organisms in this biosphere and they have a great diversity in shape, size and forms. It is practically not possible to examine and study every organism separately at individual level. It is, therefore, advisable to study the diversity of organism by classifying them in orderly manner.

❖ Basis of Five Kingdom Classification :

- Complexity of structure
- Mode of nutrition
- Level of organization

- (i) **Kingdom Monera** : Unicellular, prokaryotic, microscopic, most ancient, can live in deep oceans, hot springs, deserts, high salt concentrations etc. They include bacteria, filamentous and photosynthetic blue green algae (Cyanobacteria) etc.

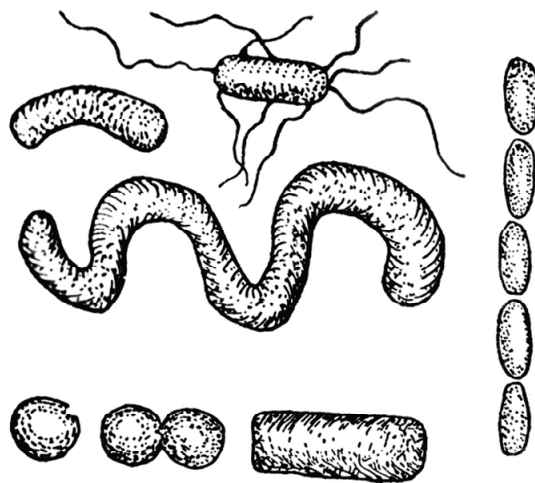


Fig. : Different Types of Bacteria

- (ii) **Kingdom Protista** : Unicellular, colonial, eukaryotic. They include photosynthetic algae, decomposers (slime moulds) and protozoa (predators) etc.

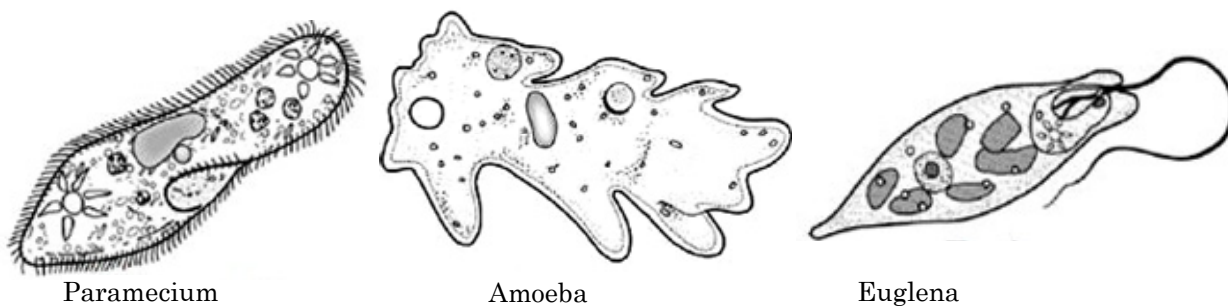


Fig. : Protista

- (iii) **Kingdom Fungi** : Unicellular or multicellular eukaryotic organisms. They are heterotrophic, parasitic or saprotrophic. They include ascomycetes and basidiomycetes.

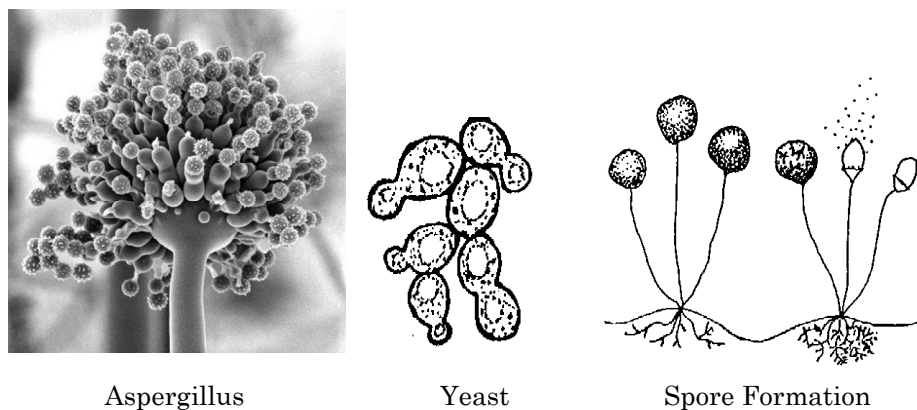


Fig. : Fungi

(iv) **Kingdom Plantae** : They are multicellular, eukaryotic, autotrophic (photosynthetic), some are heterotrophic and parasitic.

They include photosynthetic algae, green plants etc.

(v) **Kingdom Animalia** : Multicellular, eukaryotic, heterotrophic.

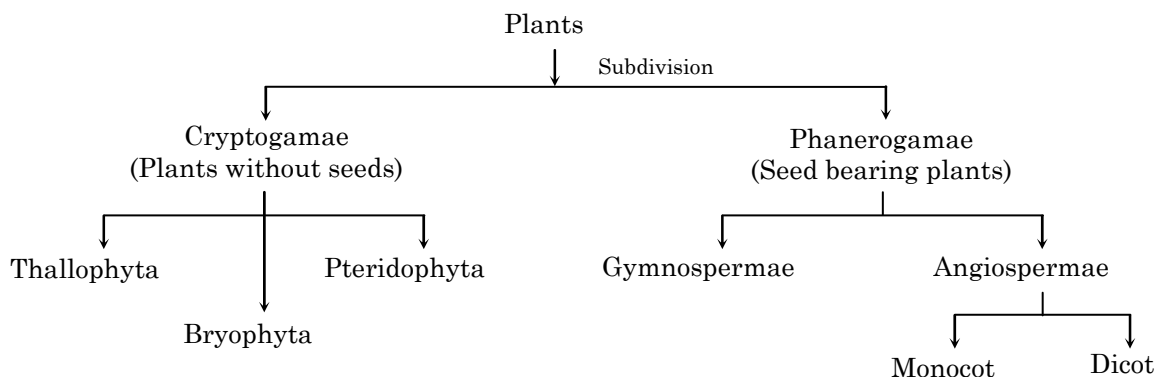
Ex.5 Give three examples of the range of variations that you see in life forms around you.

Ans. The three examples are :

- (i) The living organisms vary in size from a few micrometers (e.g., microscopic bacteria) to more than 30 metres long (e.g., blue whale) and more than 100 metres tall (e.g., red wood trees of California).
- (ii) The living organisms vary in longevity from a few days (like mosquitoes) to several thousand years (pine trees).
- (iii) The living organisms range from colourless or transparent to brightly coloured birds and flowers.

◆ Plant Classification :

Plant kingdom was divided into two sub kingdoms by Eichler.



A. Sub kingdom cryptogamae :

(Cryptos = hidden gamous = marriage): It includes lower plants which do not bear flower or seeds.

(a) Division Thallophyta :

- It comprises of algae.
- They are most primitive and simple kind of plants in which plant body is not differentiated into root, stem and leaves. Xylem and phloem are absent. i.e. vascular system is absent.
- Reproductive organs are single-celled and there is no embryo formation after fertilization.
- These are aquatic or terrestrial, fresh water or marine. Algae are photosynthetic containing various pigments like chlorophyll, carotenoids, xanthophylls etc. These can be unicellular, colonial, filamentous and multicellular.
- Cell wall is made up of cellulose and stored food is starch. E.g. Green algae (*Ulothrix*, *Chara*, *Cladophora*, *Ulva*, *Spirogyra*), Brown algae, Red algae etc.

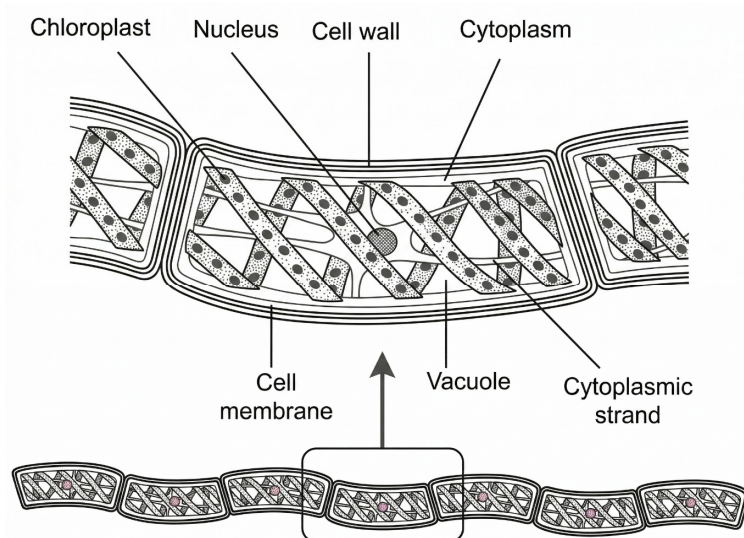
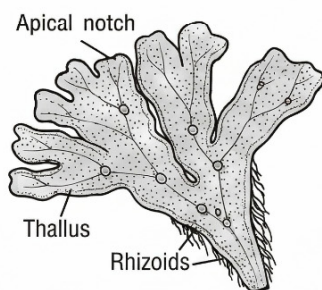


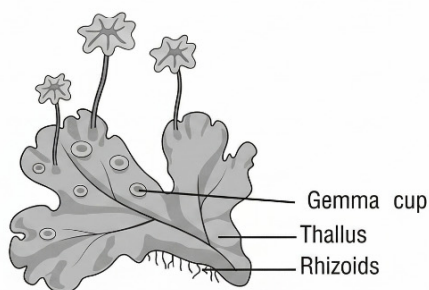
Fig. : Spirogyra

(b) Division Bryophyta :

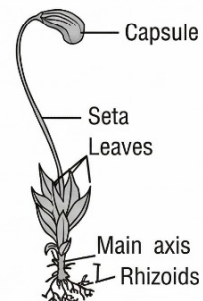
- The division Bryophyta (Greek word *Bryon* = moss ; *phyton* = plant) includes the simplest and primitive plant.
- Xylem and phloem are absent (non – vascular). Plant body is differentiated into leaf and stem like structures and root like structures (**rhizoids**).
- They are also known as amphibian of plant kingdom because they require water for the completion of their life cycle. An embryo is formed upon fertilization. Sporophyte lives as a parasite over gametophyte. Sex organs are multicellular. The male sex organs are **antheridia** and female sex organs are **archegonia**.
- E.g. Liverworts (*Riccia*, *Marchantia*), hornworts (*Anthoceros*) and Mosses (*Funaria*, *Polytrichum*).



Riccia



Marchantia



Funaria

Fig. : Bryophytes

(c) Division Pteridophyta

- Plant body is differentiated into root, stem and leaf. Xylem and phloem are present.
- These plants have no flowers and do not produce seeds. Sex organs are multicellular and jacketed by sterile cells.
- Club mosses [*Selaginella*, *Lycopodium* (“ground pine”)], horsetails (*Equisetum*) and ferns (*Marsilea*).

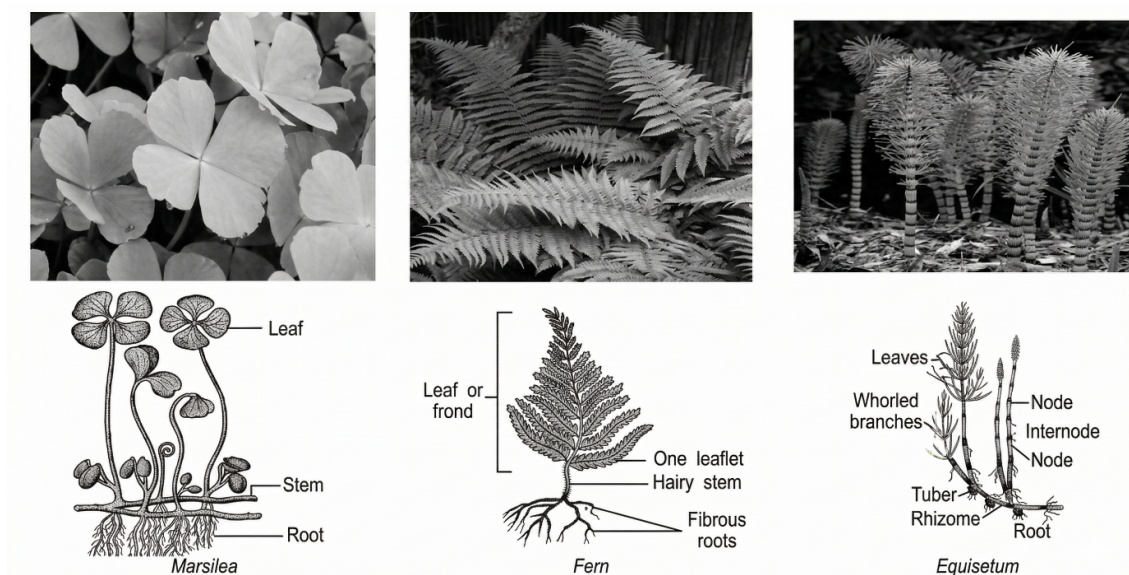


Fig. : Pteridophyta

B. Sub kingdom : Phanerogamae

(Phaneros = visible : gamous = marriage.) : This is the highest group of plants which includes seed bearing plants. They are of two types :

(a) Gymnosperms :

- The seeds produced by these plants are naked and are not enclosed within fruits. They have well developed vascular system but xylem lacks vessels and phloem lacks companion cells.
- Plants are commonly tall trees or shrubs. The reproductive organs are represented by unisexual cones. Often both cones are present on the different plant except pine (monoecious). E.g. : *Cycas* , *Pinus*, *Ginkgo*

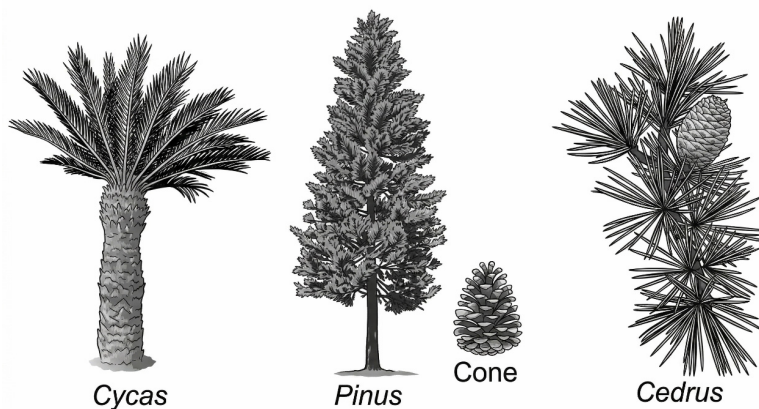


Fig. : Gymnosperms

(b) Angiosperms :

- Angiosperms are highly evolved plants and they produce seeds that are enclosed within the fruit. The reproductive organs are aggregated in a flower.
- These are represented by trees, herbs, shrubs. They have highly developed vascular system. Seeds remain enclosed in ovary. Plant embryos in seeds have structures called cotyledons. Cotyledones are called “seed leaves”.
- It is divided into two classes on the basis of number of cotyledons.

(i) Dicotyledonae :

- The seeds produced by these plants have embryos with two fleshy leaves, the cotyledons.
- Their leaves have reticulate venation, with a network of veins. The root system has a prominent tap root.
- E.g. : Pea (*Pisum sativum*), potato (*Solanum tuberosum*), sunflower (*Helianthus annuus*), rose (*Rosa indica*), banyan (*Ficus religiosa*), neem (*Melia indica*), apple (*Malus silvestris*).

(ii) Monocotyledonae :

- The seeds of these plants have only one cotyledon.
- Their leaves have parallel venation. The root system consists of fibrous roots.
- The vascular bundles are scattered and closed (i.e. lack cambium). Secondary growth does not occur.
- **Examples :** Maize (*Zea mays*), wheat (*Triticum vulgare*), rice (*Oryza sativa*), onion (*Allium cepa*), sugarcane (*Saccharum officinarum*), barley (*Hordeum vulgare*), banana (*Pandanus*), Coconut and grasses.

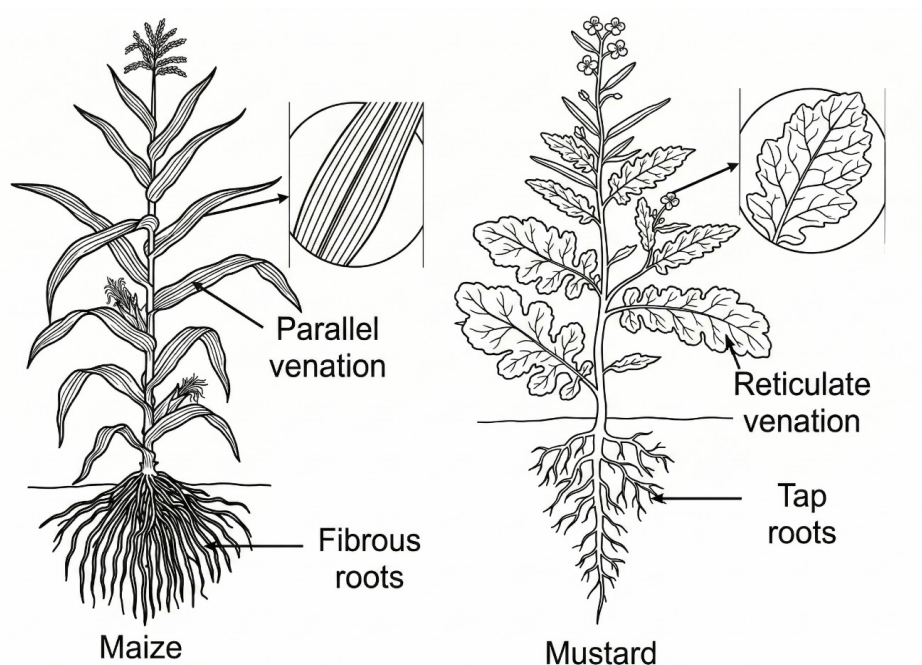


Fig. : Monocot plant and Dicot Plant

❖ Animal Kingdom :

Basis of Classification :

- Organization and differentiation of cells to form tissues and organs.
- Body symmetry.
- Formation of body cavities and blood vascular system.
- Features of embryonic development.

(a) Level of organization in living beings : It is structural differentiation of animal body.

(i) Cellular level of organization : Tissues do not differentiate. Different types of cells are present, e.g., Porifera (sponges).

(ii) Tissue level of organization : Multicellular body cells organised into tissues but organs are absent e.g., Coelenterates.

(iii) Organ level of organization : Cells are organized into tissues and tissues into organ but organ systems are absent e.g., Platyhelminthes.

(iv) Organ system level of organization : Cells are organised into tissues, tissues into organs and organ into organ systems e.g., Annelids, arthropods, chordates.

(b) Body symmetry : It is similarity in arrangement of parts. Symmetry is of two types, radial and bilateral.

(i) Radial symmetry : The body is cylindrical or discoid when similar parts occur all around the central axis. Any vertical plane passing through the central axis will divide the body into two equal halves, e.g., ctenophore, coelenterates and echinoderms.

(ii) Bilateral symmetry : The body has a head. Organs and limbs are paired. They are arranged laterally. Body is divisible into two equal halves by only one plane (mid-sagittal plane). Bilateral symmetry is found in Platyhelminthes, Nematoda, Annelida, Mollusca, Arthropoda and Chordata.

COMPETITIVE LEVEL

- Carl Woese (1977) modified the five kingdom classification by dividing the Monera into Archaeobacteria (or Archaea) and Eubacteria (True Bacteria). This is known as six kingdom classification.
- **Cephalization :** It is development of head in the anterior part of the animal body.

(c) Germ or germinal layers :

- A germ layer is group of cells in an embryo that interact with each other as the embryo develop and contribute to formation of all organs and tissue.
- Germinal layers can be two or three in number. On this basis of the same, the animals are of two types, diploblastic and triploblastic.
 - (i) Diploblastic animals :** Animals having two germinal layers, outer ectoderm and inner endoderm. Mesoderm is absent e.g. Porifera, Coelenterata.
 - (ii) Triploblastic animals :** Animals having three germinal layers-outer ectoderm, middle mesoderm and inner endoderm, e.g. Platyhelminthes to chordates.

(d) **Coelom (body cavity):** It is mesoderm lined fluid filled space that occurs between alimentary canal and body wall. Depending upon the absence or presence of coelom, animals are of three types-acoelomate, pseudocoelomate and eucoelomates.

(i) **Acoelomate :** Coelom is absent in Porifera, Coelenterata, Platyhelminthes. In Platyhelminthes, a mesoderm is present but it does not form cavity.

(ii) **Pseudocoelomate :** A cavity called pseudocoelom is present which is not lined by mesoderm. It is generally endodermal in origin. Mesoderm occurs but forms small separate pouches e.g., Aschelminthes.

(iii) **Coelomate or Eucoelomate :** A true coelom lined by mesoderm is present. E.g. Annelids to chordates.

(e) Notochord is a mesodermally derived rod-like structure formed on the dorsal side during embryonic development in some animals. It is solid rod of polygonal shaped cells, present on dorsal side between alimentary canal and nervous system in some animals.

On the basis of presence or absence of notochord animals are of two types :

(i) **Chordates :** Animals with notochord are called chordates, e.g. Amphibians, Reptiles, Aves, Mammals.

(ii) **Non chordates :** Animals which do not form notochord are called non-chordates, e.g., Porifera to echinoderms.

COMPETITIVE LEVEL

- In some animals, the body is externally and internally divided into segments with a serial repetition of at least some organs. For example, in earthworm, this phenomena is known as metamerism.
- Annelids are the first animal to develop true body cavity or coelom.
- Cnidaria is the first phylum to be organized at tissue level organization and Platyhelminthes is the first to be organized at organ system level of organization but it lacks well developed organs due to absence of coelom.
- On the basis of origin, true coelom is of two types, schizocoelom and enterocoelom.

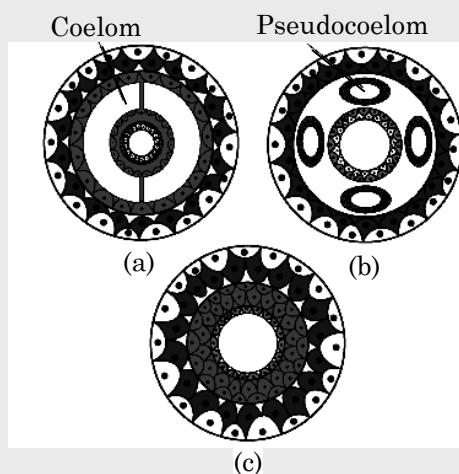


Fig. : Diagrammatic sectional view of :
(a) Coelomate
(b) Pseudocoelomate
(c) Acoelomate

◆ Non Chordates :

(a) Phylum Porifera :

- Sessile (stalk-less) and marine (*Sycon*) except one group that lives in fresh water (*Spongilla*).
- Simplest multicellular, that have cellular level of organization.
- Hard exoskeleton present.
- Sponges are mostly asymmetrical. Body is perforated by numerous pores, the ostia that open into a canal system having canals and chambers lined with collared flagellated cells or choanocytes.
- **Examples :** *Sycon*, *Euplectelea*, (Venus flower basket), *Spongilla* (Fresh water sponge).

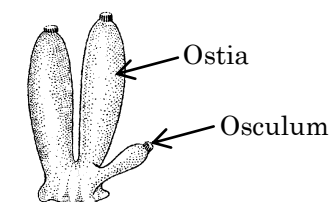


Fig. : Sycon



Fig. : Euplectelea

(b) Phylum : Cnidaria (Coelenterata) :

- All members are aquatic, mostly marine, a few such as *Hydra* are fresh water solitary or colonial forms.
- Cnidarians or coelenterates are multicellular, diploblastic animals with tissue grade of organization. Their body shows radial symmetry.
- **Examples :** *Hydra*, *Obelia* (sea fur), *Aurelia* (jelly fish), *Metridium* (sea anemone).

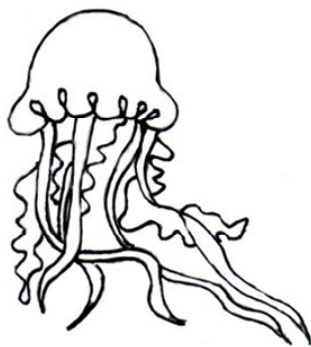


Fig. : Aurelia (Jelly fish)

COMPETITIVE LEVEL

- Body of poriferans may be vase-like, rounded, sac-like and branched. Body contains a cavity called as spongocoel.
- Coelenterates possess specialized cells (cnidoblasts or cnidocytes) bearing stinging capsules called nematocysts present on tentacles and body. Nematocysts serve the functions of paralysing the prey by injecting poison or to hold the prey.
- Coelenterates exhibit the phenomenon of polymorphism means body shows two main forms, the polyps and the medusae.

◆ **Phylum Ctenophora :**

- These organisms are marine, solitary and free swimming with radially symmetrical and transparent body.
- They possess comb plates that are ciliated which help in locomotion. They also possess tentacles.
- **Examples :** *Cestum*, *Ctenoplana*, *Pleurobrachia*.

◆ **Bioluminescence** is the property of a living organism to emit light which is found in ctenophores.

(c) Phylum Platyhelminthes :

- These animals are bilaterally symmetrical, acoelomate and dorsoventrally flattened, hence are also called as flatworm. Body is thin, soft, leaf-like or ribbon-like.
- Triploblastic animals with organs level of organization. Digestive cavity is with a single opening, the mouth. Suckers and hooks are usually present.
- **Examples :** *Dugesia* (*Planaria*), *Fasciola* (liver fluke), *Schistosoma* (blood fluke), *Taenia solium* (pork tape worm).

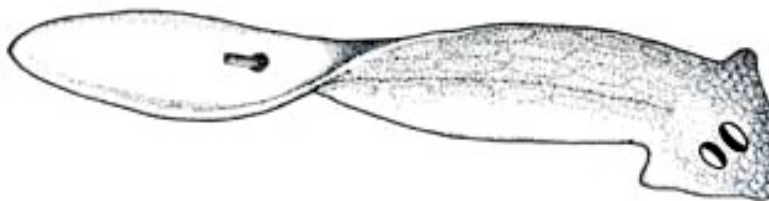


Fig. : Planaria

(d) Phylum Aschelminthes or Nematoda :

- They are parasitic or free-living with complete alimentary canal.
- They are triploblastic, unsegmented and show bilateral symmetry. Body cavity is not a true coelom but a pseudocoelom.
- Sexes are separate (sexual dimorphism).
- **Examples :** *Ascaris* (Round worm), *Enterobius* (Pin worm), *Wuchereria* (filarial worm).

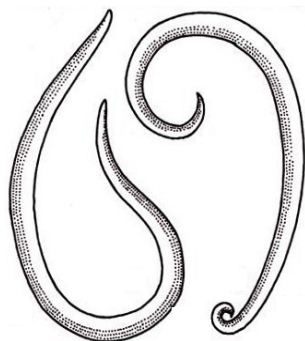


Fig. : Ascaris

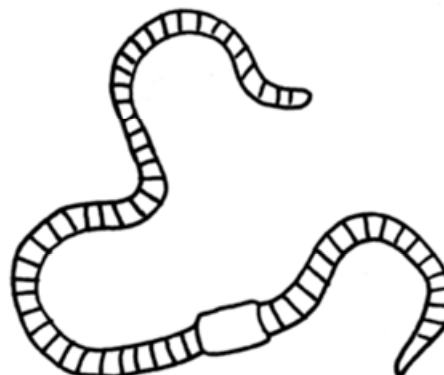


Fig. : Earthworm

(e) Phylum Annelida :

- These organisms occur in moist soil, fresh water and sea. These are elongated, with metamerically segmented body and bilateral symmetry.
- First animals with true body cavity. Body bears lateral appendages for locomotion in the form of chitinous setae or parapodia.
- **Examples :** *Nereis* (sand worm) *Aphrodite* (sea mouse), *Pheretima* (earthworm), *Hirudinaria* (cattle leech)

(f) Phylum Arthropoda :

- Triploblastic organisms with bilaterally symmetry and segmentation.
- Body segments are grouped into two regions-cephalothorax (head and thorax together) and abdomen or three regions-head, thorax and abdomen.
- Each body segment usually bears paired lateral and jointed legs or appendages.
- Coelom is reduced and body is covered with a thick chitinous covering.
- Respiration through general body surface, by gills, air tubes (tracheae) or book-lungs.
- **Example :** *Palaemon* (prawn), *Daphnia* (water flea), *Limulus* (king crab), *Palamnaeus* (scorpion)

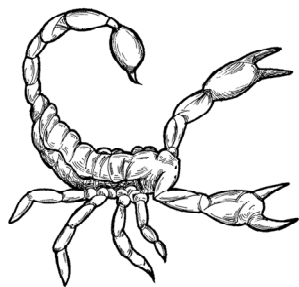


Fig. : Scorpion

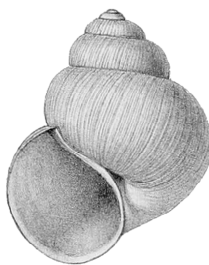


Fig. : Pila

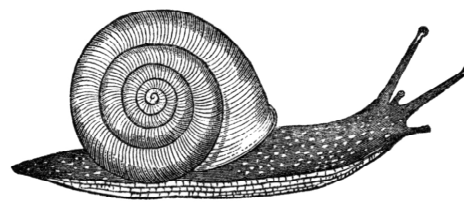


Fig. : Snail

COMPETITIVE LEVEL

- In flatworms, circulatory system, respiratory system and skeleton are absent. Excretory system occurs through specialized cells called flame cells.
- Planaria is hermaphrodite animal (an animal that contains both male and female reproductive organs).
- Arthropoda is largest phylum of animal kingdom.

(g) Phylum Mollusca :

- They have soft, unsegmented body. These are bilaterally symmetrical, triploblastic, Coelomate and having organs system level of organization.
- Body is divided into three regions head visceral mass and ventral foot. Outer surface is covered by a hard calcareous shell.
- Respiration is by gills called ctenidia. The sexes are usually separate.
- **Examples :** *Chiton*, *Pila* (Snail), *Unio* (Fresh water mussel), *Octopus* (Devil fish).

(h) Phylum Echinodermata :

- Radially symmetrical, triploblastic, coelomate animal with organ system level of organization.
- They are marine, gregarious (live in groups) and free-living animals.
- Shape may be star-like, spherical or elongate. Body surface is covered all over by calcareous spines.
- Tube feet for locomotion. These are unsegmented.
- Body cavity is modified into a water-vascular system or ambulacral system with tube like outward extension for locomotion, called tube feet.
- **Examples :** *Asterias* (star fish), *Echinus* (sea urchin), *Holothuria* (sea cucumber), *Antedon* (feather star)

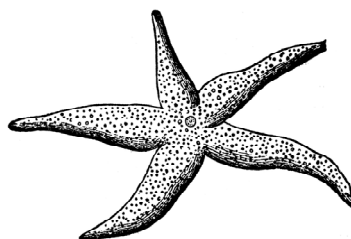


Fig. : Starfish

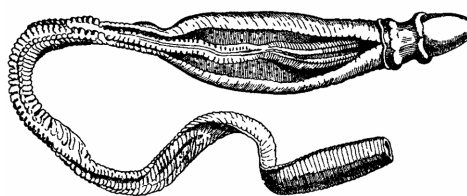


Fig. : Balanoglossus

(i) Phylum Protochordata :

- Triploblastic, coelomate animal with organ system level of organization.
- They are placed in between nonchordates and chordates as they possess some characters of both.
- They include worm like, unsegmented, bilaterally symmetrical animals which are exclusively marine.
- Their body is divided into three regions proboscis, collar and trunk.
- They do not possess notochord, which is a flexible, rod like structure running through the length of the body, above alimentary canal.
- They possess gill slit or gill cleft which is meant for respiration.
- Nerve cord in collar region is present but it is not a true dorsal nerve cord.
- **Examples :** *Balanoglossus* (tongue worm), *Saccoglossus*.

COMPETITIVE LEVEL

- Dorsal nerve chord is solid strand of nervous tissue forming a part of central nervous system.
- Difference between nerve chord and notochord is that notochord leads to form vertebral column while nerve chord leads to form spinal cord.

(j) Vertebrata :

- They are advanced animals, having a cranium (brain box) around the brain. Nervous system is well developed.
- Notochord is replaced by a vertebral column (backbone) in the adults. Endoskeleton is highly developed.

- There are two pairs of limbs or appendages.
- Head is well differentiated.
- The heart is situated ventrally. The circulatory system is closed consisting of blood vascular system and lymphatic system. Red coloured pigment haemoglobin is present in red blood corpuscles.
- Respiratory organs may be gills (in aquatic animals), skin, buccopharyngeal cavity (in amphibians) or lungs (in land animals).
- Excretion occurs through kidneys.
- Sexes are separate.

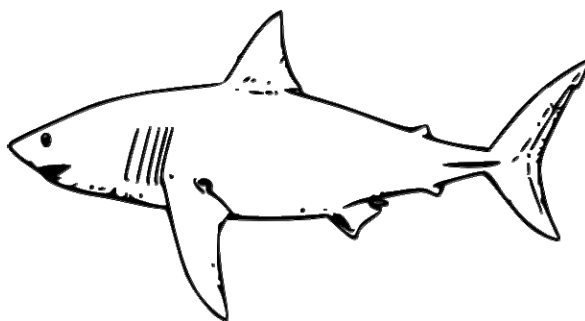


Fig. : Shark

(i) Super class Pisces :

- True fishes are included in this class which respire through gills.
- Their body is stream lined and covered by scales / plates.
- They have muscular tail and fins for movement.
- Endo skeleton is either made up of cartilage or bone.
- They are unisexual and lay eggs.
- They are cold blooded with two chambered heart.

COMPETITIVE LEVEL

S.No.	CHONDRICHTHYES	OSTEICHTHYES
1.	Mouth ventral.	Mouth terminal.
2.	Tail fin asymmetrical (heterocercal).	Tail fin symmetrical (homocercal).
3.	5-7 pairs of gills are present.	4-5 pairs of gills are present.
4.	Gills are naked.	Gills are covered by operculum.
5.	Cloacal aperture is present.	Anus and urinogenital apertures are separate.
6.	No swim bladder.	Swim bladder usually present.
7.	Fertilization is internal.	Fertilization is external.
8.	e.g. Scoliodon – Dog fish, Trygon - Sting ray, Torpedo - Electric ray, Rhineodon - Whale shark.	e.g. Labeo rohita - Rohu or Indian carp, Anabas - Climbing perch, Caulopharyne jordani- Angler fish, Hippocampus - Sea horse, Pterois valitans -Lion fish, Exocoetus - Flying fish.

(ii) Class Amphibia :

(Greek, amphi = both, bios = life, the vertebrates leading two lives / dual life)

- They can live on land as well as in water.
- Skin is smooth or rough, rich in glands which keep it moist and with pigmented cells, i.e. chromatophores.
- Respiration occurs by lungs, skin or buccal lining. Gills are present at least during larval stage for respiration.
- Heart is three chambered with two auricles and a ventricle. Red blood corpuscles are large, biconvex, oval and nucleated. Brain is not much developed. Cranial nerves are 10 pairs.
- Eggs with gelatinous covering and are usually laid in water. Fertilization is external.
- Development is indirect with a tadpole larva which undergoes metamorphosis to become adult.
- **Example :** Salamanders, frogs and toads, *Necturus* (mud puppy), *Triturus* (newt), *Rana* (frog), *Bufo* (toad).

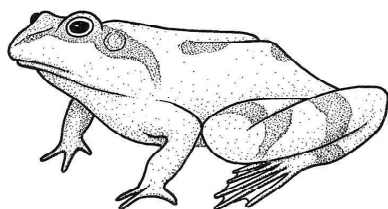


Fig. : Frog

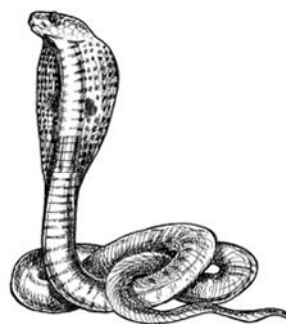


Fig. : Snake

(iii) Class Reptilia :

- In these animal two pairs of pentadactyl limbs are present, but in snakes limbs are reduced or absent.
- Body is covered with epidermal horny scales. Skin is dry, impermeable and devoid of glands.
- Respiration takes place by lungs only. Gills are absent.
- Heart is incompletely four – chambered, having two auricles and incompletely divided ventricles. In crocodile, heart is completely four chambered.
- Sexes are separate. Fertilization is internal.
- **Examples:** *Testudo* (tortoise), *Chelone* (turtle) *Draco* (flying lizard), Chameleon, *Hemidactylus* (wall lizard), *Naja* (cobra) etc.

(iv) Class Aves : (L. Aves = birds)

- The birds are described as ‘feathered reptiles’ that have developed the power of flight.
- The body is covered with soft feathers called plumage. The body is divisible into head, neck, trunk and tail.
- There are two pairs of limbs. The fore limbs are modified to form wings or are reduced. Hind limbs are strongly developed for perching, walking.
- Endoskeleton is light. The bones have air cavities. This makes the bird light.

- Respiration is by lungs only. Lungs have additional bag like membranous extensions called as air sacs.
- Heart is completely four chambered.
- Sexes are separate.
- Birds are oviparous, i.e. egg laying.
- **Examples :** *Columba* (pigeon), *Pavo* (peacock), *Corvus* (crow), *Passer* (sparrow). *Struthio camelus* (ostrich), white stork (*Ciconia ciconia*), tufted duck (*Aythya fligula*). *Kiwi* and *Penguin* are flightless birds.



Fig. : Sparrow



Fig. : Penguin

COMPETITIVE LEVEL

- Body is without scales. Endoskeleton is mostly bony. Notochord does not persist in adults.
- Head and trunk are distinct. Neck and tail may or may not be present.
- They are tetrapods (four – limbed) having digits without nails.
- Excrete either ammonia (by tadpole) or urea (by adults).
- Sexes are separate, i.e., dioecious. Male is without copulatory organ.
- Jaws are modified to form a strong beak. Teeth are absent.
- High degree of parental care is exhibited. There is no larval stage in development.
- Fertilization is internal. Fertilized eggs are laid with a yolk (stored food) and with a hard calcareous shell.

(v) Class Mammalia :

- Mammalia is the most evolved group of organisms and are found in diverse habitats ranging from deserts, polar ice caps, oceans, mountains, forests and grasslands.
- Skin is covered with an exoskeleton of hair. Hair are provided with sweat glands which help in the regulation of body temperature. In aquatic mammals, hair being negligible, the subcutaneous layer of fats provides insulation.
- Mammals have two pairs of pentadactyl limbs.

- The body cavity is unequally divided into two parts by a muscular partition called as diaphragm.
- Eyes are provided with movable lids.
- Ears have fleshy external ears or pinnae.
- Teeth are embedded in sockets (thecodont). Two sets of teeth develop in the life time of a mammal, milk teeth and permanent teeth (diphyodont). Teeth are of different types (heterodont).
- Respiration occurs by lungs. Heart is four chambered. R.B.Cs are non nucleated and usually circular.
- Sexes are separate. Gonads are paired. Testes lie commonly in the scrotal sacs outside the abdomen.
- Fertilization is internal. Eggs are small, microscopic without shells and are retained in uterus of female for development.
- Mammals are divided into three main groups :

(1) **Egg-laying mammals (monotremes)** : These mammals show characters of both reptiles and mammals. They lay hard shelled eggs (oviparous). E.g., Spiny ant eater, duck billed platypus

(2) **Marsupial mammals (pouched mammals)** :

- Pouched or marsupial mammals (Latin marsupium = pouch) They are viviparous.
- The young ones, when born, are carried in pouch called marsupium present on the mother's abdomen.
- In the pouch, they feed on the mother milk. Example : Kangaroo (*Macropus*), kola bear.

(3) **Placental mammals (true mammals)** : These mammals with true placenta.

- The embryo is retained in the uterus.
- These are the very successful group of land animals, occurring in diverse climatic conditions.
- Example : Bat, lion, tiger, camel, giraffe, whale, dolphin, monkey, humans etc.

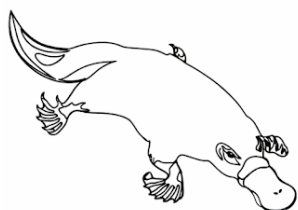


Fig. : Platypus



Fig. : Kangaroo

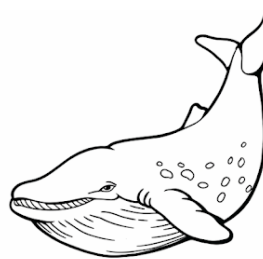


Fig. : Whale

Note : They are named mammals as all of them possess mammary glands (milk producing glands). Mammals are the only animals which feed their young ones with milk.

Ex.6 Will advanced organisms be the same as complex organisms? Why?

Ans. The complexity of body design will increase over evolutionary time. This is the process by which new modifications occur in the organisms. So, in this way we can say that advanced organisms will be complex as compared to complex organisms.

Ex.7 Define the main characteristics of Vertebrates.

Ans. These animals have a true vertebral column and internal skeleton, which allows a completely different distribution of muscle attachment points to be used for movement. Vertebrates are bilaterally symmetrical, triploblastic, coelomic and segmented, with complex differentiation of body tissues and organs.

Ex.8 Bat can fly still it is placed in mammals. Why?

Ans. Bat can fly still it is placed in mammals because:

- (i) Gives birth to the young ones.
- (ii) Presence of mammary glands.
- (iii) Presence of external ears.
- (iv) Presence of teeth in beak.
- (v) Presence of hair on body.
- (vi) Presence of diaphragm between thoracic and abdominal cavity.

Ex.9 Why are whales not grouped in fishes ?

Ans. Whales belong to Class Mammalia while fishes are included in class Pisces. Both are aquatic animals but differ in several basic characteristics, such as :

- (a) Whales are warm-blooded animals while fishes are cold-blooded.
- (b) Whales respire through lungs while fishes respire through gills.
- (c) Whales possess mammary glands which are absent in fishes.
- (d) Whales have four-chambered heart, while fishes have a two chambered heart.

Fossils as Evidence

Fossils are preserved remains or impressions of ancient organisms found in rocks.

Importance of Fossils

- Provide evidence of evolution
- Show changes in organisms over time
- Help scientists study extinct organisms

Older rock layers generally contain simpler organisms, while newer layers contain more complex organisms.

Biodiversity Under Threat

Human activities are destroying biodiversity rapidly.

Causes of Biodiversity Loss

1. Deforestation
2. Pollution
3. Climate change
4. Overuse of natural resources
5. Habitat destruction

The extinction of one species affects many others connected to it through food chains and ecological interactions.

◆ Need for Conservation

Conservation of biodiversity is necessary for:

- Ecological balance
- Sustainable development
- Survival of future generations

Viruses

Biological diversity includes a vast range of organisms differing in size, structure, complexity, and mode of life. Most living organisms are composed of one or more cells and are therefore classified as **cellular organisms**. However, certain biological entities lack cellular organization. These are termed **acellular entities**.

Among acellular entities, **viruses** occupy a unique and significant position. They do not possess a cellular structure, yet they contain genetic material and exhibit certain characteristics associated with living organisms. Because of this dual nature, viruses are often regarded as existing at the boundary between living and non-living matter.

A virus is an ultramicroscopic, infectious acellular particle composed of genetic material enclosed within a protein coat. It can replicate only inside a living host cell by utilizing the host's metabolic machinery.

Viruses are described as **obligate intracellular parasites**, meaning that they are completely dependent on living cells for reproduction and survival.

Outside the host cell, viruses remain inert and show no metabolic activity.

◆ Discovery of Viruses

The discovery of viruses dates back to the late nineteenth century. In 1892, the Russian scientist **Dmitri Ivanovsky** observed that the infectious agent causing tobacco mosaic disease could pass through filters that retained bacteria.

Later, in 1898, the Dutch microbiologist **Martinus Beijerinck** concluded that the agent was a new type of infectious entity and named it "virus."

This discovery marked the beginning of virology, the scientific study of viruses.

◆ Structural Organization of Viruses

Viruses are extremely small, generally ranging from 20 to 300 nanometres in size, and can be observed only under an electron microscope.

A typical virus consists of the following components :

1. **Genetic Material** : The core of a virus contains nucleic acid, which may be either :

- DNA (Deoxyribonucleic acid), or
- RNA (Ribonucleic acid)

Unlike cellular organisms, a virus contains only one type of nucleic acid.

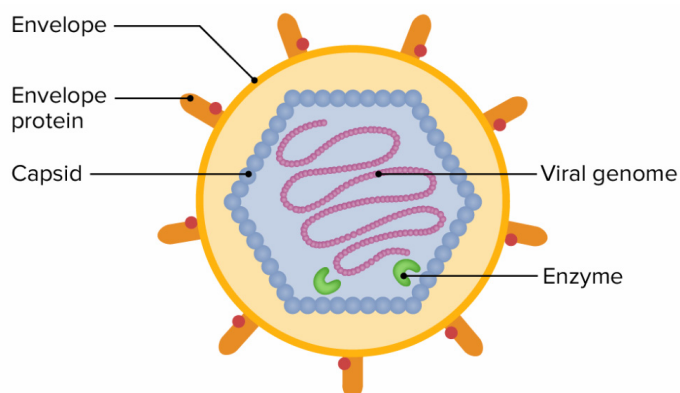
The genetic material carries the instructions necessary for viral replication.

2. **Capsid (Protein Coat)** : The nucleic acid is surrounded by a protective protein covering known as the **capsid**. The capsid is made up of protein subunits called capsomeres.

Functions of the capsid include :

- Protecting the genetic material.
- Helping the virus attach to the host cell.
- Determining the shape of the virus.

3. **Envelope (in Some Viruses)** : Certain viruses possess an outer lipid membrane called an **envelope**, derived from the host cell membrane. This envelope contains specialized proteins that assist in entering host cells.



◆ Types and Shapes of Viruses

Viruses exhibit different structural forms :

1. **Helical Viruses** : Rod-shaped; genetic material is coiled within a cylindrical capsid.
2. **Spherical (Icosahedral) Viruses** : Appear round due to symmetrical arrangement of capsomeres.
3. **Complex Viruses** : Possess a head-tail structure, such as bacteriophages that infect bacteria.

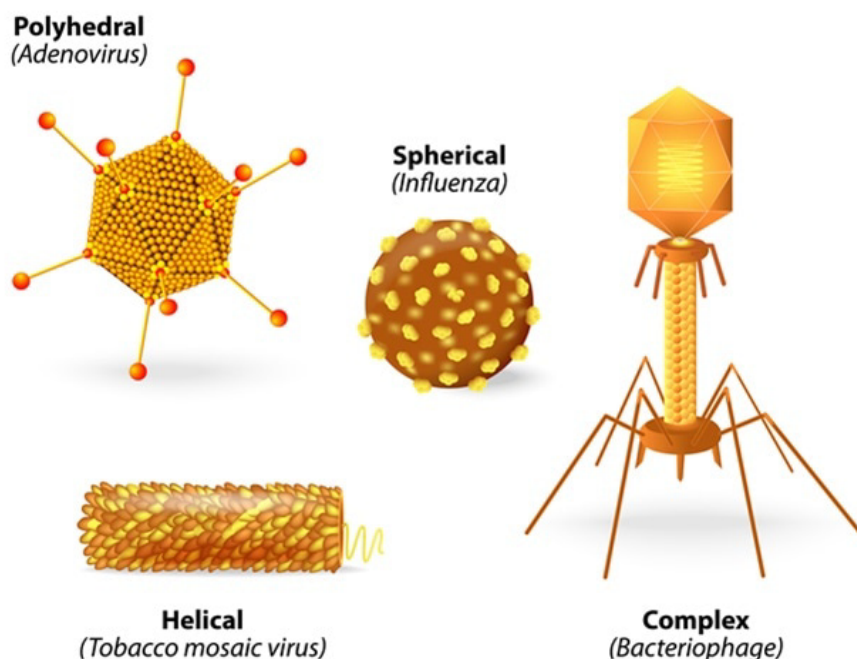


Fig. : Different Shapes of Virus

◆ Characteristics of Viruses

Viruses display both living and non-living characteristics.

Living Characteristics :

- Presence of genetic material.
- Ability to reproduce within host cells.
- Capability to mutate and evolve.

Non-Living Characteristics :

- Absence of cellular organization.
- Lack of cytoplasm and organelles.
- No independent metabolism.
- Ability to be crystallized like chemical substances.

Due to this dual nature, viruses are considered transitional forms between living and non-living entities.

Mode of Reproduction

Viruses reproduce only within living cells through a process involving several stages:

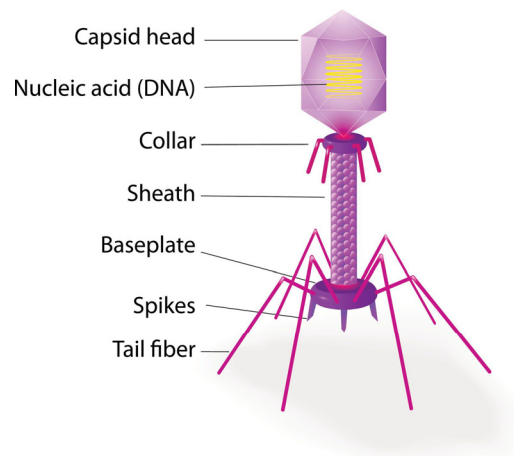
1. **Attachment** – The virus binds to specific receptors on the host cell surface.
2. **Penetration** – The viral genetic material enters the host cell.
3. **Replication** – The host cell machinery synthesizes viral components.
4. **Assembly** – New viral particles are assembled.
5. **Release** – Newly formed viruses are released, often destroying the host cell.

Host Specificity

Viruses are highly specific to their hosts. A virus that infects plants generally does not infect animals. Similarly, bacteriophages infect only bacteria.

Host specificity depends on the presence of suitable receptors on host cells.

Structure of bacteriophage



Diseases Caused by Viruses : Viruses are responsible for numerous diseases affecting plants, animals, and humans.

In Humans :

- Influenza
- Measles
- Polio
- Hepatitis
- COVID-19

In Plants :

- Tobacco mosaic disease
- Leaf curl disease

In Animals :

- Rabies
- Foot-and-mouth disease

◆ Economic and Scientific Importance

Although viruses are pathogenic in many cases, they have significant scientific importance :

- Used in vaccine development.
- Applied in gene therapy and genetic engineering.
- Useful in molecular biology research.
- Bacteriophages are employed to control harmful bacteria.

EXERCISE-1

➤ Very Short Answer Type Questions

- Q.1 Name the group of plants which produce seeds but not fruits.
- Q.2 In which animal phylum body cavity appears for the first time ?
- Q.3 What is the basic unit of classification ?
- Q.4 Name the highest taxonomic category.
- Q.5 Who proposed binomial nomenclature ?
- Q.6 What are viruses?
- Q.7 Why are viruses called obligate intracellular parasites?
- Q.8 Name the two types of genetic material found in viruses.
- Q.9 What is a capsid?
- Q.10 Name the virus that infects bacteria.

➤ Short Answer Type Questions – Type I

- Q.11 Define prokaryotes.
- Q.12 Name the reproductive organs of -
(i) Gymnosperms (ii) Angiosperms.
- Q.13 Which phylum includes segmented worms ?
- Q.14 Name two groups of warm blooded animals with four-chambered heart.
- Q.15 What are bryophytes ?
- Q.16 Explain why viruses are considered at the boundary of living and non-living.
- Q.17 Describe the structure of a virus.

Q.18 Write the steps involved in viral reproduction.

Q.19 Differentiate between viruses and bacteria.

Q.20 Mention four viral diseases in humans.

➤ Short Answer Type Questions – Type II

Q.21 Differentiate between dicotyledonae and monocotyledonae plants.

Q.22 Write any two important features found in platyhelminthes.

Q.23 Describe the body structure of sponges.

Q.24 Describe the 4 characteristic features of the largest phylum of kingdom Animalia.

Q.25 With the help of flow chart depict five kingdoms classification.

Q.26 How do animals of Porifera differ from animals of Cnidaria ?

Q.27 What is difference between bilateral symmetry and radial symmetry ?

Q.28 What do you mean by the term coelomate animals ? Give two examples.

Q.29 What are adaptations in fishes due to which they are aquatic ?

Q.30 Describe some flight adaptations of the birds.

➤ Long Answer Type Questions

Q.31 What is hierarchy of categories ? Describe the same.

Q.32 Describe the characteristics of kingdom Protista. Give two examples.

Q.33 Define pteridophytes. Compare between bryophytes and Pteridophytes.

- Q.34** Explain general characteristics of phylum Platyhelminthes and Nematoda. Give two examples of each.
- Q.35** What are differences between the following :
(a) Cartilaginous fishes and bony fishes ;
(b) Amphibia and Reptilia ;
- Q.36** Explain the structure, characteristics, and mode of reproduction of viruses.
- Q.37** Describe the economic importance and harmful effects of viruses.

Practical & Value Based Type Questions

- Q.38** During a field-trip some students visited an agricultural farm and saw a few birds eating earthworms. They enjoyed the scene and then they also started picking and killing the earthworms for pleasure. The farmer strongly objected and asked them to leave the field.
(a) What could be the reason behind such a behavior of the farmer ?
(b) What values do you find missing in the student's behavior ?
(c) Which phylum do earthworms belong to ?
(d) Write two identifying features of earthworm.
- Q.39** Rahul is having a beautiful pet dog 'Ginger'. One day, he observed a small insect between his toes. He removed it carefully from the toe and observed it curiously as chapter titled 'Diversity In Living Organisms' was being taught in the school.
(a) Identify the phylum to which the insect belongs to.
(b) Enlist any two characteristic features of this phylum.
(c) Comment on Rahul's behavior.
- Q.40** Ajay and mother were walking in the garden. Ajay saw mushrooms growing on decaying leaves. He tried to uproot them. He was stopped by his mother.
(a) Identify the kingdom to which the organism belongs to.
(b) Write the mode of nutrition exhibited by the mushrooms.
(c) Why did Ajay's mother stop him ?
- Q.41** Rohit was playing cricket in the ground with his friends. One of his friend who was standing near the boundary felt something climbing upon his feet. Subsequently he started crying in pain. Rohit being class nine students identified the climbing creature as leech and helped removing the leech.
(a) Name the phylum to which leech belongs.
(b) Write two identifying features of the phylum.
(c) Enlist two values shown by Rohit.
- Q.42** A group of children were playing a heap of sand. One of them collected some shelled creatures from the sand. Being taught lesson on diversity in School, one of them identified them as snail.
(a) Write two characteristic features of the phylum.
(b) What positive attitude towards learning is shown by the students ?

EXERCISE-2

- Q.1** Algae are –
(A) Autotrophic (B) Heterotrophic
(C) Both (D) None
- Q.2** Chlamydomonas is the example of -
(A) Algae (B) Fungi
(C) Lichens (D) None
- Q.3** Which class is called amphibians of the plant kingdom -
(A) Thallophyta
(B) Bryophyta
(C) Pteridophyta
(D) Gymnosperm
- Q.4** Example of pteridophyta is -
(A) Selaginella (B) Cycas
(C) Pinus (D) None
- Q.5** Cnidoblasts is characteristic feature of -
(A) Coelenterata (B) Porifera
(C) Protozoa (D) Arthropoda
- Q.6** Prawn (prawn) is the example of -
(A) Arthropoda (B) Annelida
(C) Coelenterata (D) Protozoa
- Q.7** Four chambered heart is found in -
(A) Draco
(B) Hemidactylus
(C) Tortoises and turtles
(D) Crocodile
- Q.8** In mollusca respiratory organ is called -
(A) Ctenidia (B) Lungs
(C) Book-lungs (D) All the above
- Q.9** In larval stage which symmetry is present in echinodermata -
(A) Radial (B) Biradial
(C) Bilateral (D) None of these
- Q.10** Whittaker failed to give any place to one of the following in his classification -
(A) Cyanobacteria (B) Virus
(C) Slime moulds (D) all above
- Q.11** Parazoa includes -
(A) Protozoans
(B) Porifera
(C) Parasites of invertebrates (mosozoa)
(D) None above
- Q.12** Viruses are essentially made up of -
(A) Proteins and nucleic acid
(B) Proteins and carbohydrates
(C) Lipids and nucleic acids
(D) Starch, proteins and lipid
- Q.13** Prokaryotes are included in the group -
(A) Monera (B) Basidiomycetes
(C) Bryophytes (D) Tracheophyta
- Q.14** Four kingdom system of classification was given by -
(A) Copeland (B) Whittaker
(C) Linnaeus (D) Von mohl
- Q.15** Which group of plants are called “Reptiles of plant world” -
(A) Pteridophytes (B) Gymnosperms
(C) Algae (D) Fungi
- Q.16** Which of the following organism has characters of both animals and plants -
(A) Blue-green algae (B) Euglena
(C) Moss (D) Cycas
- Q.17** Vascular bundle are found in -
(A) Thallophyta (B) Bryophyta
(C) Pteridophyta (D) Lichens
- Q.18** A branch of biology which deals with the identification, nomenclature and classification of organisms is called -
(A) Morphology (B) Ecology
(C) Taxonomy (D) Phytogeography
- Q.19** Binomial nomenclature was introduced by -
(A) John Ray
(B) A. P. de Candolle
(C) A. L. de Jussieu
(D) Carolus Linnaeus

- Q.20** Plants with hidden reproductive organs are included in -
 (A) Gymnosperms (B) Angiosperms
 (C) Phanerogams (D) Cryptogams
- Q.21** The basic unit of classification is -
 (A) Family (B) Genus
 (C) Species (D) Order
- Q.22** Which of the following kingdoms includes many kinds of unicellular eukaryotic organisms ?
 (A) Monera (B) Protista
 (C) Fungi (D) Animalia
- Q.23** Whale belongs to the class -
 (A) Pisces (B) Amphibia
 (C) Reptilia (D) Mammalia
- Q.24** Which group of animals are called 'flat worms'?
 (A) Porifera (B) Coelenterata
 (C) Platyhelminthes (D) Nematoda
- Q.25** Sweat and oil glands are present in the skin of -
 (A) Amphibians (B) Reptiles
 (C) Aves (D) Mammals
- Q.26** You are identifying a plant that possesses seeds but not fruits. It may belong to -
 (A) Pteridophyta (B) Gymnosperm
 (C) Bryophyta (D) Angiosperm
- Q.27** Seeds are naked in -
 (A) Gymnosperm (B) Angiosperm
 (C) Both (A) and (B) (D) Monocots
- Q.28** Two chambered heart occurs in -
 (A) crocodiles (B) fish
 (C) aves (D) amphibians
- Q.29** Skeleton is made entirely of cartilage in -
 (A) Sharks (B) Tuna
 (C) Rohu (D) None of these
- Q.30** One of the following is not an Annelid -
 (A) Nereis (B) Earthworm
 (C) Leech (D) Urchins
- Q.31** Viruses are called acellular because they:
 (A) Are very small
 (B) Lack cellular organization
 (C) Cause diseases
 (D) Contain DNA
- Q.32** A virus contains :
 (A) Both DNA and RNA
 (B) Only DNA
 (C) Only RNA
 (D) Either DNA or RNA
- Q.33** The protein coat of a virus is called :
 (A) Capsule (B) Capsid
 (C) Cytoplasm (D) Envelope
- Q.34** Viruses reproduce :
 (A) By binary fission
 (B) By budding
 (C) Only inside host cells
 (D) Independently
- Q.35** The virus that infects bacteria is called:
 (A) Retrovirus (B) Bacteriophage
 (C) Retroplasm (D) Pathogen
- Q.36** Who first discovered viruses?
 (A) Robert Hooke
 (B) Louis Pasteur
 (C) Dmitri Ivanovsky
 (D) Darwin
- Q.37** Viruses can be seen only with:
 (A) Simple microscope
 (B) Compound microscope
 (C) Electron microscope
 (D) Naked eye
- Q.38** Which of the following is a viral disease?
 (A) Malaria (B) Tuberculosis
 (C) Measles (D) Typhoid
- Q.39** The outer lipid layer present in some viruses is called :
 (A) Capsomere (B) Envelope
 (C) Core (D) Cytoplasm
- Q.40** Viruses are considered non-living outside the host because they :
 (A) Are too small
 (B) Cannot move
 (C) Show no metabolic activity
 (D) Lack DNA

EXERCISE-3

(Previous Year Questions - NTSE & NSO)

- Q.1** Planaria is kept in which group –
 (A) Coelentrata (B) Platyhelminthes
 (C) Nematoda (D) Annelida
- Q.2** Which of the following is a example of Bryophyte -
 (A) Moss (B) Fern
 (C) Pinus (D) Algae
- Q.3** Dissimilarity found in Aves and Mammalia is -
 (A) Warm Blooded Animal
 (B) Lay eggs
 (C) Breathe through Lungs
 (D) Four chambered Heart
- Q.4** Which one of the following plant belong to division Pteridophyta ?
 (A) Funaria (B) Cycas
 (C) Riccia (D) Fern
- Q.5** Locomotion by tube feet is found in phylum
 (A) Mollusca (B) Coelenterata
 (C) Echinodermata (D) Annelida
- Q.6** Example of coprophagy is
 (A) horse (B) man
 (C) cow (D) rabbit
- Q.7** Which group of animals are found only in sea ?
 (A) Mollusca (B) Echinodermata
 (C) Porifera (D) Protozoa
- Q.8** Match the name of phylum given under column-I, with the name of animal given under column-II and state the correct sequence alphabetically.
- | Column I | | Column II | |
|--------------------|------------|-----------|--|
| (i) Protozoa | (a) Sponge | | |
| (ii) Porifera | (b) Apis | | |
| (iii) Coelenterata | (c) Amoeba | | |
| (iv) Arthropoda | (d) Hydra | | |
- | | (i) | (ii) | (iii) | (iv) |
|-----|-----|------|-------|------|
| (A) | a | b | c | d |
| (B) | d | c | b | a |
| (C) | c | a | d | b |
| (D) | b | d | a | c |
- Q.9** If a particular animal has shelled egg, hair and teats on the body and has cloaca, then it may be a connecting link between -
 (A) Reptile and aves
 (B) Aves and mammal
 (C) Reptile and mammal
 (D) Mammal and aves
- Q.10** In national parks, conservation is provided to
 (A) only plant species
 (B) only animal species
 (C) both plants and animals
 (D) complete ecosystem
- Q.11** Botanical name of amla is :
 (A) Medicago sativa
 (B) Emblica officinalis
 (C) Zinblica officinate
 (D) Ocimum sanctum
- Q.12** Binomial nomenclature was introduced by :
 (A) John Ray
 (B) Aristotle
 (C) A.P De Candolle
 (D) Carolus Linnaeus
- Q.13** Which one of the following class of animals has coelomic cavity filled with blood ?
 (A) Nematoda (B) Annelida
 (C) Arthropoda (D) Protozoa
- Q.14** Which two of the following animals belong to the same Phylum ?
 (A) Starfish and Nereis
 (B) Antedon and Starfish
 (C) Antedon and Chiton
 (D) Nereis and Chiton
- Q.15** Which one of the following is having conducting tissue ?
 (A) Fern (B) Funaria
 (C) Riccia (D) Marchantia
- Q.16** Which of the following two are only applicable for birds ?
 (A) Warm blooded and four chambered heart
 (B) Feather and breathing by lungs
 (C) Warm blooded and egg laying
 (D) Egg laying and with Feather

- Q.17** Which of the following statement is true about dicotyledons.
 (A) Their root system consists of similar fibrous roots
 (B) Their leaves have paralalled venation
 (C) Their vascular bundles are arranged in a ring
 (D) Their flowers are trimerous
- Q.18** A plant has body differentiated into root, stem & leaves. It also has tissue for conduction of water and other substances, but has naked embryo. Under which type will such plants be classified ?
 (A) Gymnosperms (B) Angiosperms
 (C) Bryophyta (D) Pteridophyta
- Q.19** Choose the correct hierarchy in classification group.
 (A) Phylum - class - order - family - species
 (B) Phylum - order - class - family - species
 (C) Phylum - class - family - order - species
 (D) Phylum - order - family - class - species
- Q.20** All the cats in the world belong to the same species.
 (A) Panthera leo (B) Panthera tigris
 (C) Panthera pardus (D) Phelis domestics
- Q.21** this animal lays eggs as reptiles but has mammary glands and hair on the skin like mammals.
 (A) Duckbill (B) Peripatus
 (C) Bear (D) Lungfish
- Q.22** A fragment of DNA that provides complete information about one protein is referred to as for that protein.
 (A) Nucleotide (B) Enzyme
 (C) Nucleoside (D) Gene
- Q.23** The endosperm of angiosperms is
 (A) haploid (B) diploid
 (C) triploid (D) polyploid
- Q.24** Match the following
- | Column I | Column II |
|-------------------------------|------------------------|
| (1) collar cells | (a) coelenterate |
| (2) diploblast | (b) porifera |
| (3) closed circulatory system | (c) echinodermata |
| (4) water vascular system | (d) Annelida |
| (A) 1-a, 2-b, 3-c, 4-d | (B) 1-a, 2-b, 3-c, 4-d |
| (C) 1-b, 2-a, 3-d, 4-c | (D) 1-a, 2-b, 3-d, 4-c |
- Q.25** Botanical name of Tulsi is
 (A) Ocimum sanctum
 (B) Saraca indica
 (C) Ficus benghalensis
 (D) Eagle marmelos
- Q.26** The group of amphibian plants is
 (A) Funaria, Marchantia
 (B) Marsilia, Horse-tail
 (C) Pinus, Cycas
 (D) Typha, Hydrilla.
- Q.27** The example of an egg laying mammal is
 (A) Bat (B) Whale
 (C) Echidna (D) Kangaroo
- Q.28** Green gland is present in
 (A) the body of pigeon
 (B) the body of prawn
 (C) the body of rohu fish
 (D) in cactus plant
- Q.29** Botanical name of Amla is.....
 (A) Ocimum sanctum
 (B) Phyllanthus emblica
 (C) Saraca indica
 (D) Ficus bengalensis
- Q.30** Which plant leaves have a parallel venation ?
 (A) Peepal leaves
 (B) Hibiscus leaves
 (C) Banana leaves
 (D) Banyan tree leaves
- Q.31** Obelia belongs to –
 (A) Coelenterata (B) Porifera
 (C) Annelida (D) Arthropoda
- Q.32** In binomial nomenclature first word indicates –
 (A) Species (B) Genus
 (C) Sub species (D) None of these
- Q.33** Whale belongs to -
 (A) Mammalia (B) Amphibia
 (C) Annelida (D) Reptilia
- Q.34** Botanical name of Margosa (Neem) is:
 (A) Azadirachta indica
 (B) Pisum sativum
 (C) Cassia fistula
 (D) Brassica compestris

- Q.35** Which is the odd one ?
 (A) Planaria (B) Liver-fluke
 (C) Ascaris (D) Tape-worm
- Q.36** An egg laying mammal is
 (A) Kangaroo (B) Bat
 (C) Whale (D) Echidna
- Q.37** Animals of which phylum are pseudocoelomate ?
 (A) Porifera (B) Platyhelminthes
 (C) Aschelminthes (D) Mollusca
- Q.38** The example of hydrophytes are
 (A) Hydrilla, Calotropis
 (B) Lotus, Salsola
 (C) Moss, Lichen
 (D) Segetaria, Trapa
- Q.39** Water vascular system is found in
 (A) Cnidaria
 (B) Echinodermata
 (C) Mollusca
 (D) Annelida
- Q.40** Amphibians of plant kingdoms are
 (A) Bacteria (B) Gymnosperm
 (C) Bryophyta (D) Algae
- Q.41** From the following which animal is warm blooded, presence of mammary glands and body divided into head, neck, trunk and tail.
 (A) Penguin (B) Tortoise
 (C) Pigeon (D) Bat
- Q.42** Body structure of different animals is given below. Identify to which phylum the animal belongs.
 (1) Long, cylindrical, metamerically segmented.
 (2) Triploblastic, bilaterally symmetrical, eucoelomate.
 (3) They have setae or parapodia or suckers for locomotion.
 (A) Arthropoda (B) Annelida
 (C) Aschelminthes (D) Mollusca

ANSWER KEY

EXERCISE - 2

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	A	B	A	A	A	D	A	B	B	A	A	A	A	A
Ques.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	B	C	C	D	D	C	B	D	C	D	B	A	B	A	D
Ques.	31	32	33	34	35	36	37	38	39	40					
Ans.	B	D	B	C	B	C	C	C	B	C					

EXERCISE - 3

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	A	B	D	C	D	B	C	C	D	B	D	C	B	A
Ques.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	B	C	A	A	D	A	D	C	C	A	A	C	A	B	C
Ques.	31	32	33	34	35	36	37	38	39	40	41	42			
Ans.	A	B	A	A	C	D	C	D	B	C	D	B			

SOLUTIONS

EXERCISE-1

➤ Very Short Answer Type Questions

- Sol.1** Gymnosperm produce seeds but not fruits.
- Sol.2** In Annelida body cavity appeared for the first time.
- Sol.3** Species is the basic unit of classification.
- Sol.4** Kingdom is the highest taxonomic category.
- Sol.5** Carolus Linnaeus proposed binomial nomenclature.
- Sol.6** Viruses are acellular, microscopic infectious agents that contain genetic material enclosed within a protein coat and can reproduce only inside a living host cell.
- Sol.7** Because they can reproduce only inside the living cells of a host organism.
- Sol.8** DNA or RNA (only one type in a single virus).
- Sol.9** The protective protein covering that surrounds the genetic material of a virus.
- Sol.10** Bacteriophage.

➤ Short Answer Type Questions – Type I

- Sol.11** Prokaryotes are microscopic organisms that have neither a distinct nucleus nor other specialized organelles.
- Sol.12** Reproductive organs in gymnosperms are male and female cones.
In angiosperms, flowers are the reproductive organs.
- Sol.13** Annelida includes segmented worms.
- Sol.14** Aves and mammalia are two groups of warm blooded animals with four-chambered heart.
- Sol.15** Bryophytes are simplest, primitive and non-vascular plant. They are also known as amphibian of plant kingdom.

Sol.16 Viruses show both living and non-living characteristics.

Living characteristics include presence of genetic material, ability to reproduce (inside host), and mutation. Non-living characteristics include absence of cellular structure, lack of metabolism, and ability to be crystallized. Therefore, they are placed at the borderline of living and non-living organisms.

Sol.17 A virus consists of genetic material (DNA or RNA) enclosed within a protein coat called capsid. The capsid is made of protein subunits called capsomeres. Some viruses also possess an outer lipid envelope derived from the host cell membrane. The structure is simple and lacks cytoplasm or organelles.

Sol.18 The steps of viral reproduction are:

1. Attachment to host cell.
2. Entry of genetic material.
3. Replication using host cell machinery.
4. Assembly of new viral particles.
5. Release of new viruses, often destroying the host cell.

Sol.19

Feature	Viruses	Bacteria
Cellular structure	Acellular	Cellular
Size	Smaller	Larger
Genetic material	DNA or RNA (one type)	Both DNA and RNA
Reproduction	Only inside host	Independent reproduction
Living nature	Borderline	Living organism

- Sol.20**
1. Influenza
 2. Measles
 3. Polio
 4. COVID-19

➤ Short Answer Type Questions – Type II

Sol.21

Dicotyledonae	Monocotyledonae
(i) The seeds have two cotyledons	(i) They have only one cotyledon
(ii) Leaves have reticulate venation	(ii) Leaves show parallel venation
(iii) They have tap root system	(iii) They have fibrous root system

Sol.22 Two important features found in platyhelminthes are :

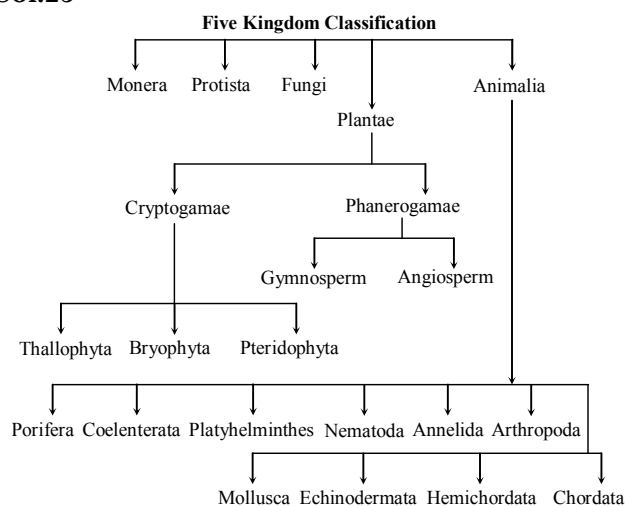
- body is dorsoventrally flattened.
- body is soft and unsegmented.

Sol.23 Sponges are simplest, multicellular organism with cellular level of organization. They are mostly asymmetrical and body is perforated by numerous pores called ostia. Body contains a cavity called as spongocoel.

Sol.24 Largest phylum of kingdom animalia is Arthropoda. The characteristic features are :

- Triploblastic organism with bilaterally symmetry and segmentation.
- Each body segment usually bears paired lateral and jointed legs or appendages
- Coelom is reduced and body is covered with a thick chitinous covering
- Respiration through general body surface by gills, air tubes or book-lungs.

Sol.25



Sol.26

Porifera	Cnidaria
(i) They contain numerous pores or holes in the body.	(i) They contain a body cavity, which consist of a single opening
(ii) They possess an exoskeleton	(ii) They do not possess an exoskeleton
(iii) They are mostly asymmetrical	(iii) They are radially symmetrical
(iv) They have cellular level of organisation	(iv) They have tissue level of organisation

Sol.27

Bilateral	Radial
The animal's body can be divided into two equal halves by one plane Eg- Earthworm and cockroach	The animal's body can be divided into two equal halves by any vertical plane passing through the central axis Eg- Hydra and starfish

Sol.28 Animals that possess a true body cavity called coelom lined by mesoderm are termed as coelomate animals.
Eg. : Annelida, Arthropoda etc.

Sol.29 Adaptations in fishes for aquatic life are :

- Body is stream-lined to cut water current
- Respiratory organ are gills
- They have muscular tail and fins for movement.

Sol.30 Flight adaptations in birds :

- Bones have air cavities. This makes the bird light
- Body is covered with soft feathers
- The forelimbs are modified to form wings
- Lungs have additional bag like membranous extensions called as air sacs

➤ Long Answer Type Questions

Sol.31 Hierarchy means ranking a living creature according to their growing similarities. The hierarchical order of classifying organism is :

Kingdom → Phylum/ Division → Class → Order → Family → Genus → Species.

- (i) Kingdom : It is the highest category.
- (ii) Phylum/ Division : Many classes with some common characters
- (iii) Class : Similar orders are placed together in a class
- (iv) Order : A number of families having several common characters
- (v) Family : Group of Genera having several common characters
- (vi) Genus : It is a group of closely related species with common ancestry
- (vii) Species : Group of organisms capable of interbreeding to produce offspring

Sol.32 Characteristics of kingdom protista are :

- (i) They are unicellular eukaryotes
 - (ii) Most of them are aquatic
 - (iii) They are autotroph as well as heterotroph
- Eg. : Amoeba, Euglena and Slime moulds.

Sol.33 Pteridophyta is a vascular plant which does not have flower and does not produce seeds.

Pteridophytes	Bryophytes
1. Vascular tissues are present	1. Vascular tissue are absent
2. Body is differentiated into leaf, stem and root. Eg-Fern	2. Plant body is differentiated into leaf, stem and root like structures Eg -Moss

Sol.34 Platyhelminthes :

- (i) They are dorsoventrally flat hence called as "Flat-worm"
 - (ii) They exhibit bilateral symmetry
 - (iii) They are acoelomate
 - (iv) They are triploblastic animals with organs level of organization
 - (v) Suckers and hooks are usually present
- Eg. : Liver flukes, Tapeworm, Planaria

Nematoda :

- (i) They are cylindrical in shape
 - (ii) They are parasitic or free-living with complete alimentary canal
 - (iii) They are triploblastic, unsegmented and show bilateral symmetry
 - (iv) Body cavity is not a true coelom, but a pseudocoelom
 - (v) Sexes are separate
- Eg- Ascaris, Wuchereria

Sol.35

(a) Cartilaginous	Bony fishes
1. Endoskeleton is made up of cartilage	1. Endoskeleton is entirely made up of bone
2. Mouth ventral	2. Mouth terminal
3. Gills are naked	3. Gills are covered by operculum
4. No swim bladder present	4. Swim bladder is usually present
Eg : Scoliodon, Trygon, Torpedo	Eg : Labeorohita, Anabas

(b) Amphibia	Reptilia
1. They live on land as well as in water	1. They live usually on land
2. Skin is smooth and moist	2. Skin is dry, hard and scaly
3. Fertilization is external	3. Fertilization is internal

Sol.36 Viruses are acellular infectious particles composed of genetic material enclosed within a protein coat called capsid. The genetic material may be DNA or RNA. Some viruses possess an outer lipid envelope. They lack cytoplasm, organelles, and cellular machinery.

Viruses show both living and non-living characteristics. They contain genetic material and can reproduce inside host cells, but they lack metabolism and cellular organization.

The reproduction process includes attachment to host cell, penetration, replication using host machinery, assembly of viral components, and release of new viruses. This process often damages or destroys the host cell.

Thus, viruses are unique biological entities that depend entirely on host cells for survival.

Sol.37 Viruses cause many diseases in plants, animals, and humans, leading to economic loss. In humans, they cause influenza, measles, hepatitis, and COVID-19. In plants, they cause tobacco mosaic disease

and leaf curl disease, reducing agricultural productivity. In animals, they cause rabies and foot-and-mouth disease.

Despite their harmful effects, viruses are useful in biotechnology and medicine. They are used in vaccine production, gene therapy, and molecular biology research. Bacteriophages help control harmful bacteria.

Therefore, viruses have both harmful and beneficial impacts on society.

➤ Practical & Value Based Type Questions

- Sol.38** (a) The possible reason could be farmer knows the value of earthworm that how they help in making soil fertile for agriculture
(b) Students showed merciless behaviour towards earthworm and do not know the value of every species present in nature
(c) Earthworm belongs to annelida
(d) Two identifying feature of earthworm are :
(i) These are elongated with metamerically segmented body
(ii) They have true coelom
- Sol.39** (a) Insect belongs to phylum Arthropoda
(b) Two characteristic features of this phylum are
(i) Jointed appendages
(ii) Coelom is reduced and body is covered with a thick chitinous covering
(c) Rahul showed a caring behaviour towards his dog and also a behavior of curious student who wants to learn from nature
- Sol.40** (a) Organism belongs to Kingdom fungi
(b) Mushrooms are saprotrophic because they fed on dead and decaying matter
(c) Ajay's mother stopped him because mushroom may have been poisonous and some toxic substances may have caused irritation in Rahul's hands

- Sol.41** (a) Leech belongs to Annelida
(b) Two characteristic features of the phylum are
(i) They have true body cavity
(ii) They have metamerically segmented body
(c) Rohit showed caring nature for his friend and also he showed that he is a studious student who studies his chapter well.

- Sol.42** (a) Two features of phylum mollusca are:
(i) They have soft and unsegmented body
(ii) Body is divided into head, visceral mass and ventral foot
(b) They study well and have better understanding of chapter

EXERCISE-2

- Sol.1** [A]
Algae are able to produce their own food.
- Sol.2** [A]
Chlamydomonas is the example of algae.
- Sol.3** [B]
Bryophyta is called amphibians of plant kingdom.
- Sol.4** [A]
Selaginella is an example of pteridophyta
- Sol.5** [A]
Coelenterates possess special cells bearing stinging capsules called cnidoblast.
- Sol.6** [A]
Prawn belongs to Arthropoda.
- Sol.7** [D]
Crocodile being reptile possess 4-chambered heart.
- Sol.8** [A]
Respiration in mollusca is by ctenidia.

- Sol.9 [C]**
In larval stage bilateral symmetry is present in echinodermata and radially symmetrical in adult.
- Sol.10 [B]**
Whittaker failed to give any place to virus in his classification.
- Sol.11 [B]**
Parazoa includes porifera, where cells are loosely aggregated.
- Sol.12 [A]**
Viruses are essentially made up of proteins and nucleic acid.
- Sol.13 [A]**
Monera consist of unicellular prokaryotes.
- Sol.14 [A]**
Four kingdom system of classification was given by copeland.
- Sol.15 [A]**
Pteridophytes are called reptiles of plant world.
- Sol.16 [B]**
Euglena shows both autotrophic and heterotrophic nutrition.
- Sol.17 [C]**
Vascular bundles are found in Pteridophytes.
- Sol.18 [C]**
A branch of biology which deals with the identification, nomenclature and classification of organisms is called taxonomy.
- Sol.19 [D]**
Binomial nomenclature was introduced by carolus linnaeus.
- Sol.20 [D]**
Crytogams have hidden reproductive organs.
- Sol.21 [C]**
The basic unit of classification is species.
- Sol.22 [B]**
Protista contain unicellular eukaryotes.
- Sol.23 [D]**
Whale belongs to the class mammalia.
- Sol.24 [C]**
Platyhelminthes are also called flatworms.
- Sol.25 [D]**
Sweat and oil glands are present in the skin of mammals.
- Sol.26 [B]**
Gymnosperm are group of plans producing naked seeds..
- Sol.27 [A]**
Seeds are naked in gymnosperm.
- Sol.28 [B]**
Two chambered heart occurs in fish.
- Sol.29 [A]**
Sharks are chondriochythes.
- Sol.30 [D]**
Urchins comes under phylum Echinodermata.
- Sol.31 [B]**
Viruses do not have cellular structure such as cytoplasm or organelles.
- Sol.32 [D]**
A virus contains only one type of nucleic acid—either DNA or RNA.
- Sol.33 [B]**
Capsid is the protein covering protecting viral genetic material.
- Sol.34 [C]**
Viruses depend on host cell machinery for reproduction.
- Sol.35 [B]**
Bacteriophages specifically infect bacteria.
- Sol.36 [C]**
Ivanovsky first observed virus-like agents in tobacco mosaic disease.
- Sol.37 [C]**
Viruses are too small to be seen under a light microscope.

Sol.38 [C]
Measles is caused by a virus; others are caused by bacteria or protozoa.

Sol.39 [B]
Some viruses possess a lipid envelope derived from host cell membrane.

Sol.40 [C]
Outside host cells, viruses remain inactive and show no metabolism.

EXERCISE-3

Sol.1 [B]
Planaria are flat-worm which comes under the group of platyhelminthes.

Sol.2 [A]
Moss is an example of bryophyte.

Sol.3 [B]
Mammals are viviparous and aves are oviparous.

Sol.4 [D]
Fern belong to division pteridophyta.

Sol.5 [C]
Tube feet is found in phylum Echinodermata.

Sol.6 [D]
They eat their own faeces.

Sol.7 [B]
Echinodermata are exclusively marine.

Sol.8 [C]
Protozoa–Amoeba, Porifera–Sponge, Coelenterata–Hydra, Arthropoda–Apis.

Sol.9 [C]
These character shows connecting link between reptiles and mammal.

Sol.10 [D]
In national parks, conservation is provided to complete ecosystem.

Sol.11 [B]
Botanical name of amla is *emblica officinalis*.
Medicago sativa–Alfalfa, Ocimum sanctum–Tulsi, Zingiber officinale–Ginger.

Sol.12 [D]
Binomial nomenclature was introduced by carolus linnaeus.

Sol.13 [C]
Arthropoda has coelomic cavity filled with blood.

Sol.14 [B]
They belong to phylum echinodermata.

Sol.15 [A]
Fern belongs to pteridophyta.

Sol.16 [B]
Presence of feather and breathing by lungs having airsacs are present only in birds.

Sol.17 [C]
In dicots vascular bundles are arranged in a ring.

Sol.18 [A]
Gymnosperms have naked seeds.

Sol.19 [A]
The hierarchy in classification is kingdom. Phylum - Class - Order - Family - Genus - species.

Sol.20 [D]
All cats in the world belong to *Felis domesticus*.

Sol.21 [A]
Duckbill shows characters of both reptiles and mammals.

Sol.22 [D]
A fragment of DNA that provides complete information about one protein is referred to as genes.

Sol.23 [C]
Endosperm is formed due to triple fusion.

Sol.24 [C]
Collar cells - porifera, diploblast - coelenterate, closed circulatory system - annelida, water vascular system - echinodermata.

- Sol.25 [A]**
Botanical name of Tulsi is *Ocimum sanctum*.
Saraca indica-Ashoka, *Ficus benghalensis*-Banyan tree, *Aegle marmelos*-Indian bael.
- Sol.26 [A]**
They are examples of bryophytes.
- Sol.27 [C]**
The example of an egg laying mammals is echidna.
- Sol.28 [A]**
Preen gland are oil glands present in birds.
- Sol.29 [B]**
The botanical name of Amla is *Phyllanthus emblica*, *Ocimum sanctum*-Tulsi, *Saraca indica*-Ashoka, *Ficus bengalensis*-Banyan tree.
- Sol.30 [C]**
Banana are monocots having parallel venation.
- Sol.31 [A]**
Obelia belongs to coelenterata.
- Sol.32 [B]**
In binomial nomenclature first word indicates Genus and second word indicate Species.
- Sol.33 [A]**
Whale belongs to mammalia.
- Sol.34 [A]**
Botanical name of Margosa is *Azadirachta indica*. *Pisum sativum*-Pea, *Cassia fistula*-laburnum, *Brassica campestris*-Fields mustard.
- Sol.35 [C]**
Ascaris belongs to Nematoda and rest all belong to Platyhelminthes.
- Sol.36 [D]**
Echidna is an egg laying mammal.
- Sol.37 [C]**
Animals of aschelminthes are pseudocoelomate.
- Sol.38 [D]**
Segetaria and *trapa* are the examples of hydrophytes.
- Sol.39 [B]**
Echinodermata has water vascular system.
- Sol.40 [C]**
Amphibians of plant kingdom are bryophyta.
- Sol.41 [D]**
Bat is a mammal.
- Sol.42 [B]**
Annelida are metamerically segmented, triploblastic, bilaterally symmetrical, coelomate. They have parapodia or setae or suckers for locomotion.

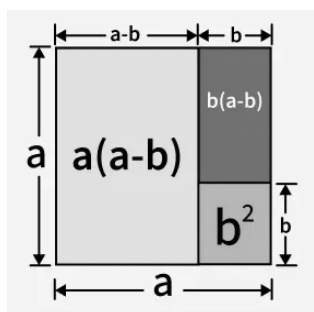
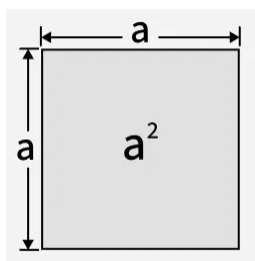
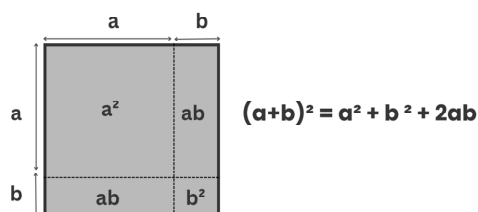
MATHEMATICS

EXPLORING ALGEBRAIC IDENTITIES

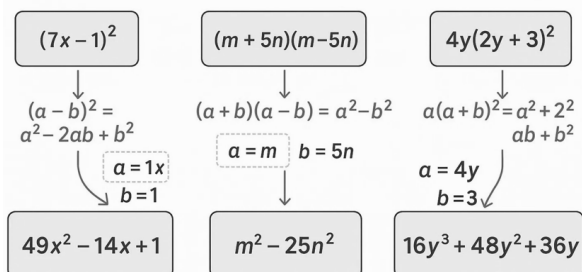
Chapter Outline

- ✧ Algebraic Identities
- ✧ Visualising Algebraic identities
Using Geometric Models
- ✧ Factorization
- ✧ Rational Expressions
- ✧ H.C.F. and L.C.M. of Polynomials

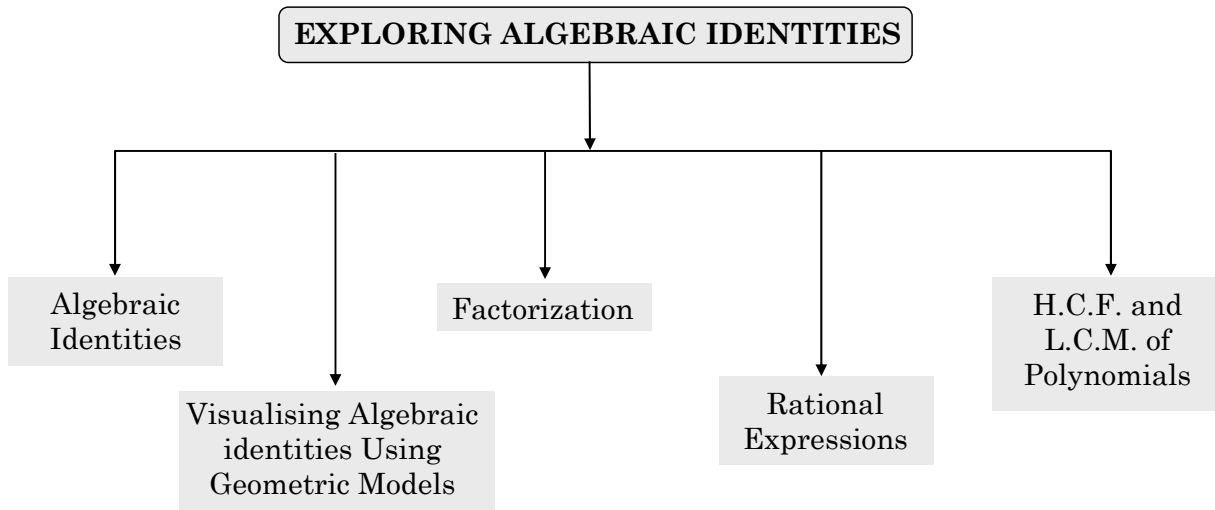
Geometric Representation of $(a+b)^2$



Applying Algebraic Identities



MIND MAP



EXPLORING ALGEBRAIC IDENTITIES

Algebraic Identities

- (i) $(a + b)^2 = a^2 + 2ab + b^2$
- (ii) $(a - b)^2 = a^2 - 2ab + b^2$
- (iii) $a^2 - b^2 = (a + b)(a - b)$
- (iv) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
- (v) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- (vi) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
- (vii) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
- (viii) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
- (ix) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

Special case : if $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$.

Visualising Algebraic identities Using Geometric Models

Algebraic identities are equations that are always true for all values of variables. In the previous classes, we have read about the following identities

$$(a + b)^2 = a^2 + 2ab + b^2, (a - b)^2 = a^2 - 2ab + b^2 \text{ and } a^2 - b^2 = (a + b)(a - b).$$

In this section, we shall revise previous identities and learn some new ones. We will also visualize identities geometrically and deal with various problems based on these identities.

Identity 1 : $(a + b)^2 = a^2 + 2ab + b^2$

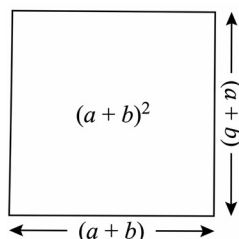
Proof : LHS $= (a + b)^2 = (a + b)(a + b) = a(a + b) + b(a + b)$
 $= a^2 + ab + ab + b^2 = a^2 + 2ab + b^2 = \text{RHS}.$

Geometric Model

Algebraic identity $(a + b)^2$ can be derived in algebraic form by the geometrical approach.

Take a square whose side length is $(a + b)$ units.

Therefore, the area of the square is $(a + b)^2$ sq. units.

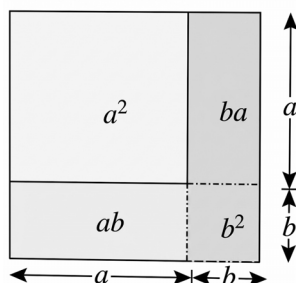


Now divide this square into four different parts vertically and horizontally such that the lengths of the parts are a units and b units.

Draw lines inside the square so that it forms :

- One larger square of side a units.
- One small square of side b units.
- Two rectangles of dimensions $a \times b$ and $b \times a$.

The area of the square $(a + b)^2$ is also equal to the sum of the areas of the individual squares and rectangles.



So, sum of areas of parts = $a^2 + ba + ab + b^2$

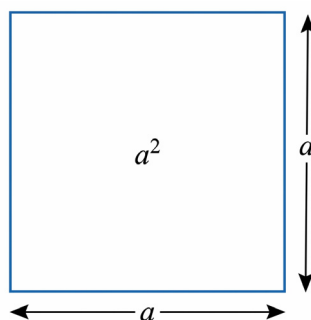
Therefore, $(a + b)^2 = a^2 + ba + ab + b^2 = a^2 + 2ab + b^2$

Identity 2 : $(a - b)^2 = a^2 - 2ab + b^2$

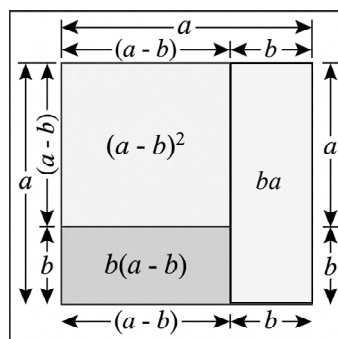
Proof : LHS = $(a - b)^2 = (a - b)(a - b) = a(a - b) - b(a - b)$
 $= a^2 - ab - ab + b^2 = a^2 - 2ab + b^2 = \text{RHS.}$

Geometric Model

Let us consider a square whose sidelength is a units, such that its area is equal to a^2 sq. units. For proving $(a - b)^2$ geometrically, we have to split the square into three different geometrical shapes. First, take a small square from this, whose sidelength is $(a - b)$ as shown in the following figure.



Here, we get two rectangles: one is horizontal and the other is vertical. Let's add the areas of all these three figures.



Area of square with side $(a - b) = (a - b)^2$

Area of vertical rectangle = $a \times b = ab$

Area of horizontal rectangle = $(a - b) \times b = b(a - b)$

Area of square with side $a = \text{Area of square with side } (a - b) + \text{Area of two rectangles}$

$$\Rightarrow a^2 = (a - b)^2 + ab + b(a - b)$$

$$\Rightarrow a^2 = (a - b)^2 + ab + ab - b^2$$

$$\Rightarrow a^2 = (a - b)^2 + 2ab - b^2$$

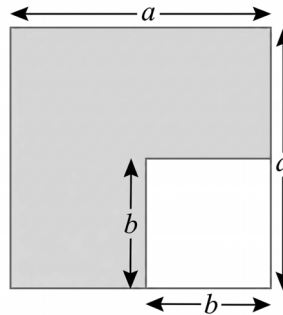
Therefore, $(a - b)^2 = a^2 - 2ab + b^2$

Identity 3 : $a^2 - b^2 = (a + b)(a - b)$

Proof : $\text{RHS} = (a + b)(a - b) = a(a - b) + b(a - b)$
 $= a^2 - ab + ab - b^2 = a^2 - b^2 = \text{LHS}.$

Geometric Model

Take a square whose sidelength is a units. Therefore, the area of the square is a^2 sq. units.



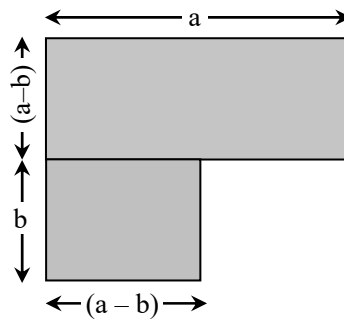
Draw a small square with the sidelength of b units at a corner of the square.

So, the area of the small square is b^2 . Therefore, area of remaining shape (new geometric shape) $= a^2 - b^2$.

Now split the shape into two different rectangles. The width of the larger rectangle is $(a - b)$, and that of the small rectangle is b .

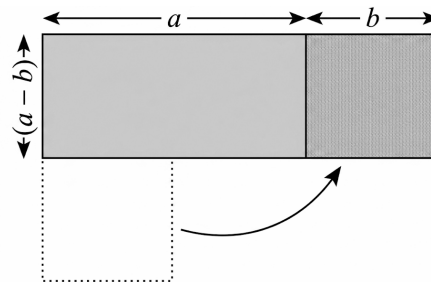
Now, cut out the small rectangle and put it next to the larger rectangle as shown.

$\therefore$ The area of the combined rectangle $= (a + b)(a - b)$



This rectangle has the same area as the original shape.

$\therefore a^2 - b^2 = (a + b)(a - b)$

**Identity 4: $(x + a)(x + b) = x^2 + (a + b)x + ab$**

Proof : $\text{LHS} = (x + a)(x + b) = x(x + b) + a(x + b)$
 $= x^2 + bx + ax + ab = x^2 + (a + b)x + ab = \text{RHS}.$

Geometric Model

Take a rectangle. Divide it into two parts horizontally such that their lengths become x units and a units.

Now, divide the same rectangle vertically such that the length become x units and b units as shown.



The length of the side of the square is x units.

So, area of the square = x^2 units.

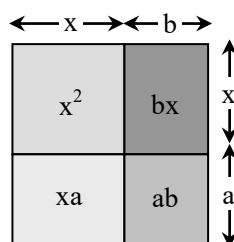
The length and width of rectangle are b units and x units respectively.

Then its area = bx sq. units.

The length and width of another rectangle are x units and a un respectively.

Then its area = ax sq. units.

The length and width of another rectangle are b units and a un Area of this rectangle = ba sq. units.



$$\begin{aligned}\text{So, } (x + a)(x + b) &= x^2 + xa + bx + ba \\ &= x^2 + (a + b)x + ab\end{aligned}$$

Identity 5: $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

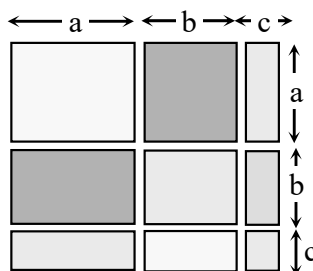
Proof : $\text{LHS} = (a + b + c)^2 = (a + b + c)(a + b + c)$

$$= a(a + b + c) + b(a + b + c) + c(a + b + c) = a^2 + ab + ac + ab + b^2 + bc + ac + bc + c^2$$

$$= a^2 + b^2 + c^2 + ab + ab + ac + ac + bc + bc = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc = \text{RHS}$$

Geometric Model

Take a square and divide it vertically into three different parts by drawing two lines. The lengths are a units, b units and c units respectively as shown in the figure.



Divide the square horizontally into three parts such that their sidelengths become a units, b units and c units respectively, as shown in the figure.

Now the sidelength of the original square is $(a + b + c)$ units.

Therefore, the area of the square is

$$(a + b + c) \times (a + b + c) = (a + b + c)^2$$

Here, the square whose area is $(a + b + c)^2$, is divided into three square and six rectangle.

Length of each side of three squares are a units, b units and c units.

So, their areas are a^2 sq. units, b^2 sq. units and c^2 sq. units respectively.

The lengths of sides of two rectangles are a units and b units.

So, the area of each rectangle is ab sq. units.

The lengths of sides of two rectangles are c units and a units.
 So, the area of each rectangle is ca sq. units.
 The lengths of sides of two rectangles are b units and c units.
 So, the area of each rectangle is bc sq. units.
 So, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

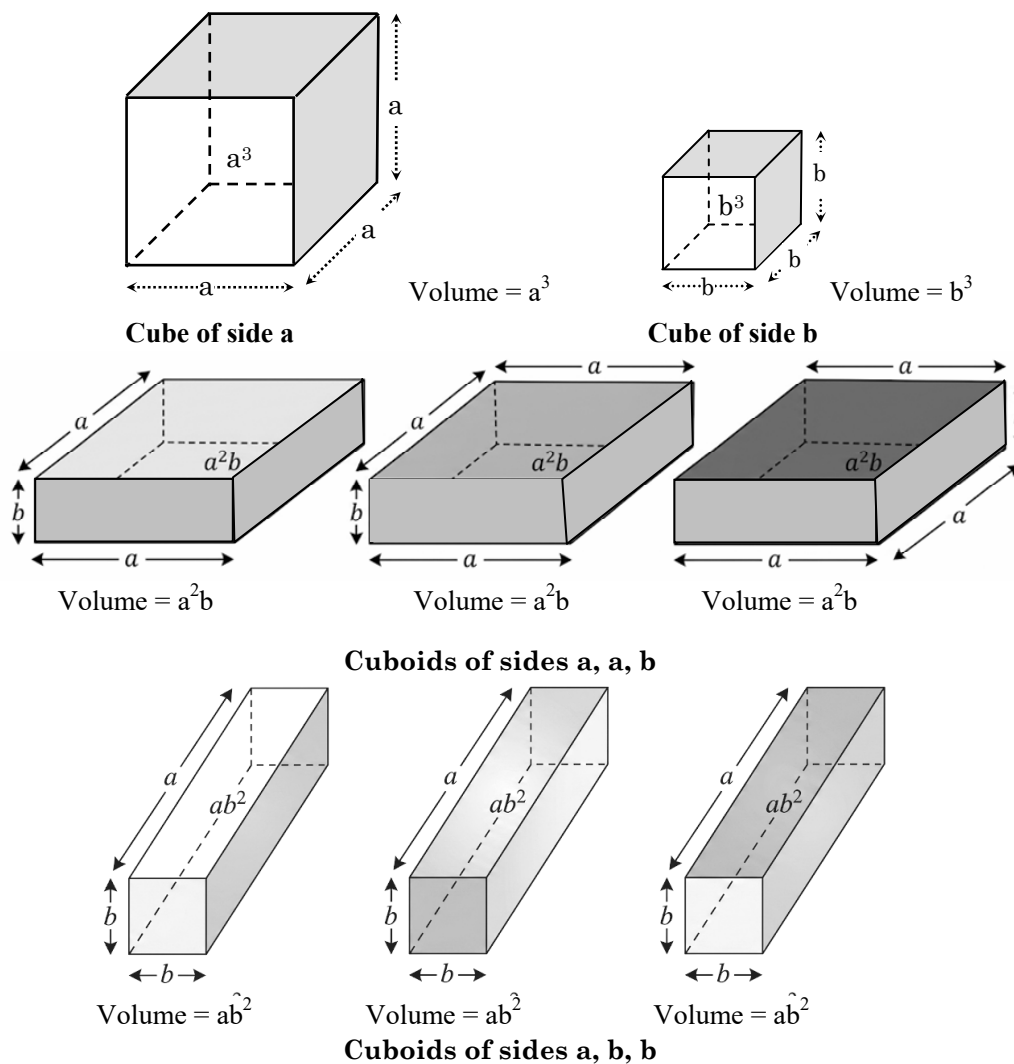
a^2	ab	ca
ab	b^2	bc
ca	bc	c^2

Identity 6: $(a + b)^3 = a^3 + b^3 + 3ab(a + b) = a^3 + b^3 + 3a^2b + 3ab^2$

Proof : LHS $= (a + b)^3 = (a + b)(a + b)(a + b) = (a + b)(a + b)^2$
 $= a(a^2 + b^2 + 2ab) + b(a^2 + b^2 + 2ab) = a^3 + ab^2 + 2a^2b + a^2b + b^3 + 2ab^2$
 $= a^3 + b^3 + 3a^2b + 3ab^2 + a^3 + b^3 + 3ab(a + b) = \text{RHS}.$

Geometric Model

Take a cube of side a units and a cube of side b units. Then take three cuboids each of whose side measure a , a and b units. Now take three cuboids each of whose sides measure b , b and a units.



Ex.3 Evaluate each of the following by using identities :

(i) 103×97

(ii) 103×103

(iii) $(97)^2$

(iv) $185 \times 185 - 115 \times 115$

Sol. (i) We have,

$$\begin{aligned} 103 \times 97 &= (100 + 3)(100 - 3) \\ &= (100)^2 - (3)^2 = 10000 - 9 = 9991 \end{aligned}$$

(ii) We have,

$$\begin{aligned} 103 \times 103 &= (103)^2 \\ &= (100 + 3)^2 = (100)^2 + 2 \times 100 \times 3 + (3)^2 \\ &= 10000 + 600 + 9 = 10609 \end{aligned}$$

(iii) We have,

$$\begin{aligned} (97)^2 &= (100 - 3)^2 \\ &= (100)^2 - 2 \times 100 \times 3 + (3)^2 \\ &= 10000 - 600 + 9 = 9409 \end{aligned}$$

(iv) We have,

$$\begin{aligned} 185 \times 185 - 115 \times 115 \\ &= (185)^2 - (115)^2 = (185 + 115)(185 - 115) \\ &= 300 \times 70 = 21000 \end{aligned}$$

Ex.4 If $x^2 + \frac{1}{x^2} = 34$, find : $x^4 + \frac{1}{x^4}$

Sol. We have,

$$\begin{aligned} x^2 + \frac{1}{x^2} = 34 &\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = (34)^2 \\ \Rightarrow (x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2 \times x^2 \times \frac{1}{x^2} &= 1156 \\ \Rightarrow x^4 + \frac{1}{x^4} + 2 &= 1156 \\ \Rightarrow x^4 + \frac{1}{x^4} &= 1156 - 2 \\ \Rightarrow x^4 + \frac{1}{x^4} &= 1154 \end{aligned}$$

Ex.5 If $x + y = 12$ and $xy = 32$, find the value of $x^2 + y^2$.

Sol. We have,

$$\begin{aligned} (x + y)^2 &= x^2 + y^2 + 2xy \\ \Rightarrow 144 &= x^2 + y^2 + 2 \times 32 && [\text{Putting } x + y = 12 \text{ and } xy = 32] \\ \Rightarrow 144 &= x^2 + y^2 + 64 \\ \Rightarrow 144 - 64 &= x^2 + y^2 \\ \Rightarrow x^2 + y^2 &= 80 \end{aligned}$$

Ex.6 Prove that : $2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = [(a - b)^2 + (b - c)^2 + (c - a)^2]$

Sol. We have,

$$\begin{aligned}\text{L.H.S.} &= 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca \\ &= (a^2 - 2ab + b^2) + (b^2 - 2bc + c^2) + (c^2 - 2ca + a^2) \text{ [Re-arranging the terms]} \\ &= (a - b)^2 + (b - c)^2 + (c - a)^2 = \text{R.H.S.}\end{aligned}$$

$$\begin{aligned}\text{Hence, } 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca \\ = [(a - b)^2 + (b - c)^2 + (c - a)^2]\end{aligned}$$

Ex.7 Write the following in expanded form :

$$\begin{array}{ll} \text{(i) } (9x + 2y + z)^2 & \text{(ii) } (3x + 2y - z)^2 \\ \text{(iii) } (x - 2y - 3z)^2 & \text{(iv) } (-x + 2y + z)^2 \end{array}$$

Sol. Using the identity

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$\begin{aligned} \text{(i) We have, } (9x + 2y + z)^2 &= (9x)^2 + (2y)^2 + z^2 + 2 \times 9x \times 2y + 2 \times 2y \times z + 2 \times 9x \times z \\ &= 81x^2 + 4y^2 + z^2 + 36xy + 4yz + 18xz \\ \text{(ii) We have, } (3x + 2y - z)^2 &= [3x + 2y + (-z)]^2 \\ &= (3x)^2 + (2y)^2 + (-z)^2 + 2 \times 3x \times 2y + 2 \times 2y \times (-z) + 2 \times 3x \times (-z) \\ &= 9x^2 + 4y^2 + z^2 + 12xy - 4yz - 6xz \\ \text{(iii) We have, } (x - 2y - 3z)^2 &= [x + (-2y) + (-3z)]^2 \\ &= x^2 + (-2y)^2 + (-3z)^2 + 2 \times x \times (-2y) + 2 \times (-2y) \times (-3z) + 2 \times (-3z) \times x \\ &= x^2 + 4y^2 + 9z^2 - 4xy + 12yz - 6zx \\ \text{(iv) We have, } (-x + 2y + z)^2 &= [(-x) + 2y + z]^2 \\ &= (-x)^2 + (2y)^2 + z^2 + 2 \times (-x) \times (2y) + 2 \times 2y \times z + 2 \times (-x) \times z \\ &= x^2 + 4y^2 + z^2 - 4xy + 4yz - 2zx \end{aligned}$$

Ex.8 If $a^2 + b^2 + c^2 = 250$ and $ab + bc + ca = 3$, find $a + b + c$.

Sol. We know that

$$\begin{aligned} (a + b + c)^2 &= a^2 + b^2 + c^2 + 2(ab + bc + ca) \\ \Rightarrow (a + b + c)^2 &= 250 + 2 \times 3 \\ \Rightarrow (a + b + c)^2 &= 256 \\ \Rightarrow (a + b + c) &= \pm \sqrt{256} \\ \Rightarrow a + b + c &= \pm 16 \end{aligned}$$

Ex.9 Write each of the following in expanded form:

$$\text{(i) } (2x + 3y)^3 \quad \text{(ii) } (3x - 2y)^3$$

Sol. (i) Replacing a by $2x$ and b by $3y$ in the identity

$$\begin{aligned} (a + b)^3 &= a^3 + b^3 + 3ab(a + b), \text{ we have} \\ (2x + 3y)^3 &= (2x)^3 + (3y)^3 + 3 \times 2x \times 3y \times (2x + 3y) \\ &= 8x^3 + 27y^3 + 18xy \times 2x + 18xy \times 3y \\ &= 8x^3 + 27y^3 + 36x^2y + 54xy^2 \end{aligned}$$

(ii) Replacing a by $3x$ and b by $2y$ in the identity $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$, we have

$$\begin{aligned}(3x - 2y)^3 &= (3x)^3 - (2y)^3 - 3 \times 3x \times 2y \times (3x - 2y) \\ &= 27x^3 - 8y^3 - 18xy \times (3x - 2y) \\ &= 27x^3 - 8y^3 - 54x^2y + 36xy^2\end{aligned}$$

Ex.10 If $x + y = 12$ and $xy = 27$, find the value of $x^3 + y^3$.

Sol. We know that

$$(x + y)^3 = x^3 + y^3 + 3xy(x + y)$$

Putting $x + y = 12$ and $xy = 27$ in the above identity, we get

$$12^3 = x^3 + y^3 + 3 \times 27 \times 12$$

$$\Rightarrow 1728 = x^3 + y^3 + 972$$

$$\Rightarrow x^3 + y^3 = 1728 - 972$$

$$\Rightarrow x^3 + y^3 = 756$$

Ex.11 If $x + y + z = 1$, $xy + yz + zx = -1$ and $xyz = -1$, find the value of $x^3 + y^3 + z^3$.

Sol. We know that :

$$x^3 + y^3 + z^3 - 3xyz$$

$$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) \Rightarrow x^3 + y^3 + z^3 - 3xyz$$

$$= (x + y + z)(x^2 + y^2 + z^2 + 2xy + 2yz + 2zx - 3xy - 3yz - 3zx)$$

$$[\text{Adding and subtracting } 2xy + 2yz + 2zx]$$

$$\Rightarrow x^3 + y^3 + z^3 - 3xyz = (x + y + z) \{(x + y + z)^2 - 3(xy + yz + zx)\}$$

$$\Rightarrow x^3 + y^3 + z^3 - 3 \times -1 = 1 \times \{(1)^2 - 3 \times -1\}$$

$$[\text{Putting the values of } x + y + z, xy + yz + zx, \text{ and } xyz]$$

$$\Rightarrow x^3 + y^3 + z^3 + 3 = 4$$

$$\Rightarrow x^3 + y^3 + z^3 = 4 - 3$$

$$\Rightarrow x^3 + y^3 + z^3 = 1$$

Factorization

Type I : Factorization by taking out the common factors.

Ex.12 Factorize the following expression : $2x^2y + 6xy^2 + 10x^2y^2$

Sol. $2x^2y + 6xy^2 + 10x^2y^2 = 2xy(x + 3y + 5xy)$

Type II : Factorization by grouping the terms.

Ex.13 Factorize the following expression : $a^2 - b + ab - a$

Sol. $a^2 - b + ab - a$

$$= a^2 + ab - b - a = (a^2 + ab) - (b + a)$$

$$= a(a + b) - (a + b) = (a + b)(a - 1)$$

Type III : Factorization by making a perfect square.

Ex.14 Factorize the following expression : $9x^2 + 12xy + 4y^2$

Sol. $9x^2 + 12xy + 4y^2$

$$= (3x)^2 + 2 \times (3x) \times (2y) + (2y)^2 = (3x + 2y)^2$$

Ex.15 Factorize the following expression : $\frac{x^2}{y^2} + 2 + \frac{y^2}{x^2}$, $x \neq 0, y \neq 0$

Sol.
$$\frac{x^2}{y^2} + 2 + \frac{y^2}{x^2}$$
$$= \left(\frac{x}{y}\right)^2 + 2\left(\frac{x}{y}\right)\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2 = \left(\frac{x}{y} + \frac{y}{x}\right)^2$$

Ex.16 Factorize the following expression : $\left(5x - \frac{1}{x}\right)^2 + 4\left(5x - \frac{1}{x}\right) + 4$, $x \neq 0$

Sol.
$$\left(5x - \frac{1}{x}\right)^2 + 4\left(5x - \frac{1}{x}\right) + 4$$
$$= \left(5x - \frac{1}{x}\right)^2 + 2 \times \left(5x - \frac{1}{x}\right) \times 2 + 2^2$$
$$= \left(5x - \frac{1}{x} + 2\right)^2$$

Type IV : Factorizing by difference of two squares.

Ex.17 Factorize each of the following expressions :

(i) $36x^2 - 12x + 1 - 25y^2$ (ii) $a^2 - \frac{9}{a^2}$, $a \neq 0$

Sol. (i) $36x^2 - 12x + 1 - 25y^2$
$$= (6x)^2 - 2 \times 6x \times 1 + 1^2 - (5y)^2 = (6x-1)^2 - (5y)^2$$
$$= \{(6x-1) - 5y\} \{(6x-1) + 5y\}$$
$$= (6x-1-5y)(6x-1+5y)$$
$$= (6x-5y-1)(6x+5y-1)$$

(ii) $a^2 - \frac{9}{a^2} = (a)^2 - \left(\frac{3}{a}\right)^2$
$$= \left(a - \frac{3}{a}\right)\left(a + \frac{3}{a}\right)$$

Ex.18 Factorize the following algebraic expression : $x^4 - 81y^4$

Sol. $x^4 - 81y^4 = [(x)^2]^2 - (9y^2)^2$
$$= (x^2 - 9y^2)(x^2 + 9y^2)$$
$$= \{x^2 - (3y)^2\}(x^2 + 9y^2)$$
$$= (x-3y)(x+3y)(x^2 + 9y^2)$$

Ex.19 Factorize the following expression: $x(x+z) - y(y+z)$

Sol. $x(x+z) - y(y+z) = (x^2 - y^2) + (xz - yz)$
$$= (x-y)(x+y) + z(x-y)$$
$$= (x-y)\{(x+y) + z\}$$
$$= (x-y)(x+y+z)$$

Ex.20 Factorize the following expression : $x^4 + x^2 + 1$

Sol.
$$\begin{aligned}x^4 + x^2 + 1 &= (x^4 + 2x^2 + 1) - x^2 \\&= (x^2 + 1)^2 - x^2 = (x^2 + 1 - x)(x^2 + 1 + x) \\&= (x^2 - x + 1)(x^2 + x + 1)\end{aligned}$$

Type V : Factorizing the sum and difference of cubes of two quantities.

(i) $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$

(ii) $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$

Ex.21 Factorize the following expression : $a^3 + 27$

Sol. $a^3 + 27 = a^3 + 3^3 = (a + 3)(a^2 - 3a + 9)$

Ex.22 Simplify : $(x + y)^3 - (x - y)^3 - 6y(x^2 - y^2)$

Sol. Let $x + y = a$ and $x - y = b$.

Then, $ab = (x + y)(x - y) = x^2 - y^2$ and $a - b = (x + y) - (x - y) = 2y$

$\therefore (x + y)^3 - (x - y)^3 - 6y(x^2 - y^2)$

$= a^3 - b^3 - 3ab(a - b) = (a - b)^3$

$= \{(x + y) - (x - y)\}^3 = (2y)^3 = 8y^3$

◆ **Factorization of quadratic polynomial by splitting the middle term**

Type I : Factorization of Quadratic polynomials of the form $x^2 + bx + c$.

Ex.23 Factorize the following quadratic polynomials : $x^2 - 21x + 108$

Sol. In order to factorize $x^2 - 21x + 108$, we have to find two numbers such that their sum is -21 and the product 108 .

Clearly, $-21 = -12 - 9$ and $-12 \times -9 = 108$

$\therefore x^2 - 21x + 108 = x^2 - 12x - 9x + 108$

$= (x^2 - 12x) - (9x - 108)$

$= x(x - 12) - 9(x - 12) = (x - 12)(x - 9)$

Ex.24 Factorize the following by splitting the middle term : $x^2 + 3\sqrt{3}x + 6$

Sol. In order to factorize $x^2 + 3\sqrt{3}x + 6$, we have to find two numbers p and q such that $p + q = 3\sqrt{3}$ and $pq = 6$

Clearly, $2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$ and $2\sqrt{3} \times \sqrt{3} = 6$

So, we write the middle term $3\sqrt{3}x$ as $2\sqrt{3}x + \sqrt{3}x$, so that

$x^2 + 3\sqrt{3}x + 6$

$= x^2 + 2\sqrt{3}x + \sqrt{3}x + 6$

$= (x^2 + 2\sqrt{3}x) + (\sqrt{3}x + 6)$

$= (x^2 + 2\sqrt{3}x) + (\sqrt{3}x + 2\sqrt{3} \times \sqrt{3})$

$= x(x + 2\sqrt{3}) + \sqrt{3}(x + 2\sqrt{3})$

$= (x + 2\sqrt{3})(x + \sqrt{3})$

Type II : Factorization of polynomials reducible to the form $x^2 + bx + c$.

Ex.25 Factorize : $(a^2 - 2a)^2 - 23(a^2 - 2a) + 120$.

Sol. Let $a^2 - 2a = x$. Then,

$$(a^2 - 2a)^2 - 23(a^2 - 2a) + 120 = x^2 - 23x + 120$$

$$\text{Now, } x^2 - 23x + 120 = x^2 - 15x - 8x + 120$$

$$= (x^2 - 15x) - (8x - 120)$$

$$= x(x - 15) - 8(x - 15)$$

$$= (x - 15)(x - 8)$$

Replacing x by $a^2 - 2a$ on both sides, we get

$$(a^2 - 2a)^2 - 23(a^2 - 2a) + 120$$

$$= (a^2 - 2a - 15)(a^2 - 2a - 8)$$

$$= (a^2 - 5a + 3a - 15)(a^2 - 4a + 2a - 8)$$

$$= \{(a(a - 5) + 3(a - 5))\} \{a(a - 4) + 2(a - 4)\}$$

$$= \{(a - 5)(a + 3)\} \{(a - 4)(a + 2)\}$$

$$= (a - 5)(a + 3)(a - 4)(a + 2)$$

Ex.26 Factorize the expressions by splitting the middle term : $9(x - 2y)^2 - 4(x - 2y) - 13$

Sol. The given expression is $9(x - 2y)^2 - 4(x - 2y) - 13$.

Putting $x - 2y = a$, we get

$$9(x - 2y)^2 - 4(x - 2y) - 13 = 9a^2 - 4a - 13$$

$$\text{Now, } 9a^2 - 4a - 13 = 9a^2 - 13a + 9a - 13$$

$$= (9a^2 - 13a) + (9a - 13)$$

$$= a(9a - 13) + (9a - 13)$$

$$= (a + 1)(9a - 13)$$

Replacing a by $x - 2y$ on both sides, we get

$$9(x - 2y)^2 - 4(x - 2y) - 13 = (x - 2y + 1)(9(x - 2y) - 13)$$

$$= (x - 2y + 1)(9x - 18y - 13)$$

Ex.27 If $x^2 + px + q = (x + a)(x + b)$, then factorize $x^2 + pxy + qy^2$.

Sol. We have,

$$x^2 + px + q = (x + a)(x + b)$$

$$\Rightarrow x^2 + px + q = x^2 + x(a + b) + ab$$

On equating the coefficients of like powers of x , we get

$$p = a + b \text{ and } q = ab$$

$$\therefore x^2 + pxy + qy^2 = x^2 + (a + b)xy + aby^2$$

$$= (x^2 + axy) + (bxy + aby^2)$$

$$= x(x + ay) + by(x + ay)$$

$$= (x + ay)(x + by)$$

Type III : Factorization of quadratic polynomials of the form $ax^2 + bx + c$, $a \neq 0, 1$ **Ex.28** Factorize the following expression : $6x^2 - 5x - 6$ **Sol.** The given expression is of the form $ax^2 + bx + c$, where, $a = 6$, $b = -5$ and $c = -6$.In order to factorize the given expression, we have to find two numbers ℓ and m such that

$$\ell + m = b = -5 \text{ and } \ell m = ac \text{ i.e. } \ell m = 6 \times -6 = -36$$

i.e., we have to find two factors of -36 such that their sum is -5 . Clearly,

$$-9 + 4 = -5 \text{ and } -9 \times 4 = -36$$

$$\therefore \ell = -9 \text{ and } m = 4$$

Now, we split the middle term $-5x$ of $6x^2 - 5x - 6$ as $-9x + 4x$, so that

$$6x^2 - 5x - 6 = 6x^2 - 9x + 4x - 6$$

$$= (6x^2 - 9x) + (4x - 6)$$

$$= 3x(2x - 3) + 2(2x - 3) = (2x - 3)(3x + 2)$$

Ex.29 Factorize : $\sqrt{3}x^2 + 11x + 6\sqrt{3}$ **Sol.** The given quadratic expression is of the form $ax^2 + bx + c$, where $a = \sqrt{3}$, $b = 11$ and $c = 6\sqrt{3}$.In order to factorize it, we have to find two numbers ℓ and m such that

$$\ell + m = b = 11 \text{ and } \ell m = ac = \sqrt{3} \times 6\sqrt{3} = 18$$

$$\text{Clearly, } 9 + 2 = 11 \text{ and } 9 \times 2 = 18$$

$$\therefore \ell = 9 \text{ and } m = 2$$

$$\text{Now, } \sqrt{3}x^2 + 11x + 6\sqrt{3}$$

$$= \sqrt{3}x^2 + 9x + 2x + 6\sqrt{3}$$

$$= (\sqrt{3}x^2 + 9x) + (2x + 6\sqrt{3})$$

$$= (\sqrt{3}x^2 + 3\sqrt{3} \times \sqrt{3}x) + (2x + 6\sqrt{3})$$

$$= \sqrt{3}x(x + 3\sqrt{3}) + 2(x + 3\sqrt{3})$$

$$= (\sqrt{3}x + 2)(x + 3\sqrt{3})$$

$$\text{Hence, } \sqrt{3}x^2 + 11x + 6\sqrt{3}$$

$$= (\sqrt{3}x + 2)(x + 3\sqrt{3})$$

Ex.30 Factorize the following by splitting the middle term : $\frac{1}{3}x^2 - 2x - 9$ **Sol.** In order to factorize $\frac{1}{3}x^2 - 2x - 9$, we have to find two numbers ℓ and m such that

$$\ell + m = -2 \text{ and } \ell m = \frac{1}{3} \times -9 = -3$$

$$\text{Clearly, } -3 + 1 = -2 \text{ and } -3 \times 1 = -3$$

So, we write the middle term $-2x$ as $-3x + x$, so that

$$\frac{1}{3}x^2 - 2x - 9 = \frac{1}{3}x^2 - 3x + x - 9$$

$$= \left(\frac{1}{3}x^2 - 3x\right) + (x - 9) = \left(\frac{1}{3}x^2 - \frac{9}{3}x\right) + (x - 9)$$

$$= \frac{1}{3}x(x - 9) + (x - 9) = (x - 9)\left(\frac{1}{3}x + 1\right)$$

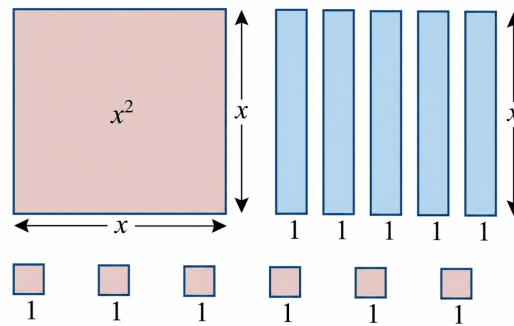
◆ Visualising Factorisation of quadratic expressions through algebra tiles

A quadratic expression like $x^2 + bx + c$ can be interpreted as the area of a rectangle, which can be decomposed into smaller rectangles and squares.

- We break the expression into parts (areas).
- By rearranging these shapes, we can determine the **length** and **breadth**, which represent the factors.

Thus, factorisation means **finding dimensions of a rectangle whose area equals the given algebraic expression**.

Consider the expression: $x^2 + 5x + 6$



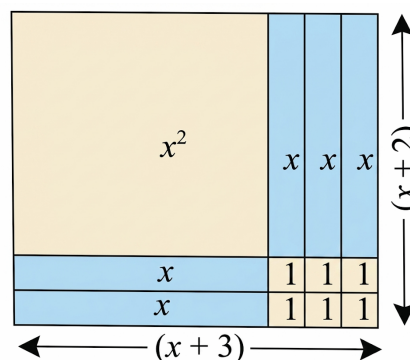
Step 1 : We interpret each term geometrically.

Term	Geometric Meaning
x^2	Area of square of side x
$5x$	Area of rectangle with side x and 5
6	Area of rectangle 2×3 or constant area

Thus, the total area represents a large rectangle formed by combining these shapes.

Step 2 : Represent each term as area

- Square of area = x^2 sq. units
- Rectangle of area = $5x$ sq. units.
- Rectangle of area = 6 sq. units



Step 3 : Split the middle term

We split ($5x$) so that arranging rectangles becomes easy. We need two numbers such that :

Sum = 5 and product = 6 . These numbers are 2 and 3 .

So, $5x = 2x + 3x$

$$x^2 + 5x + 6 = x^2 + 2x + 3x + 6$$

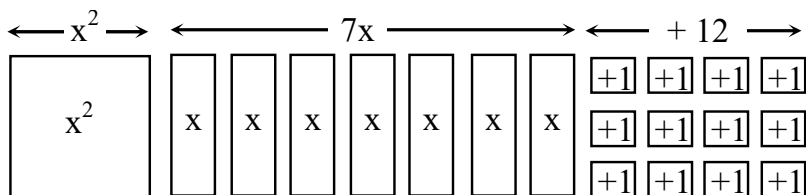
$$= (x^2 + 2x) + (3x + 6) = x(x + 2) + 3(x + 2) = (x + 2)(x + 3)$$

Ex.31 Solving $x^2 + 7x + 12$ using algebra tiles.

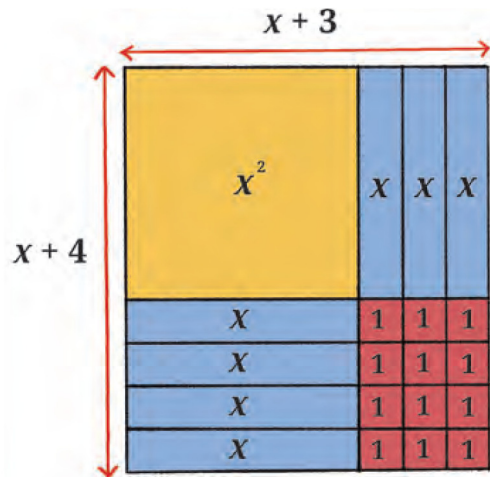
Sol. Given

$$x^2 + 7x + 12$$

So, algebra tiles for this expression are



Now from these tiles use form a rectangle or a square to solve given expression



Therefore,

$$\begin{aligned} x^2 + 7x + 12 \\ = (x + 3)(x + 4) \end{aligned}$$

◆ Factorization of $x^3 + y^3 + z^3 - 3xyz$

- In order to factorize the algebraic expressions of the form $x^3 + y^3 + z^3 - 3xyz$

We use the following identity :

$$x^3 + y^3 + z^3 - 3xyz = (x+y+z) (x^2+y^2+z^2-xy-yz-zx)$$

- If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$

Ex.32 Factorize : $(a+b)^3 + (b+c)^3 + (c+a)^3 - 3(a+b)(b+c)(c+a)$

Sol. We have,

$$(a+b)^3 + (b+c)^3 + (c+a)^3 - 3(a+b)(b+c)(c+a)$$

$$= \{(a+b) + (b+c) + (c+a)\} \{(a+b)^2 + (b+c)^2 + (c+a)^2 - (a+b)(b+c) - (b+c)(c+a) - (c+a)(a+b)\}$$

$$= (2a+2b+2c)$$

$$\{(a^2 + 2ab + b^2) + (b^2 + 2bc + c^2) + (c^2 + 2ca + a^2) - (ab + ac + b^2 + bc) - (bc + ba + c^2 + ca) - (ca + cb + a^2 + ab)\}$$

$$= 2(a+b+c) (2a^2 + 2b^2 + 2c^2 + 2ab + 2bc + 2ca - ab - ac - b^2 - bc - bc - ba - c^2 - ca - ca - cb - a^2 - ab)$$

$$= 2(a+b+c) (a^2 + b^2 + c^2 - ab - bc - ca)$$

Ex.33 Resolve $a^3 - b^3 + 1 + 3ab$ into factors.

Sol. $a^3 - b^3 + 1 + 3ab$
 $= a^3 + (-b)^3 + 1^3 - 3(a)(-b)(1)$
 $= (a-b+1)(a^2 + b^2 + 1 + ab - a + b)$
 $= (a-b+1)(a^2 + b^2 + ab - a + b + 1)$

Ex.34 Factorize : $2\sqrt{2}a^3 + 8b^3 - 27c^3 + 18\sqrt{2}abc$.

Sol. $2\sqrt{2}a^3 + 8b^3 - 27c^3 + 18\sqrt{2}abc$
 $= (\sqrt{2}a)^3 + (2b)^3 - (3c)^3 - 3(\sqrt{2}a)(2b)(-3c)$
 $= (\sqrt{2}a + 2b - 3c)(2a^2 + 4b^2 + 9c^2 - 2\sqrt{2}ab + 6bc + 3\sqrt{2}ac)$

Ex.35 Prove that : $a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a+b+c)\{(a-b)^2 + (b-c)^2 + (c-a)^2\}$

Sol. We have,
 $a^3 + b^3 + c^3 - 3abc$
 $= (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$
 $= \frac{1}{2}(a+b+c)(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca)$
 $= \frac{1}{2}(a+b+c)\{(a^2 - 2ab + b^2) + (b^2 - 2bc + c^2) + (c^2 - 2ca + a^2)\}$
 $= \frac{1}{2}(a+b+c)\{(a-b)^2 + (b-c)^2 + (c-a)^2\}$

◆ **Factorization of algebraic expression of the form $a^3 + b^3 + c^3$, when $a + b + c = 0$**

Ex.36 Simplify : $\frac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a-b)^3 + (b-c)^3 + (c-a)^3}$

Sol. We have,
 $(a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2) = 0$
 $\therefore (a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$
 $= 3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$
 $= 3(a-b)(a+b)(b-c)(b+c)(c-a)(c+a)$
 Similarly,
 $(a-b) + (b-c) + (c-a) = 0$
 $\Rightarrow (a-b)^3 + (b-c)^3 + (c-a)^3 = 3(a-b)(b-c)(c-a)$
 $\therefore \frac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a-b)^3 + (b-c)^3 + (c-a)^3}$
 $= \frac{3(a-b)(a+b)(b-c)(b+c)(c-a)(c+a)}{3(a-b)(b-c)(c-a)}$
 $= (a+b)(b+c)(c+a)$

Ex.37 Find the value of $x^3 - 8y^3 - 36xy - 216$, when $x = 2y + 6$.

Sol. We have, $x^3 - 8y^3 - 36xy - 216$
 $= x^3 + (-2y)^3 + (-6)^3 - 3(x)(-2y)(-6)$
 $= (x - 2y - 6)(x^2 + 4y^2 + 36 + 2xy - 12y + 6x)$
 $= 0 \times (x^2 + 4y^2 + 36 + 2xy - 12y + 6x)$
 $[\because x = 2y + 6 \Rightarrow x - 2y - 6 = 0] = 0$

Simplifying Rational Expressions

A rational expression is an algebraic fraction in which both the numerator and denominator are polynomials and the denominator is not zero.

Thus, $\frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$ is a rational expression.

For example : $\frac{x+2}{x}$, $\frac{x^2+2}{x+3}$ and $\frac{3x}{2x+5}$ are rational expressions.

Note: Every polynomial $p(x)$ is a rational expression, since we can write $p(x)$ as $\frac{p(x)}{1}$.

Simplifying Rational Expressions

Simplifying a rational expression means **reducing it to its simplest form** by canceling common factors from the numerator and denominator.

Important Rule :

You can cancel **common factors only**, not terms. $\frac{(x+2)(x-3)}{(x+2)} = x - 3$

Steps to Simplify Rational Expressions

- (i) Factorise numerator and denominator completely.
- (ii) Identify common factors in numerator and denominator.
- (iii) Cancel common factors.
- (iv) Write the simplified expression.

Ex.38 Simplify ; $\frac{6x}{3}$

Sol. Factorise ; $\frac{6x}{3} = \frac{3 \times 2x}{3} = 2x$ [Cancel common factor 3, we get, $2x$]

Ex.39 Simplify : $\frac{x^2 - 9}{x - 3}$

Sol. Factorising numerator, we get, $x^2 - 9 = (x - 3)(x + 3)$

Now, $\frac{x^2 - 9}{x - 3} = \frac{(x - 3)(x + 3)}{x - 3} = (x + 3)$ [Cancel common factor $(x - 3)$, we get $(x + 3)$]

Ex.40 Simplify : $\frac{2x^2 + 4x}{2x}$

Sol. Factorising numerator, $\frac{2x(x + 2)}{2x} = (x + 2)$ [Cancel common factor $2x$, we get $(x + 2)$]

◆ Application of Factor Theorem in the Factorization of Polynomials

- (i) Obtain the polynomial $p(x)$
- (ii) Obtain the constant term in $p(x)$ and find its all possible factors. For example, in the polynomial $x^4 + x^3 - 7x^2 - x + 6$ the constant term is 6 and its factors are $\pm 1, \pm 2, \pm 3, \pm 6$.
- (iii) Take one of the factors, say a and replace x by it in the given polynomial. If the polynomial reduces to zero, then $(x - a)$ is a factor of polynomial.
- (iv) Obtain the factors equal in no. to the degree of polynomial. Let these are $(x-a), (x-b), (x-c), \dots$
- (v) Write $p(x) = k (x-a) (x-b) (x-c) \dots$ where k is constant.
- (vi) Substitute any value of x other than $a, b, c, \dots$ and find the value of k .

Ex.41 Using factor theorem, factorize the polynomial $x^3 - 6x^2 + 11x - 6$.

Sol. Let $f(x) = x^3 - 6x^2 + 11x - 6$

The constant term in $f(x)$ is equal to -6 and factors of -6 are $\pm 1, \pm 2, \pm 3, \pm 6$.

Putting $x = 1$ in $f(x)$, we have

$$\begin{aligned} f(1) &= 1^3 - 6 \times 1^2 + 11 \times 1 - 6 \\ &= 1 - 6 + 11 - 6 = 0 \end{aligned}$$

$\therefore (x - 1)$ is a factor of $f(x)$

Similarly, $x - 2$ and $x - 3$ are factors of $f(x)$.

Since $f(x)$ is a polynomial of degree 3. So, it can not have more than three linear factors.

Let $f(x) = k (x-1) (x-2) (x-3)$. Then,

$$x^3 - 6x^2 + 11x - 6 = k(x-1)(x-2)(x-3)$$

Putting $x = 0$ on both sides, we get

$$\begin{aligned} -6 &= k (0 - 1) (0 - 2) (0 - 3) \\ \Rightarrow -6 &= -6k \Rightarrow k = 1 \end{aligned}$$

Putting $k = 1$ in $f(x) = k (x - 1) (x - 2) (x - 3)$, we get

$$f(x) = (x - 1) (x - 2) (x - 3)$$

Hence, $x^3 - 6x^2 + 11x - 6 = (x - 1) (x - 2) (x - 3)$

Ex.42 Factorize, $2x^4 + x^3 - 14x^2 - 19x - 6$

Sol. Let $f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$ be the given polynomial. The factors of the constant term -6 are $\pm 1, \pm 2, \pm 3$ and ± 6 , we have,

$$\begin{aligned} f(-1) &= 2(-1)^4 + (-1)^3 - 14(-1)^2 - 19(-1) - 6 \\ &= 2 - 1 - 14 + 19 - 6 = 21 - 21 = 0 \end{aligned}$$

and,

$$f(-2) = 2(-2)^4 + (-2)^3 - 14(-2)^2 - 19(-2) - 6 = 32 - 8 - 56 + 38 - 6 = 0$$

So, $x + 1$ and $x + 2$ are factors of $f(x)$.

$\Rightarrow (x + 1) (x + 2)$ is also a factor of $f(x)$

$\Rightarrow x^2 + 3x + 2$ is a factor of $f(x)$

Now, we divide

$$f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6 \text{ by}$$

$x^2 + 3x + 2$ to get the other factors.

	$2x^2 - 5x - 3$	
$x^2 + 3x + 2$	$2x^4 + x^3 - 14x^2 - 19x - 6$	
	$2x^4 + 6x^3 + 4x^2$	
	$- \quad - \quad -$	
	$- 5x^3 - 18x^2 - 19x - 6$	
	$- 5x^3 - 15x^2 - 10x$	
	$+ \quad + \quad +$	
	$- 3x^2 - 9x - 6$	
	$- 3x^2 - 9x - 6$	
	$+ \quad + \quad +$	
	0	

$$\begin{aligned} \therefore 2x^4 + x^3 - 14x^2 - 19x - 6 \\ &= (x^2 + 3x + 2)(2x^2 - 5x - 3) \\ &= (x + 1)(x + 2)(2x^2 - 5x - 3) \end{aligned}$$

$$\begin{aligned} \text{Now } 2x^2 - 5x - 3 &= 2x^2 - 6x + x - 3 \\ &= 2x(x - 3) + 1(x - 3) \\ &= (x - 3)(2x + 1) \end{aligned}$$

$$\begin{aligned} \text{Hence, } 2x^4 + x^3 - 14x^2 - 19x - 6 \\ &= (x + 1)(x + 2)(x - 3)(2x + 1) \end{aligned}$$

Ex.43 Factorize $x^2 + 4 + 9z^2 + 4x - 6xz - 12z$

Sol. The presence of the three squares viz. x^2 , $(2)^2$, and $(3z)^2$ gives a clue that identity (vii) could be used. So we write.

$$A = x^2 + (2)^2 + (3z)^2 + 4x - 6xz - 12z$$

We note that the last two of the product terms are negative and that both of these contain z . Hence we write A as

$$\begin{aligned} A &= x^2 + (2)^2 + (-3z)^2 + 2 \cdot x - 2 \cdot x \cdot (-3z) + 2 \cdot 2 \cdot (-3z) = (x + 2 - 3z)^2 \\ &= (x + 2 - 3z)(x + 2 - 3z) \end{aligned}$$

COMPETITIVE LEVEL

H.C.F. and L.C.M. of Polynomials

The relation between H.C.F. and L.C.M. of two polynomials is the product of the two polynomials is equal to the product of their H.C.F. and L.C.M.

If $p(x)$ and $q(x)$ are two polynomials, then $p(x) \cdot q(x) = \{\text{H.C.F. of } p(x) \text{ and } q(x)\} \times \{\text{L.C.M. of } p(x) \text{ and } q(x)\}$.

Ex. 44 Find the H.C.F. and L.C.M. of the expressions $a^2 - 12a + 35$ and $a^2 - 8a + 7$ by factorization.

Sol. First expression $= a^2 - 12a + 35 = a^2 - 7a - 5a + 35 = a(a - 7) - 5(a - 7) = (a - 7)(a - 5)$

Second expression $= a^2 - 8a + 7 = a^2 - 7a - a + 7 = a(a - 7) - 1(a - 7) = (a - 7)(a - 1)$

Therefore, the H.C.F. $= (a - 7)$ and L.C.M. $= (a - 7)(a - 5)(a - 1)$

Note:

- (i) The product of the two expressions is equal to the product of their factors.
 (ii) The product of the two expressions is equal to the product of their H.C.F. and L.C.M.

$$\begin{aligned}
 \text{Product of the two expressions} &= (a^2 - 12a + 35)(a^2 - 8a + 7) \\
 &= (a - 7)(a - 5)(a - 7)(a - 1) \\
 &= (a - 7)(a - 7)(a - 5)(a - 1) \\
 &= \text{H.C.F.} \times \text{L.C.M. of the two expressions}
 \end{aligned}$$

Ex.45 Find the L.C.M. of the two expressions $a^2 + 7a - 18$, $a^2 + 10a + 9$ with the help of their H.C.F.

Sol. First expression = $a^2 + 7a - 18 = a^2 + 9a - 2a - 18 = a(a + 9) - 2(a + 9) = (a + 9)(a - 2)$
 Second expression = $a^2 + 10a + 9 = a^2 + 9a + a + 9 = a(a + 9) + 1(a + 9) = (a + 9)(a + 1)$

Therefore, the H.C.F. = $(a + 9)$

Therefore, L.C.M. = Product of the two expressions/H.C.F.

$$= \frac{(a^2 + 7a - 18)(a^2 + 10a + 9)}{(a + 9)} = \frac{(a + 9)(a - 2)(a + 9)(a + 1)}{(a + 9)} = (a - 2)(a + 9)(a + 1)$$

Ex.46 $m^2 - 5m - 14$ is an expression. Find out another similar expression such that their H.C.F. is $(m - 7)$ and L.C.M. is $m^3 - 10m^2 + 11m + 70$.

Sol. According to the problem,

$$\begin{aligned}
 \text{Required Expression} &= \frac{\text{L.C.M.} \times \text{H.C.F.}}{\text{Given expression}} \\
 &= \frac{(m^3 - 10m^2 + 11m + 70)(m - 7)}{m^2 - 5m - 14} \\
 &= \frac{(m^2 - 5m - 14)(m - 5)(m - 7)}{m^2 - 5m - 14} \\
 &= (m - 5)(m - 7) = m^2 - 12m + 35
 \end{aligned}$$

Therefore, the required expression = $m^2 - 12m + 35$

EXERCISE-1

➤ Very Short Answer Type Questions

- Q.1** Use suitable identities to find the following products :
 (i) $(x + 4)(x + 10)$
 (ii) $(4m + 7n)^2$
- Q.2** Evaluate the following products without multiplying directly.
 (i) 95×96 (ii) 104×96
- Q.3** Write the following cubes in expanded form.
 (i) $(2x + 1)^3$ (ii) $\left[x - \frac{2}{3}y\right]^3$
- Q.4** Without actually calculating the cubes, find the value of $(28)^3 + (-15)^3 + (-13)^3$
- Q.5** Simplify : $\frac{x^2 + 5x}{x}$
- Q.6** For what value of k is $y^3 + ky + 2k - 2$ exactly divisible by $(y + 1)$?

➤ Short Answer Type Questions – Type I

- Q.7** Evaluate the following using suitable identities :
 (i) $(99)^3$ (ii) $(102)^3$
- Q.8** Simplify : $\frac{x^2 + 2x - 5}{x^2 - 25}$

Factorize each of the following expression

- Q.9(a)** (i) $6 - 5y - y^2$
 (ii) $a^2 + 46a + 205$
 (iii) $ab + ac - b^2 - bc$
- (b) (i) $\sqrt{3}x^2 + 11x + 6\sqrt{3}$
 (ii) $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$
- (c) (i) $x^2y^2 - xy - 72$
 (ii) $x^4 - 5x^2 + 4$
 (iii) $(x^2 - 4x)(x^2 - 4x - 1) - 20$
- (d) (i) $2x^3 - 3x^2 - 17x + 30$
 (ii) $x^3 - 6x^2 + 11x - 6$
 (iii) $x^3 + x^2 - 4x - 4$
 (iv) $3x^3 - x^2 - 3x + 1$
 (v) $x^3 - 23x^2 + 142x - 120$

- Q.10** If $a + b + c = 9$ and $ab + bc + ca = 40$, find $a^2 + b^2 + c^2$.

- Q.11** Find positive square root of $36x^2 + 60x + 25$.

- Q.12** Simplify : $\sqrt{2a^2 + 2\sqrt{6}ab + 3b^2}$

➤ Short Answer Type Questions – Type II

- Q.13** Find the following products :
 (i) $\left(\frac{x}{2} + 2y\right)\left(\frac{x^2}{4} - xy + 4y^2\right)$
 (ii) $(x^2 - 1)(x^4 - 1)(x^4 + x^2 + 1)$
- Q.14** Factorize : $4(x-y)^2 - 12(x-y)(x+y) + 9(x+y)^2$
- Q.15** Find the value of $1 - a^2 + 14ab - 49b^2$.
- Q.16** Prove that :

$$\frac{0.77 \times 0.77 \times 0.77 + 0.23 \times 0.23 \times 0.23}{0.77 \times 0.77 - 0.77 \times 0.23 + 0.23 \times 0.23} = 1$$
- Q.17** Prove that
 $(a + b)^3 + (b + c)^3 + (c + a)^3 - 3(a + b)(b + c)(c + a) = 2(a^3 + b^3 + c^3 - 3abc)$
- Q.18** Factorize : (i) $x^6 - y^6$ (ii) $x^{12} - y^{12}$
- Q.19** If $x^4 + \frac{1}{x^4} = 47$. Find the value of $x^3 + \frac{1}{x^3}$.
- Q.20** If $a + b = 10$ and $a^2 + b^2 = 58$, find the value of $a^3 + b^3$.
- Q.21** If $a^2 + b^2 + c^2 - ab - bc - ca = 0$, prove that $a = b = c$.
- Q.22** Solving $x^2 + 5x + 6$ using algebra tiles.

➤ Long Answer Type Questions

- Q.23** Simplify :

$$\frac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a - b)^3 + (b - c)^3 + (c - a)^3}$$
- Q.24** If $a + b + c = 0$, then find the value of

$$\frac{(b+c)^2}{bc} + \frac{(c+a)^2}{ca} + \frac{(a+b)^2}{ab}$$
- Q.25** Simplify $(2x - 5y)^3 - (2x + 5y)^3$.

Q.26 Multiply $x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$ by $(-z + x - 2y)$.

Q.27 If a, b, c are all non-zero and $a + b + c = 0$, prove that $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$.

Q.28 If $a + b + c = 5$ and $ab + bc + ca = 10$, then prove that $a^3 + b^3 + c^3 - 3abc = -25$.

Q.29 Prove that $(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$.

Q.30 Solving $2x^2 - 7x + 3$ using algebra tiles.

EXERCISE-2

- Q.1** The polynomial $11a^2 - 12\sqrt{2}a + 2$ on factorisation gives :
 (A) $(11a + \sqrt{2})(a - \sqrt{2})$
 (B) $(a - \sqrt{2})(11a - \sqrt{2})$
 (C) $(a + 11)(a + \sqrt{2})$
 (D) $(11a - \sqrt{2})(a + \sqrt{2})$
- Q.2** The polynomial $x^5 - a^2x^3 - x^2y^3 + a^2y^3$ on factorisation gives :
 (A) $(x - y)(x - a)(x + a)(x^2 + y^2 + xy)$
 (B) $(x + a)(x - y)(x - a)(x^2 - y^2 + xy)$
 (C) $(x + a)(x + y)(x - a)(x^2 + y^2 + xy)$
 (D) None of these
- Q.3** Factorise : $a^3 + b^3 + 3ab - 1$.
 (A) $(a + b - 1)(a^2 + b^2 + a + b + 1 - ab)$
 (B) $(a + b - 1)(a^2 + b^2 + a + b - 1 + ab)$
 (C) $(a + b - 1)(a^2 + b^2 - a - b + 1 + ab)$
 (D) None of these
- Q.4** Find the square root of $\frac{x^2}{9} + \frac{9}{4x^2} - \frac{x}{3} - \frac{3}{2x} + \frac{5}{4}$.
 (A) $\frac{2x}{3} + \frac{3}{2x} - \frac{1}{2}$ (B) $\frac{x}{3} - \frac{3}{2x} + 1$
 (C) $\frac{3}{x} + \frac{2}{3x} - \frac{1}{2}$ (D) $\frac{x}{3} + \frac{3}{2x} - \frac{1}{2}$
- Q.5** If $\sqrt{4x^4 + 12x^3 + 25x^2 + 24x + 16} = ax^2 + bx + c$, then which of the following is true?
 (A) $2b = a - c$ (B) $2a = b + c$
 (C) $2b = a + c$ (D) $2b = c - a$
- Q.6** If $a + b + c = 6$, the value of $(2 - a)^3 + (2 - b)^3 + (2 - c)^3 - 3(2 - a)(2 - b)(2 - c)$ is :
 (A) 1 (B) 0 (C) -1 (D) 2
- Q.7** If $\left(x + \frac{1}{x}\right) = 2$, then the value of $\left(x^6 + \frac{1}{x^6}\right)$ is -
 (A) 2 (B) 4 (C) 8 (D) None
- Q.8** If $(x + y + z) = 6$ and $(xy + yz + zx) = 11$, then the value of $(x^3 + y^3 + z^3 - 3xyz)$ is-
 (A) 81 (B) 54
 (C) 18 (D) None of these
- Q.9** Find the value of $\frac{a^3 + b^3 + c^3 - 3abc}{ab + bc + ca - a^2 - b^2 - c^2}$, when $a = -5$, $b = -6$, $c = 10$.
 (A) 1 (B) -1 (C) 2 (D) -2
- Q.10** If $a^2 - b^2 = 21$ and $a^2 + b^2 = 29$, which of the following could be the value of ab ?
 I. -10 II. $5\sqrt{2}$ III. 10
 (A) I only (B) II only
 (C) III only (D) I and III only
- Q.11** On simplifying $(a + b)^3 + (a - b)^3 + 6a(a^2 - b^2)$ we get :
 (A) $8a^2$ (B) $8a^2b$
 (C) $8a^3b$ (D) $8a^3$
- Q.12** If $(x + y + z) = 1$, $xy + yz + zx = -1$, $xyz = -1$, then value of $x^3 + y^3 + z^3$ is :
 (A) -1 (B) 1
 (C) 2 (D) -2
- Q.13** Evaluate:
 $\frac{(a - b)^2}{(b - c)(c - a)} + \frac{(b - c)^2}{(a - b)(c - a)} + \frac{(c - a)^2}{(a - b)(b - c)}$
 (A) 0 (B) 1 (C) 2 (D) 3
- Q.14** The value of $\frac{0.76 \times 0.76 \times 0.76 + 0.24 \times 0.24 \times 0.24}{0.76 \times 0.76 - 0.76 \times 0.24 + 0.24 \times 0.24}$ is :
 (A) 0.52 (B) 1
 (C) 0.01 (D) 0.1
- Q.15** Which of the following is a factor of $(x + y)^3 - (x^3 + y^3)$?
 (A) $x^3 + y^2 + 2xy$
 (B) $x^2 + y^2 - xy$
 (C) xy^2
 (D) $3xy$

- Q.16** If $\frac{x}{y} + \frac{y}{x} = -1$ ($x, y \neq 0$), then value of $x^3 - y^3$ is
(A) 1 (B) -1
(C) 0 (D) $\frac{1}{2}$

- Q.17** If $49x^2 - b = \left(7x + \frac{1}{2}\right) \left(7x - \frac{1}{2}\right)$, then the value of b is :
(A) 0 (B) $\frac{1}{\sqrt{2}}$
(C) $\frac{1}{4}$ (D) $\frac{1}{2}$

- Q.18** If $x^2 + kx + 6 = (x + 2)(x + 3)$, for all x, then the value of k is :
(A) 1 (B) -1
(C) 5 (D) 3

- Q.19** The value of k, if $x - 1$ is a factor of $p(x) = kx^2 - \sqrt{2}x + 1$ is :
(A) $\sqrt{2}$ (B) $\sqrt{2} + 1$
(C) $\sqrt{2} - 1$ (D) $\frac{\sqrt{3}}{2}$

- Q.20** If $(x + 2)$ and $(x - 1)$ are factors of the polynomial $10x^2 + px + q$, then $\frac{p}{q}$, is :

- (A) $-\frac{1}{2}$ (B) $\frac{1}{4}$
(C) -1 (D) -2

- Q.21** Which of the following is not a rational expression ?

- (A) $x - \frac{1}{2}$ (B) $\frac{x^2 - \sqrt{3}}{x}$
(C) $\frac{x - 5}{x^{1/2}}$ (D) $\frac{3}{x}$

EXERCISE-3

(Previous Year Questions – NTSE, IJSO & IMO)

- Q.1** If $x + \frac{1}{x} = a + b$, $x - \frac{1}{x} = a - b$ then
 (A) $ab = 1$ (B) $ab = 2$
 (C) $a + b = 0$ (D) $a = b$
- Q.2** H.C.F. and L.C.M. of expressions $(x^3 - 1)$ and A are $(x - 1)$ and $(x^6 - 1)$ respectively. Then the value of A is:
 (A) $x^3 + 1$
 (B) $x^4 - x^3 + x - 1$
 (C) $(x - 1)(x^2 - x + 1)$
 (D) $(x - 1)(x^2 + x + 1)$
- Q.3** H.C.F. of $x^2 + 5x + 6$ and $x^3 + 27$ is :
 (A) $x + 2$ (B) $x - 2$
 (C) $x - 3$ (D) $x + 3$
- Q.4** One of the factors of the expression $(2x - 3y)^2 - 7(2x - 3y) - 30$ is :
 (A) $2x - 3y - 10$ (B) $2x - 3y + 10$
 (C) $3x - 2y + 5$ (D) $6x - 4y - 15$
- Q.5** L.C.M. of $x^3 + x^2 + x + 1$ and $x^3 - x^2 + x - 1$ is:
 (A) $x^4 + 1$ (B) $x^4 - 1$
 (C) $x^2 + 1$ (D) $x^2 - 1$
- Q.6** If $x + \frac{1}{x} = 3$, then the value of $x^6 + \frac{1}{x^6}$ is :
 (A) 927 (B) 114 (C) 364 (D) 322
- Q.7** If a, b, c and d are natural numbers such that $a^5 = b^6$, $c^3 = d^4$, and $d - a = 61$, then the smallest value of $c - b$ is :
 (A) 61 (B) 122 (C) 239 (D) 593
- Q.8** If x, y, z are positive real numbers and a, b, c are rational numbers, then the value of

$$\frac{1}{1 + x^{b-a} + x^{c-a}} + \frac{1}{1 + x^{a-b} + x^{c-b}} + \frac{1}{1 + x^{b-c} + x^{a-c}}$$

 (A) -1 (B) 0
 (C) 1 (D) None of these
- Q.9** If $x + 1$ is a factor of $ax^3 + x^2 - 2x + 4a - 9$, then the value of $a =$
 (A) 1 (B) 2
 (C) 3 (D) 1
- Q.10** If $x - y = 1$ and $x^2 + y^2 = 41$, then $x + y =$
 (A) ± 6 (B) ± 9
 (C) ± 16 (D) ± 25
- Q.11** If $x^4 + \frac{1}{x^4} = 47$, then the value of $x + \frac{1}{x} =$
 (A) 3 (B) 5
 (C) 7 (D) 11
- Q.12** If $a + b + c = 4$ and $ab + bc + ca = 6$, then $a^3 + b^3 + c^3 - 3abc =$
 (A) -6 (B) -7
 (C) -8 (D) 10
- Q.13** If $\ell + m + n = 0$, then $\frac{\ell^2}{mn} + \frac{m^2}{n\ell} + \frac{n^2}{\ell m} =$
 (A) 1 (B) 2
 (C) 3 (D) 4
- Q.14** If $a + b + c = 5$ and $ab + bc + ca = 10$, then $a^3 + b^3 + c^3 - 3abc =$
 (A) -15 (B) -20
 (C) -25 (D) 30
- Q.15** If $a^2 + b^2 + c^2 = 90$ and $a + b + c = 20$, then the value of $ab + bc + ca =$
 (A) 155 (B) 160
 (C) 165 (D) 170
- Q.16** The rational expression $\frac{x^2 + 4x - 5}{x^2 - 1}$ in its simplest form is :
 (A) $x + 5$ (B) $\frac{x+5}{x-1}$
 (C) $\frac{x+1}{x+5}$ (D) $\frac{x+5}{x+1}$

ANSWER KEY

EXERCISE - 1

1. (i) $x^2 + 14x + 40$ (ii) $16m^2 + 56mn + 49n^2$ 2. (i) 9120 (ii) 9984
3. (i) $8x^3 + 12x^2 + 6x + 1$ (ii) $x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$ 4. 16380 5. $(x + 5)$ 6. 3
7. (i) 970299 (ii) 1061208 8. $\frac{(x-3)}{(x-5)}$
9. (a) (i) $(1 - y)(y + 6)$ (ii) $(a + 41)(a + 5)$ (iii) $(a - b)(b + c)$
 (b) (i) $(\sqrt{3}x + 2)(x + 3\sqrt{3})$ (ii) $(\sqrt{3}x + 2)(4x - \sqrt{3})$
 (c) (i) $(xy - 9)(xy + 8)$ (ii) $(x - 2)(x + 2)(x - 1)(x + 1)$ (iii) $(x - 5)(x + 1)(x - 2)^2$
 (d) (i) $(x - 2)(x + 3)(2x - 5)$ (ii) $(x - 1)(x - 2)(x - 3)$ (iii) $(x + 1)(x + 2)(x - 2)$
 (iv) $(x - 1)(x + 1)(3x - 1)$ (v) $(x - 1)(x - 10)(x - 12)$
10. 1 11. $(6x + 5)$ 12. $(\sqrt{2}a + \sqrt{3}b)$ 13. (i) $\frac{x^3}{8} + 8y^3$ (ii) $x^6 - 1$ 14. $(x + 5y)^2$
15. $(1 + a - 7b)(1 - a + 7b)$
18. (i) $(x + y)(x - y)(x^2 - xy + y^2)(x^2 + xy + y^2)$
 (ii) $(x - y)(x + y)(x^2 + y^2)(x^4 + y^4 - x^2y^2)(x^2 - xy + y^2)(x^2 + xy + y^2)$
19. ± 18 20. 370 22. $(x + 3)(x + 2)$ 23. $(a + b)(b + c)(c + a)$ 24. 3
25. $(-10y)(12x^2 + 25y^2)$ 26. $x^3 - 8y^3 - z^3 - 6xyz$ 30. $(x - 3)(2x - 1)$

EXERCISE - 2

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	A	A	D	C	B	A	C	A	D	D	B	D	B	D
Ques.	16	17	18	19	20	21									
Ans.	C	C	C	C	A	C									

EXERCISE - 3

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	B	D	A	B	D	D	C	B	B	A	C	C	C	A
Ques.	16														
Ans.	D														

SOLUTIONS

EXERCISE-1

➤ Very Short Answer Type Questions

1. (i) $(x + 4)(x + 10) = x^2 + (4 + 10)x + (4 \times 10) = x^2 + 14x + 40$
 [Using $(x + a)(x + b) = x^2 + (a + b)x + ab$]
 (ii) $(4m + 7n)^2 = [(4m)^2 + 2(4m)(7n) + (7n)^2]$
 $= 16m^2 + 56mn + 49n^2$
 [Using $(a + b)^2 = a^2 + 2ab + b^2$]
2. (i) $95 \times 96 = (100 - 5) \times (100 - 4) = [(100) + (-5)][(100) + (-4)]$
 $= (100)^2 + (-5 - 4)(100) + \{-5 \times (-4)\}$
 [Using $(x + a)(x + b) = x^2 + (a + b)x + ab$]
 $= 10000 - 900 + 20 = 9120$
 (ii) $104 \times 96 = (100 + 4)(100 - 4)$
 $= (100)^2 - (4)^2$
 [Using $(a + b)(a - b) = a^2 - b^2$]
 $= 10000 - 16 = 9984$
3. (i) $(2x + 1)^3 = (2x)^3 + 1^3 + 3(2x)^2(1) + 3(2x)(1)^2$
 [Using $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$]
 $= 8x^3 + 1 + 12x^2 + 6x = 8x^3 + 12x^2 + 6x + 1$
 (ii) $\left(x - \frac{2}{3}y\right)^3 = (x)^3 - \left(\frac{2}{3}y\right)^3 - 3(x)^2\left(\frac{2}{3}y\right) + 3(x)\left(\frac{2}{3}y\right)^2$
 [Using $(a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$]
 $= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$
4. $(28)^3 + (-15)^3 + (-13)^3$
 Let $a = 28$, $b = -15$ and $c = -13$. Then,
 $a + b + c = 28 - 15 - 13 = 28 - 28 = 0$
 $\therefore a^3 + b^3 + c^3 = 3abc$
 $\Rightarrow (28)^3 + (-15)^3 + (-13)^3 = 3 \times 28 \times (-15) \times (-13) = 16380$
5. Factorise numerator : $\frac{x(x+5)}{x} = (x+5)$
 [Cancel common factor, x we get $(x+5)$]
6. If any polynomial $p(x)$ is exactly divisible by $(x - a)$, then remainder will be zero.
 $\therefore p(a) = 0$; $p(y) = y^3 + ky + 2k - 2$

Here, $a = -1$

$$\begin{aligned} \therefore p(-1) &= 0 \\ (-1)^3 + k(-1) + 2k - 2 &= 0 \\ -1 - k + 2k - 2 &= 0 \\ k - 3 &= 0 \Rightarrow \boxed{k = 3} \end{aligned}$$

Ans.

➤ Short Answer Type Questions – Type I

7. (i) $(99)^3 = (100 - 1)^3 = (100)^3 - (1)^3 - 3(100)^2(1) + 3(100)(1)^2$
 $= 1000000 - 1 - 30000 + 300$
 $[(a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2]$
 $= 1000000 - 30000 + 300 - 1 = 970299$
 (ii) $(102)^3 = (100 + 2)^3 = (100)^3 + (2)^3 + 3(100)^2(2) + 3(1000)(2)^2$
 $[Using (a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2]$
 $= 1000000 + 8 + 60000 + 1200 = 1000000 + 60000 + 1200 + 8 = 1061208$
8. We have, $\frac{x^2 + 2x - 5}{x^2 - 25} = \frac{x^2 + 5x - 3x - 15}{x^2 - 5^2}$
 $= \frac{(x - 5)(x + 3)}{(x + 5)(x - 5)} = \frac{(x + 3)}{(x - 5)}$
9. (a) (i) $6 - 5y - y^2$
 $= -(y^2 + 5y - 6)$
 $= -(y^2 + 6y - y - 6)$
 $= -(y(y + 6) - 1(y + 6))$
 $= -(y - 1)(y + 6)$
 (ii) $a^2 + 46a + 205$
 $= a^2 + 41a + 5a + 205$
 $= a(a + 41) + 5(a + 41)$
 $= (a + 5)(a + 41)$
 (iii) $\overbrace{ab + ac - b^2} - bc$
 $= a(a - b) + c(a - b)$
 $= (a - b)(a + c)$
 (b) (i) $\sqrt{3}x^2 + 11x + 6\sqrt{3}$
 $= \sqrt{3}\left(x^2 + \frac{11}{\sqrt{3}}x + 6\right)$
 $= \sqrt{3}\left(x^2 + \frac{4}{2\sqrt{3}}x + \frac{18}{2\sqrt{3}}x + 6\right)$
 $= \sqrt{3}\left(x^2 + \frac{4}{2\sqrt{3}}\right)\left(x + \frac{9}{\sqrt{3}}\right)$

Ans.

Ans.

Ans.

$$\begin{aligned}
 \text{(ii)} \quad & 4\sqrt{3}x^2 + 5x - 2\sqrt{3} \\
 &= \sqrt{3} \left[4x^2 + \frac{5x}{\sqrt{3}} - 2 \right] \\
 &= \sqrt{3} \left[4x^2 + \frac{5x}{\sqrt{3}} - 2 \right] \\
 &= 4\sqrt{3} \left[x^2 + \frac{5x}{4\sqrt{3}} - \frac{2}{4} \right] \\
 &= 4\sqrt{3} \left[x^2 - \frac{16x}{8\sqrt{3}} - \frac{6x}{8\sqrt{3}} - \frac{1}{2} \right] \\
 &= 4\sqrt{3} \left[\left(x - \frac{2}{\sqrt{3}} \right) \cdot \left(x + \frac{3}{4\sqrt{3}} \right) \right]
 \end{aligned}$$

$$\begin{aligned}
 \text{(c) (i)} \quad & x^2y^2 - xy - 72 \\
 &= x^2y^2 - 9xy + 8xy - 72 \\
 &= xy(xy - 9) + 8(xy - 9) \\
 &= (xy + 8)(xy - 9) \\
 \text{(ii)} \quad & x^4 - 5x^2 + 4 \\
 &= x^4 - 4x^2 - x^2 + 4 \\
 &= x^2(x^2 - 4) - 1(x^2 - 4) \\
 &= (x^2 - 1)(x^2 - 4) \\
 &= (x - 1)(x + 1)(x - 2)(x + 2)
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad & (x^2 - 4x)(x^2 - 4x - 1) - 20 \\
 &= (x^2 - 4x)^2 - (x^2 - 4x) - 20 \\
 &= (x^2 - 4x)^2 + 4(x^2 - 4x) - 5(x^2 - 4x) - 20 \\
 &= (x^2 - 4x)(x^2 - 4x - 5) + 4(x^2 - 4x - 5) \\
 &= (x^2 - 4x + 4)(x^2 - 4x - 5) \\
 &= (x - 2)^2(x^2 - 5x + 4x - 5) \\
 &= (x - 2)^2(x + 1)(x - 5)
 \end{aligned}$$

$$\begin{aligned}
 \text{(d) (i)} \quad & 2x^3 - 3x^2 - 17x + 30 \\
 & \text{put } x = \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6 \\
 & \text{Here,} \\
 & (x - 2) \text{ is a factor of } p(x) \text{ because} \\
 & p(2) = 2.2^3 - 3.2^2 - 17.2 + 30 \\
 & p(2) = 16 - 12 - 34 + 30 \\
 & \boxed{p(2) = 0}
 \end{aligned}$$

Now,

$$\begin{array}{r|l}
 & 2x^2 + x - 15 \\
 x - 2 & \overline{2x^3 - 3x^2 - 17x + 30} \\
 & \underline{2x^3 - 4x^2} \\
 & + \\
 & x^2 - 17x + 30 \\
 & \underline{x^2 - 2x} \\
 & - 15x + 30 \\
 & \underline{-15x + 30} \\
 & + \\
 & 0
 \end{array}$$

$$\begin{aligned}
 \therefore 2x^3 - 3x^2 - 17x + 30 \\
 &= (x - 2)(2x^2 - x - 15) \\
 &= (x - 2)(2x^2 + 6x - 5x - 15) \\
 &= (x - 2)(2x(x + 3) - 5(x + 3)) \\
 &= (x - 2)(2x - 5)(x + 3)
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & x^3 - 6x^2 + 11x - 6 \\
 & q(x) = x^3 - 6x^2 + 11x - 6 \\
 & \text{Using Factor Theorem,} \\
 & \text{Put } x = \pm 1, \pm 2, \pm 3, \\
 & \therefore p(q(1)) = 1^3 - 6.1^2 + 11.1 - 6 \\
 & \quad = 1 - 6 + 11 - 6 = 0 \\
 & q(2) = 2^3 - 6.2^2 + 11.2 - 6 \\
 & \quad = 8 - 24 + 22 - 6 = 0 \\
 & q(-1) = (-1)^3 - 6(-1)^2 + 11(-1) - 6 \\
 & \quad = -1 - 6 - 11 - 6 = -24 \\
 & q(-2) = (-2)^3 - 6(-2)^2 + 11(-2) - 6 \\
 & \quad = -8 - 24 - 22 - 6 = -60 \\
 & q(3) = 3^3 - 6.3^2 + 11.3 - 6 \\
 & q(3) = 27 - 54 + 33 - 6 = 60 - 60 = 0
 \end{aligned}$$

$\therefore (x - 1), (x - 2) \& (x - 3)$ are the factors of the given polynomial.

$$\therefore x^3 - 6x^2 + 11x - 6 = (x - 1)(x - 2)(x - 3)$$

$$\begin{aligned}
 \text{(iii)} \quad & x^3 + x^2 - 4x - 4 \\
 & \text{Using Factor Theorem,} \\
 & \text{Put } x = \pm 1, \pm 2, \pm 4 \\
 & r(1) = 1 + 1 - 4 - 4 = -6 \\
 & r(-1) = -1 + 1 + 4 - 4 = 0 \\
 & r(2) = 2^3 + 2^2 - 8 - 4 = 8 + 4 - 8 - 4 = 0 \\
 & r(-2) = (-2)^3 + (-2)^2 + 8 - 4 \\
 & \quad = -8 + 4 + 8 - 4 = 0
 \end{aligned}$$

$\therefore (x + 1), (x - 2) \& (x + 2)$ are the factors of $r(x)$.

$$\begin{aligned}
 \therefore r(x) &= x^3 + x^2 - 4x - 4 \\
 &= (x + 1)(x - 2)(x + 2)
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad & f(x) = 3x^3 - x^2 - 3x + 1 \\
 & \text{put } x = 1, \\
 & f(1) = 3.1^3 - 1^2 - 3.1 + 1 = 0 \\
 & \therefore (x - 1) \text{ is a factor of } f(x)
 \end{aligned}$$

$$\begin{array}{r|l}
 & 3x^2 + 2x - 1 \\
 \text{Now, } x - 1 & \overline{3x^3 - x^2 - 3x + 1} \\
 & \underline{3x^3 - 3x^2} \\
 & + \\
 & 2x^2 - 3x + 1 \\
 & \underline{2x^2 - 2x} \\
 & - x + 1 \\
 & \underline{-x + 1} \\
 & 0
 \end{array}$$

$$\begin{aligned}
 \therefore 3x^3 - x^2 - 3x + 1 \\
 &= (x - 1)(3x^2 + 2x - 1)
 \end{aligned}$$

$$\begin{aligned}
&= (x-1).(3x^2 + 3x - x - 1) \\
&= (x-1).(3x(x+1) - 1(x+1)) \\
&= (x-1)(3x-1)(x+1) \\
\text{(v) } g(x) &= x^3 - 23x^2 + 142x - 120 \\
\text{put } x &= \pm 1, \pm 2, \pm 3, \pm 4 \dots \pm 120 \\
\therefore g(1) &= 1 - 23 + 142 - 120 \\
g(1) &= 0 \Rightarrow (x-1) \text{ is a factor of } g(x) \\
\therefore g(x) &= (x-1).k(x)
\end{aligned}$$

To find $k(x)$, use long division method,

$$\begin{array}{r}
3x^2 - 22x + 120 \\
x-1 \overline{) x^3 - 23x^2 + 142x - 120} \\
\underline{x^3 - x^2} \\
-22x^2 + 142x - 120 \\
\underline{-22x^2 + 22x} \\
120x - 120 \\
\underline{120x - 120} \\
0
\end{array}$$

$$\begin{aligned}
\therefore g(x) &= (x-1)(x^2 - 22x + 120) \\
&= (x-1)(x^2 - 12x - 10x + 120) \\
&= (x-1)(x-10)(x-12) \quad \text{Ans.}
\end{aligned}$$

10. Given that, $a + b + c = 9$ and $ab + bc + ca = 40$,
 $a^2 + b^2 + c^2 = ?$

Now, using the Identity,

$$(a + b + c)^2 = (a^2 + b^2 + c^2 + 2(ab + bc + ca))$$

$$\Rightarrow 9^2 = (a^2 + b^2 + c^2) + 2(40)$$

$$\Rightarrow a^2 + b^2 + c^2 = 81 - 80 = 1 \quad \text{Ans.}$$

11. Square root of $36x^2 + 60x + 25$
 Using Factorization,
 $36x^2 + 60x + 25 = (6x)^2 + 2.(6x).5 + 5^2$
 $= (6x + 5)^2$

$$\therefore \sqrt{36x^2 + 60x + 25} = \pm(6x + 5) \quad \text{Ans.}$$

$$\begin{aligned}
12. \quad &\sqrt{2a^2 + 2\sqrt{6}ab + 3b^2} \\
&= \sqrt{(\sqrt{2}a)^2 + 2.(\sqrt{2}a)(\sqrt{3}b) + (\sqrt{3}b)^2} \\
&= \sqrt{(\sqrt{2}a + \sqrt{3}b)^2} = \pm(\sqrt{2}a + \sqrt{3}b)
\end{aligned}$$

➤ Short Answer Type Questions – Type II

$$\begin{aligned}
13. \quad &\left(\frac{x}{2} + 2y\right) \left(\frac{x^2}{4} - xy + 4y^2\right) \\
&= \left[\left(\frac{x}{2}\right) + (2y)\right] \left[\left(\frac{x}{2}\right)^2 - \left(\frac{x}{2}\right)(2y) + (2y)^2\right]
\end{aligned}$$

$$\begin{aligned}
&= \left(\frac{x}{2}\right)^3 + (2y)^3 \\
&\quad [\text{Using } (a+b)(a^2 - ab + b^2) = a^3 + b^3] \\
&= \frac{x^3}{8} + 8y^3 \\
\text{(ii) } &(x^2 - 1)(x^4 + x^2 + 1) \\
&= [(x^2) - (1)][(x^2)^2 + (x^2)(1) + (1)^2] \\
&= (x^2)^3 - (1)^3 = x^6 - 1 \\
&\quad [\text{Using } (a-b)(a^2 + ab + b^2) = a^3 - b^3]
\end{aligned}$$

$$\begin{aligned}
14. \quad &4(x-y)^2 - 12(x-y)(x+y) + 9(x+y)^2 \\
&= (2(x-y))^2 - 2.2(x-y).3(x+y) + (3(x+y))^2 \\
&= [2(x-y) - 3(x+y)]^2 \\
&= (2(x-y) - 3x - 3y)^2 \\
&= (-x - 5y)^2 = (x + 5y)^2 \\
&= (x + 5y)(x + 5y) \quad \text{Ans.}
\end{aligned}$$

$$\begin{aligned}
15. \quad &1 - a^2 + 14ab - 49b^2 \\
&= 1 - (a^2 - 14ab + 49b^2) \\
&= 1 - (a^2 - 2.7b.a + (7b)^2) \\
&= 1^2 - (a - 7b)^2 = (1 - (a - 7b)).(1 + (a - 7b)) \\
&= (7b - a + 1)(a - 7b + 1) \quad \text{Ans.}
\end{aligned}$$

$$\begin{aligned}
16. \quad &\frac{0.77 \times 0.77 \times 0.77 + 0.23 \times 0.23 \times 0.23}{0.77 \times 0.77 - 0.77 \times 0.23 + 0.23 \times 0.23} \\
\text{Let } x &= 0.77, y = 0.23 \\
\therefore \frac{x^3 + y^3}{x^2 + xy + y^2} &= \frac{(x+y)(x^2 - xy + y^2)}{x^2 - xy + y^2} \\
&= x + y = 0.77 + 0.23 = 1 \quad \text{Ans.}
\end{aligned}$$

$$\begin{aligned}
17. \quad &\text{P.T.} \\
&(a+b)^3 + (b+c)^3 + (c+a)^3 - 3(a+b)(b+c)(c+a) \\
&= 2(a^3 + b^3 + c^3 - 3abc) \\
\text{Let } x &= a+b, y = b+c, z = c+a \\
\text{L.H.S.} &= x^3 + y^3 + z^3 - 3xyz \\
&= (x+y+z) \cdot \left(\frac{(x-y)^2 + (y-z)^2 + (z-x)^2}{2}\right) \\
&= \frac{(a+b+b+c+c+a)}{2} \cdot ((a+b-b-c)^2 + (b+c-c-a)^2 + (c+a-a-b)^2) \\
&= \frac{2(a+b+c)}{2} ((a-c)^2 + (b-a)^2 + (c-b)^2) \\
&= (a+b+c) \cdot ((a-c)^2 + (a-b)^2 + (b-c)^2) \\
&= 2 \cdot \frac{(a+b+c)}{2} \cdot [(a-b)^2 + (b-c)^2 + (c-a)^2] \\
&= 2(a^3 + b^3 + c^3 - 3abc)
\end{aligned}$$

18. (i) $x^6 - y^6$
 $= (x^3)^2 - (y^3)^2$
 $= (x^3 - y^3)(x^3 + y^3)$
 $= (x - y)(x^2 + xy + y^2)(x + y)(x^2 - xy + y^2)$
 $= (x - y)(x + y)(x^2 + xy + y^2)(x^2 - xy + y^2)$

(ii) $x^{12} - y^{12}$
 $= (x^6)^2 - (y^6)^2$
 $= (x^6 - y^6)(x^6 + y^6)$
 $= [(x^3)^2 - (y^3)^2] \cdot [(x^3)^2 + (y^3)^2]$
 $= (x^3 - y^3)(x^3 + y^3)(x^2 + y^2)(x^4 - x^2y^2 - y^4)$
 $= (x - y)(x + y)(x^2 + xy + y^2)(x^2 - xy + y^2)(x^2 + y^2)(x^4 - x^2y^2 + y^4)$ **Ans.**

19. $x^4 + \frac{1}{x^4} = 47$; $x^3 + \frac{1}{x^3} = ?$

$$(x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2 = 47 + 2$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 49$$

$$x^2 + \frac{1}{x^2} = \pm 7$$

$$\therefore x^2 + \frac{1}{x^2} = 7$$

$$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 9$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 9$$

$$\Rightarrow x + \frac{1}{x} = \pm 3$$

$$x + \frac{1}{x} = 3 \quad \dots(1) \quad x + \frac{1}{x} = -3$$

Cubic on both side

$$\left(x + \frac{1}{x}\right)^3 = 3^3$$

$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 27$$

From Eqⁿ (1)

$$x^3 + \frac{1}{x^3} + 3(3) = 27$$

$$x^3 + \frac{1}{x^3} = 27 - 9 = 18$$

$$\boxed{x^3 + \frac{1}{x^3} = 18} \quad \text{Ans.}$$

Cubic on both sides

$$\left(x + \frac{1}{x}\right)^3 = (-3)^3$$

$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = -27$$

$$\left(x^3 + \frac{1}{x^3}\right) + 3(-3) = -27$$

$$x^3 + \frac{1}{x^3} - 9 = -27$$

$$\boxed{x^3 + \frac{1}{x^3} = -18} \quad \text{Ans.}$$

20. $a + b = 10$ & $a^2 + b^2 = 58$, $a^3 + b^3 = ?$
 $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ $(a + b)^2 = a^2 + b^2 + 2ab$
 $= (10)(58 - 21)$ $10^2 = a^2 + b^2 + 2(ab)$
 $= 370$ **Ans.** $10^2 = 58 + 2(2b)$
 $ab = \frac{100 - 58}{2}$
 $ab = \frac{42}{2}$
 $ab = 21$

21. If $a^2 + b^2 + c^2 - ab - bc - ca = 0$, P.T. $a = b = c$.

↓

$$2(a^2 + b^2 + c^2 - ab - bc - ca) = 0$$

$$(a - b)^2 + (b - c)^2 + (c - a)^2 = 0$$

$$(a - b)^2 + (b - c)^2 + (c - a)^2 = 0$$

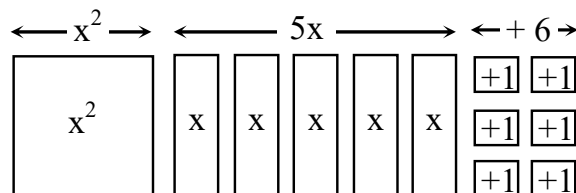
$$\Rightarrow (a - b) = 0, b - c = 0 \text{ \& } c - a = 0$$

$$\Rightarrow \boxed{a = b = c} \quad \text{H.P.}$$

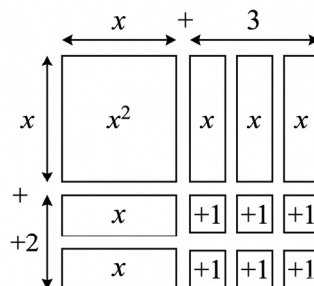
22. Given

$$x^2 + 5x + 6$$

So, algebra tiles for this expression are



Now from these tiles use form a rectangle or a square to solve given expression



Therefore,

$$x^2 + 5x + 6 = (x + 3)(x + 2)$$

➤ Long Answer Type Questions

23. Let $x = a^2 - b^2$, $y = b^2 - c^2$ and $z = c^2 - a^2$
Now, $x + y + z = a^2 - b^2 + b^2 - c^2 + c^2 - a^2 = 0$
 $x^3 + y^3 + z^3 = 3xyz = 3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$
 $= 3(a + b)(a - b)(b + c)(b - c)(c + a)(c - a)$
 $\Rightarrow (a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$
 $= 3(a + b)(a - b)(b + c)(b - c)(c + a)(c - a)$

Also let $p = a - b$, $q = b - c$ and $r = c - a$
 Now, $p + q + r = a - b + b - c + c - a = 0$
 $\therefore p^3 + q^3 + r^3 = 3pqr$
 $\Rightarrow (a - b)^3 + (b - c)^3 + (c - a)^3 = 3(a - b)(b - c)(c - a)$

Hence,

$$\begin{aligned} & \frac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a - b)^3 + (b - c)^3 + (c - a)^3} \\ &= \frac{3(a + b)(a - b)(b + c)(b - c)(c + a)(c - a)}{3(a - b)(b - c)(c - a)} \\ &= (a + b)(b + c)(c + a) \end{aligned}$$

24.
$$\frac{(b + c)^2}{bc} + \frac{(c + a)^2}{ca} + \frac{(a + b)^2}{ab} = \frac{a(b + c)^2 + b(c + a)^2 + c(a + b)^2}{abc}$$

$$= \frac{a(-a)^2 + b(-b)^2 + c(-c)^2}{abc} = \frac{a^3 + b^3 + c^3}{abc}$$

$$= \frac{3abc}{abc} = 3$$

[$\because$ If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$]

25. Let $a = 2x - 5y$, $b = 2x + 5y$
 Use identity $a^3 - b^3$

26. $(-z + x - 2y) \cdot (x^2 + 4y^2 + z^2 + 2xy + xz - 2yz)$
 $= (x - 2y - z) \cdot ((x)^2 + (2y)^2 + (-z)^2 + x(2y) - (x)(-z)) - (-2y)(-z)$

Let $p = x$, $q = -2y$, $r = -z$
 $= (p + q + r)(p^2 + q^2 + r^2 - pq - qr - rp)$
 $= p^3 + q^3 + r^3 - 3pqr$
 $= (x)^3 + (-2y)^3 + (-z)^3 - 3(x)(-2y)(-z)$
 $= x^3 - 8y^3 - z^3 - 6xyz$ **Ans.**

27. Given that, $a + b + c = 0$ P.T.

$$\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$$

$$\text{L.H.S} = \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$$

$$= \frac{a^3 + b^3 + c^3}{abc} = \frac{3abc}{abc}$$

$$= 3 = \text{RHS} \left(\because a^3 + b^3 + c^3 = 3abc \right. \\ \left. \text{if } a + b + c = 0 \right)$$

or

$$\text{L.H.S} = \frac{a^3 + b^3 + c^3}{abc} = \frac{(-b - c)^3 + b^3 + c^3}{abc}$$

$$\begin{aligned} &= \frac{(b^3 + c^3 + 3bc(b + c)) + b^3 + c^3}{abc} \\ &= \frac{-3bc(b + c)}{abc} \\ &= \frac{-3bc(-a)}{abc} \\ &= 3 \end{aligned}$$

Ans.

28. Given that,

$$\left[\begin{array}{l} a + b + c = 5 \\ ab + bc + ca = 10 \end{array} \right] \dots(1)$$

Now,

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

As we know that,

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$\Rightarrow 5^2 = (a^2 + b^2 + c^2) + 2(10)$$

$$\Rightarrow a^2 + b^2 + c^2 = 5 \dots(2)$$

From Eq (1) & (2),

$$a^3 + b^3 + c^3 - 3ab = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= (5)(5 - (10))$$

$$= -25$$

$$= \text{R.H.S}$$

29. P.T. $(a + b + c)^3 - a^3 - b^3 - c^3$
 $= 3(a + b)(b + c)(c + a)$

L.H.S $= (a + b + c)^3 - a^3 - b^3 - c^3$
 $q(a, b, c) = (a + b + c)^3 - a^3 - b^3 - c^3$
 $= c^3 - c^3 = 0$

Put $a = -b$ then

$$q(-b, b, c) = (-b + b + c)^3 - (-b)^3 - b^3 - c^3$$

$$= c^3 - c^3 = 0$$

$\therefore (a + b)$ is a factor of $q(a, b, c)$

Put $b = -c$, then

$$q(a, -c, c) = (a - c + c)^3 - a^3 - (-c)^3 - c^3$$

$$= a^3 - a^3 = 0$$

$\therefore (b + c)$ is a factor of $q(a, b, c)$

Put $c = -a$, then

$$q(a, b, -a) = (a + b - a)^3 - a^3 - b^3 - (-a)^3$$

$$= b^3 - a^3 - b^3 + a^3 = 0$$

$\therefore (c + a)$ is a factor of $q(a, b, c)$

Therefore,

$$(a + b + c)^3 - (a^3 + b^3 + c^3)$$

$$= k(a + b)(b + c)(c + a)$$

To find the value of k put $a = 1, b = +1, c = 0$

$$(1 + 1 + 0)^3 - (1^3 + 1^3 + 0^3)$$

$$= k(1 + 1)(1 + 0)(0 + 1)$$

$$8 - 2 = k.(2).1.1$$

$$\Rightarrow 2k = 6 \quad \Rightarrow \boxed{k = 3}$$

$$\therefore (a + b + c)^3 - (a^3 + b^3 + c^3)$$

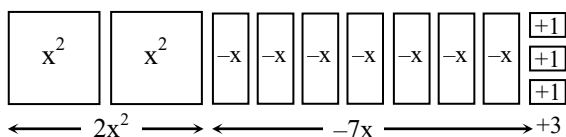
$$= 3(a + b)(b + c)(c + a) \quad \text{H.P.}$$

30.

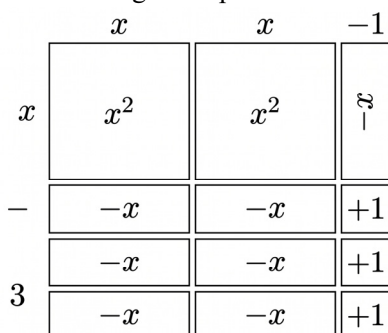
Given

$$2x^2 - 7x + 3$$

So, algebra tiles for this expression are



Now from these tiles use form a rectangle or a square to solve given expression



Therefore,

$$2x^2 - 7x + 3$$

$$= (x - 3)(2x - 1)$$

EXERCISE-2

1.[B]

$$11a^2 - 12\sqrt{2}a + 2$$

$$= 11a^2 - 11\sqrt{2}a - \sqrt{2}a + 2$$

$$= 11a(a - \sqrt{2}) - \sqrt{2}(a - \sqrt{2})$$

$$= (11a - \sqrt{2})(a - \sqrt{2})$$

2.[A]

$$\text{Let, } p(x) = x^5 - a^2x^3 - x^2y^3 + a^2y^3$$

$$= x^3(x^2 - a^2) - y^3(x^2 - a^2)$$

$$= (x^3 - y^3)(x^2 - a^2)$$

$$= (x - y)(x^2 + y^2 + xy)(x - a)(x + a)$$

$$= (x - a)(x + a)(x - y)(x^2 + y^2 + xy)$$

3.[A]

$$a^3 + b^3 + 3ab - 1$$

$$= a^3 + b^3 + (-1)^3 - 3(a)(b)(-1)$$

$$[\text{Using the identity } a^3 + b^3 + c^3 - 3abc]$$

$$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= (a + b - 1)(a^2 + b^2 + 1 - ab - b(-1) - (-1)(a))$$

$$= (a + b - 1)(a^2 + b^2 + 1 - ab + b + a)$$

4.[D]

$$\sqrt{\frac{x^2}{9} + \frac{9}{4x^2} - \frac{x}{3} - \frac{3}{2x} + \frac{5}{4}}$$

$$= \sqrt{\left(\frac{x}{3}\right)^2 + \left(\frac{3}{2x}\right)^2 - \left(\frac{3}{2}\right)^2 + 2\left(\frac{x}{3}\right)\left(-\frac{3}{2x}\right) + 2\left(-\frac{3}{2x}\right)\left(-\frac{3}{2}\right)}$$

$$= \sqrt{\left(\frac{x}{3}\right)^2 + \left(\frac{3}{x}\right)^2 + \left(\frac{1}{2}\right)^2 + 2\left(\frac{x}{3}\right)\left(\frac{3}{2x}\right) + 2\left(\frac{3}{2x}\right)\left(-\frac{1}{2}\right) + 2\left(-\frac{1}{2}\right)\left(\frac{x}{3}\right)}$$

[Using the identity,

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$$

$$= \sqrt{\left(\frac{x}{3} + \frac{3}{2x} - \frac{1}{2}\right)^2}$$

$$= \frac{x}{3} + \frac{3}{2x} - \frac{1}{2}$$

5.[C]

$$\begin{array}{r|l} 2x^2 + 3x + 4 & 4x^4 + 12x^3 + 25x^2 + 24x + 16 \\ \hline 2x^2 + 2x^2 & 4x^4 \\ \hline & 12x^3 + 25x^2 + 24x + 16 \\ 4x^2 + 3x & \underline{12x^3 + 9x^2} \\ + 3x & 16x^2 + 24x + 16 \\ 4x^2 + 6x & \underline{16x^2 + 24x + 16} \\ + 4 & 0 \end{array}$$

$$\therefore \sqrt{4x^4 + 12x^3 + 25x^2 + 24x + 16} = 2x^2 + 3x + 4$$

$$\therefore a = 2, b = 3, c = 4$$

6.[B]

If $a + b + c = 6$, then

$$(2 - a)^3 + (2 - b)^3 + (2 - c)^3 - 3(2 - a)(2 - b)(2 - c) = ?$$

$$\text{Let } 2 - a = x$$

$$2 - b = y$$

$$\Rightarrow x + y + z = (2 - a) + (2 - b) + (2 - c)$$

$$2 - c = z$$

$$\Rightarrow x + y + z = 6 - (a + b + c)$$

$$\Rightarrow x + y + z = 6 - 6 = 0 \quad \dots (1)$$

$$\Rightarrow x^3 + y^3 + z^3 - 3xyz$$

$$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) = 0$$

7.[A]

Given that,

$$x + \frac{1}{x} = 2 \quad \dots (1)$$

then, $x^6 + \frac{1}{x^6} = ?$

First, do squaring on both sides,

$$\left(x + \frac{1}{x}\right)^2 = 2^2 \Rightarrow x^2 + \frac{1}{x^2} + 2 = 4$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 2 \quad \dots (2)$$

Now, Take cubic on both sides, we get,

$$\left(x^2 + \frac{1}{x^2}\right)^3 = 2^3$$

$$\Rightarrow x^6 + \frac{1}{x^6} + 3(x)^2 + \left(\frac{1}{x^2}\right)\left(x^2 + \frac{1}{x^2}\right) = 8$$

$$\Rightarrow x^6 + \frac{1}{x^6} + 3(2) = 8$$

$$\Rightarrow x^6 + \frac{1}{x^6} = 8 - 6$$

$$\Rightarrow \boxed{x^6 + \frac{1}{x^6} = 2}$$

Ans.

Method 2 : Given that ; $x + \frac{1}{x} = 2$

$$x + 1 = 2x$$

$$\Rightarrow (x - 1)^2 = 0 \Rightarrow x = 1$$

Now, $x^6 + \frac{1}{x^6} = 1^6 + \frac{1}{1^6}$

$$\Rightarrow x^6 + \frac{1}{x^6} = 2$$

Ans.

8.[C] Given that,

$$x + y + z = 6, \quad \dots (1)$$

$$xy + yz + zx = 11, \quad \dots (2)$$

$$x^3 + y^3 + z^3 - 3xyz$$

$$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

From Eqⁿ (1),

$$(x + y + z)^2 = 6^2$$

$$(x^2 + y^2 + z^2) + 2(xy + yz + zx) = 36$$

From Eqⁿ (1) & (2)

$$(x^2 + y^2 + z^2) + 2(11) = 36$$

$$x^2 + y^2 + z^2 = 36 - 22$$

$$\boxed{x^2 + y^2 + z^2 = 14} \quad \dots (3)$$

$$\therefore x^3 + y^3 + z^3 - 3xyz = (6) \cdot (14 - (11))$$

$$= 6 \times (3) = 18$$

9.[A]
$$\frac{a^3 + b^3 + c^3 - 3abc}{-(a^2 + b^2 + c^2 - ab - bc - ca)}$$

$$= \frac{(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)}{-(a^2 + b^2 + c^2 - ab - bc - ca)}$$

Given that,
$$\begin{cases} a = -5 \\ b = -6 \\ c = 10 \end{cases}$$

$$= -(a + b + c) = -(-5 - 6 + 10)$$

$$= -(-11 + 10) = 1$$

10.[D] Given that,

$$a^2 - b^2 = 21$$

$$\frac{a^2 + b^2 = 29}{2a^2 = 50}$$

$$2a^2 = 50$$

$$a^2 = 25$$

$$\boxed{a = \pm 5}$$

$$a^2 + b^2 = 21$$

$$25 - b^2 = 21$$

$$b^4 = 4$$

$$\boxed{b = \pm 2}$$

$$\therefore ab = \pm 10$$

11.[D]
$$(a+b)^3 + (a-b)^3 + 6a(a^2 - b^2)$$

$$= (a+b)^3 + (a-b)^3 + 3((a+b) + (a-b)(a+b)(a-b))$$

$$= [(a+b) + (a-b)]^3 = (2a)^3 = 8a^3$$

12.[B] Given that,

$$\left. \begin{aligned} x + y + z &= 1 \\ xy + yz + zx &= -1 \\ xyz &= -1 \end{aligned} \right\} \quad \dots (1)$$

$$x^3 + y^3 + z^3 - 3xyz$$

$$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

From (1),

$$(x + y + z)^2 = 1^2$$

$$x^2 + y^2 + z^2 + 2(xy + yz + zx) = 1$$

$$(x^2 + y^2 + z^2) - 2(-1) = 1$$

$$x^2 + y^2 + z^2 = 3$$

$$\therefore x^3 + y^3 + z^3 - 3(-1) = (1) \cdot (3 - (-1))$$

$$\Rightarrow x^3 + y^3 + z^3 + 3 = 4$$

$$x^3 + y^3 + z^3 = 1$$

13.[D]
$$\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} + \frac{(c-a)^2}{(a-b)(b-c)} L$$

$$\left. \begin{aligned} a - b &= x \\ b - c &= y \\ c - a &= z \end{aligned} \right\}$$

$$\text{et } b - c = y$$

$$c - a = z$$

$$\therefore x + y + z = 0$$

$$\therefore x^3 + y^3 + z^3 - 3xyz = 0$$

$$\begin{aligned}
 x^3 + y^3 + z^3 &= 3xyz \\
 &= \frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)} \\
 &= \frac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3
 \end{aligned}$$

14.[B] $\frac{0.76 \times 0.76 \times 0.76 + 0.24 \times 0.24 \times 0.24}{0.76 \times 0.76 - 0.76 \times 0.24 + 0.24 \times 0.24}$
 Let $x = 0.76$, $y = 0.24$

$$= \frac{x^3 + y^3}{x^2 - xy + y^2}$$

$$= \frac{(x+y) \cdot (x^2 - xy + y^2)}{x^2 - xy + y^2} = x + y = 0.76 + 0.24 = 1$$

15. [D] $3xy$
 $(x+y)^3 - (x^3 + y^3)$
 $\therefore (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
 $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
 $= [x^3 + 3x^2y + 3xy^2 + y^3] - [(x+y)(x^2 - xy + y^2)]$
 $= [x^3 + 3x^2y + 3xy^2 + y^3] - [x(x^2 - xy + y^2) + y(x^2 - xy + y^2)]$
 $= [x^3 + 3x^2y + 3xy^2 + y^3] - [x(x^2 - xy + y^2) + y(x^2 - xy + y^2)]$
 $= [x^3 + 3x^2y + 3xy^2 + y^3] - [x^3 - x^2y + xy^2 + yx^2 - xy^2 + y^3]$
 $= [x^3 + 3x^2y + 3xy^2 + y^3] - [x^3 - x^2y + xy^2 + yx^2 - xy^2 + y^3]$
 $= x^3 + 3x^2y + 3xy^2 + y^3 - x^3 + x^2y - xy^2 - yx^2 + xy^2 - y^3$
 $= 3x^2y + 3xy^2$
 $= 3xy(x+y)$

16. [C] 0
 $\frac{x}{y} + \frac{y}{x} = -1$
 $\Rightarrow \frac{x^2 + y^2}{xy} = -1$
 $\Rightarrow x^2 + y^2 = -xy$
 $\Rightarrow x^2 + y^2 + xy = 0 \quad \dots(i)$
 $\frac{x^3 - y^3}{x^3 - y^3} \therefore a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
 $= (x-y)(x^2 + xy + y^2)$
 from eq. (i)
 $= (x-y)(0)$
 $= 0$

17.[C] $\frac{1}{4}$
 $49x^2 - b = \left(7x + \frac{1}{2}\right) \left(7x - \frac{1}{2}\right)$
 $\therefore (a+b)(a-b) = a^2 - b^2$

$$\begin{aligned}
 \Rightarrow 49x^2 - b &= (7x)^2 - \left(\frac{1}{2}\right)^2 \\
 \Rightarrow 49x^2 - b &= 49x^2 - \frac{1}{4}
 \end{aligned}$$

Compare both sides

$$b = \frac{1}{4}$$

18.[C] 5
 $x^2 + kx + 6 = (x+2)(x+3)$
 $\Rightarrow x^2 + kx + 6 = x(x+3) + 2(x+3)$
 $\Rightarrow x^2 + kx + 6 = x^2 + 3x + 2x + 6$
 $\Rightarrow x^2 + kx + 6 = x^2 + 5x + 6$
 Compare both sides
 $k = 5$

19.[C] $\sqrt{2} - 1$
 $p(x) = kx^2 - \sqrt{2}x + 1$
 $g(x) = (x-1)$
 Put $g(x) = 0$
 $\Rightarrow x - 1 = 0$
 $\Rightarrow x = 1$
 Put $x = 1$ in $p(x)$ and $p(x) = 0$
 $\Rightarrow p(1) = 0$
 $\Rightarrow k(1)^2 - \sqrt{2}(1) + 1 = 0$
 $\Rightarrow k - \sqrt{2} + 1 = 0$
 $\Rightarrow k = \sqrt{2} - 1$

20.[A] $-\frac{1}{2}$
 $p(x) = 10x^2 + px + q$
 $g(x) = (x+2)$
 Put $g(x) = 0$
 $x + 2 = 0$
 $\Rightarrow x = -2$
 Put $x = -2$ in $p(x)$ and $p(x) = 0$
 $\Rightarrow p(-2) = 0$
 $\Rightarrow 10(-2)^2 + p(-2) + q = 0$
 $\Rightarrow 40 - 2p + q = 0$
 $\Rightarrow 2p - q = 40 \quad \dots(i)$
 Now, $g(x) = (x-1)$
 Put $g(x) = 0$
 $\Rightarrow x - 1 = 0$
 $\Rightarrow x = 1$
 Put $x = 1$ in $p(x)$ and $p(x) = 0$
 $\Rightarrow p(1) = 0$

$$\begin{aligned}
&\Rightarrow 10(1)^2 + p(1) + q = 0 \\
&\Rightarrow 10 + p + q = 0 \\
&\Rightarrow p + q = -10 \quad \dots(ii) \\
&\text{eqn. (i) + eqn. (ii)} \\
&\Rightarrow 2p - q + p + q = 40 + (-10) \\
&\Rightarrow 3p = 40 - 10 \\
&\Rightarrow 3p = 30 \\
&\Rightarrow p = \frac{30}{3} = 10
\end{aligned}$$

Put value of p in eqn. (ii)

$$\begin{aligned}
&\Rightarrow p + q = -10 \\
&\Rightarrow 10 + q = -10 \\
&\Rightarrow q = -10 - 10 \\
&\Rightarrow q = -20 \\
&\frac{p}{q} \\
&= \frac{10}{-20} \\
&= -\frac{1}{2}
\end{aligned}$$

21.[C] $\frac{x-5}{x^{1/2}}$

EXERCISE-3

1.[A] Given that,

$$x + \frac{1}{x} = a + b \quad \dots (1)$$

Adding (+) $x - \frac{1}{x} = a - b \quad \dots (2)$

$$2x = 2a$$

$$\Rightarrow \boxed{x = a}$$

From Eq. (1)

$$x + \frac{1}{x} = a + b$$

$$a + \frac{1}{x} = a + b$$

$$\boxed{x = \frac{1}{b}}$$

$$\therefore x = a = \frac{1}{b}$$

$$\Rightarrow \boxed{ab = 1}$$

2.[B] H.C.F. $\times$ L.C.M. = Product of two polynomials

$$A.(x^3 - 1) = (x - 1).(x^6 - 1)$$

$$A = \frac{(x - 1).(x^3 - 1)(x^3 + 1)}{(x^3 - 1)}$$

$$A = x^4 + x - x^3 - 1$$

$$A = x^4 - x^3 + x - 1$$

3.[D] Let, $p(x) = x^2 + 5x + 6 = (x + 3).(x + 2)$

$$q(x) = x^3 + 27 = (x + 3).(x^2 + 3x + 9)$$

$\therefore$ H.C.F. of $p(x)$ & $q(x) = (x + 3)$

4.[A] $(2x - 3y)^2 - 7(2x - 3y) - 30$

$$\text{Let } 2x - 3y = m$$

$$m^2 - 7m - 30 = m^2 - 10m + 3m - 30$$

$$= m(m - 10) + 3(m - 10)$$

$$= (m + 3)(m - 10)$$

$$= (2x - 3y + 3).(2x - 3y - 10)$$

5.[B] Let

$$p(x) = x^3 + x^2 + x + 1$$

$$= x^2(x + 1) + 1(x + 1)$$

$$= (x + 1).(x^2 + 1)$$

$$q(x) = x^3 - x^2 + x - 1$$

$$= x^2(x - 1) + 1(x - 1) = (x^2 + 1).(x - 1)$$

$$\therefore \text{L.C.M. } (p(x), q(x)) = (x + 1)(x - 1)(x^2 + 1)$$

$$= (x^2 - 1).(x^2 + 1) = x^4 - 1$$

6.[D] $x + \frac{1}{x} = 3$

Squaring on both sides, $\left(x + \frac{1}{x}\right)^2 = 9$

$$x^2 + \frac{1}{x^2} + 2 = 9$$

$$x^2 + \frac{1}{x^2} = 7$$

Taking cube on both sides, we get,

$$\left(x^2 + \frac{1}{x^2}\right)^3 = 7^3$$

$$\Rightarrow x^6 + \frac{1}{x^6} + 3\left(x^2 + \frac{1}{x^2}\right) = 343$$

$$\Rightarrow x^6 + \frac{1}{x^6} = 343 - 3 \cdot 7$$

$$\Rightarrow x^6 + \frac{1}{x^6} = 343 - 21 = 322$$

7.[D] Given that, $a^5 = b^6 \Rightarrow a = b^{6/5}$... (1)

$c^3 = d^4 \Rightarrow d = c^{3/4}$... (2)

& $d - a = 61$... (3)

From (1) & (2),

$c^{3/4} - b^{6/5} = 61$

($\because$ a, b, c & d are the natural numbers)

$\therefore c = 625$ & $b = 32$

$\therefore c - b = 625 - 32 = 593$

8.[C] Given that,

$$\begin{aligned} & \frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} \\ &= \frac{1}{1+\left(\frac{x^b}{x^a}\right)+\left(\frac{x^c}{x^a}\right)} + \frac{1}{1+\left(\frac{x^a}{x^b}\right)+\left(\frac{x^c}{x^b}\right)} + \frac{1}{1+\left(\frac{x^b}{x^c}\right)+\left(\frac{x^a}{x^b}\right)} \\ &= \left(\frac{x^a}{x^a+x^b+x^c}\right) + \left(\frac{x^b}{x^a+x^b+x^c}\right) + \left(\frac{x^c}{x^a+x^b+x^c}\right) \\ &= \frac{x^a+x^b+x^c}{x^a+x^b+x^c} = 1 \end{aligned}$$

9.[B] $p(x) = ax^3 + x^2 - 2x + 4a - 9$

$g(x) = (x + 1)$

Put $g(x) = 0$

$\Rightarrow x + 1 = 0$

$\Rightarrow x = -1$

Put $x = -1$ in $p(x)$ and $p(x) = 0$

$\Rightarrow p(-1) = 0$

$\Rightarrow a(-1)^3 + (-1)^2 - 2(-1) + 4a - 9 = 0$

$\Rightarrow -a + 1 + 2 + 4a - 9 = 0$

$\Rightarrow 3a - 6 = 0$

$\Rightarrow 3a = 6$

$\Rightarrow a = \frac{6}{3} = 2$

10.[B] ± 9

$x - y = 1, \quad x^2 + y^2 = 41$

$x - y = 1$

Square both side

$\Rightarrow (x - y)^2 = (1)^2$

$\Rightarrow x^2 + y^2 - 2xy = 1$

$\Rightarrow 41 - 2xy = 1$

$\Rightarrow 2xy = 41 - 1$

$\Rightarrow 2xy = 40$

$\Rightarrow xy = \frac{40}{2} = 20$

$(x + y)^2 = x^2 + 2xy + y^2$

$\Rightarrow (x + y)^2 = x^2 + y^2 + 2xy$

$\Rightarrow (x + y)^2 = 41 + 2(20)$

$\Rightarrow (x + y)^2 = 41 + 40$

$\Rightarrow (x + y)^2 = 81$

Square root both sides

$\Rightarrow \sqrt{(x + y)^2} = \sqrt{81}$

$\Rightarrow x + y = \pm 9$

11.[A] $x^4 + \frac{1}{x^4} = 47$

$\Rightarrow (x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2(x)^2 \left(\frac{1}{x^2}\right) - 2(x)^2 \left(\frac{1}{x^2}\right) = 47$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 - 2 = 47$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 47 + 2$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 49$

Square root both sides

$\Rightarrow \sqrt{\left(x^2 + \frac{1}{x^2}\right)^2} = \sqrt{49}$

$\Rightarrow x^2 + \frac{1}{x^2} = 7$

$\Rightarrow (x)^2 + \left(\frac{1}{x}\right)^2 + 2(x) \left(\frac{1}{x}\right) - (2x) \left(\frac{1}{x}\right) = 7$

$\Rightarrow \left(x + \frac{1}{x}\right)^2 - 2 = 7$

$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 7 + 2$

Square root both sides

$\Rightarrow \sqrt{\left(x + \frac{1}{x}\right)^2} = \sqrt{9}$

$\Rightarrow x + \frac{1}{x} = 3$

12.[C] -8

$$(a + b + c) = 4$$

Square both sides

$$\Rightarrow (a + b + c)^2 = (4)^2$$

$$\Rightarrow a^2 + b^2 + c^2 + 2ab + 2bc + ca = 16$$

$$\Rightarrow a^2 + b^2 + c^2 + 2(ab + bc + ca) = 16$$

$$\Rightarrow a^2 + b^2 + c^2 + 2(6) = 16$$

$$\Rightarrow a^2 + b^2 + c^2 + 12 = 16$$

$$\Rightarrow a^2 + b^2 + c^2 = 16 - 12$$

$$\Rightarrow a^2 + b^2 + c^2 = 4$$

$$a^3 + b^3 + c^3 - 3abc$$

$$\therefore x^3 + y^3 + z^3 - 3xyz$$

$$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= (4)[4 - (ab + bc + ca)]$$

$$= 4[4 - 6]$$

$$= 4(-2)$$

$$= -8$$

13. [C] $\ell + m + n = 0$

$$\frac{\ell^2}{mn} + \frac{m^2}{n\ell} + \frac{n^2}{\ell m}$$

$$= \frac{\ell^3 + m^3 + n^3}{\ell mn}$$

$$\text{If } \ell + m + n = 0, \text{ then } \ell^3 + m^3 + n^3 = 3\ell mn$$

$$= \frac{3\ell mn}{\ell mn}$$

$$= 3$$

14.[C] $a + b + c = 5$, and $ab + bc + ca = 10$

$$a + b + c = 5$$

Square both sides

$$\Rightarrow (a + b + c)^2 = (5)^2$$

$$\Rightarrow a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 25$$

$$\therefore (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$$

$$\Rightarrow a^2 + b^2 + c^2 + 2(ab + bc + ca) = 25$$

$$\Rightarrow a^2 + b^2 + c^2 + 2(10) = 25$$

$$\Rightarrow a^2 + b^2 + c^2 + 20 = 25$$

$$\Rightarrow a^2 + b^2 + c^2 = 25 - 20$$

$$\Rightarrow a^2 + b^2 + c^2 = 5$$

$$a^3 + b^3 + c^3 - 3abc$$

$$\therefore x^3 + y^3 + z^3 - 3xyz = (x + y + z)$$

$$(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= (5)[a^2 + b^2 + c^2 - (ab + bc + ca)]$$

$$= (5)[5 - (10)]$$

$$= 5[-5]$$

$$= -25$$

Sol.15 [A]

$$a^2 + b^2 + c^2 = 90, \quad a + b + c = 20$$

$$a + b + c = 20$$

square both sides

$$\Rightarrow (a + b + c)^2 = (20)^2$$

$$\Rightarrow a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 400$$

$$\therefore (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$$

$$\Rightarrow 90 + 2(ab + bc + ca) = 400$$

$$\Rightarrow 2(ab + bc + ca) = 400 - 90$$

$$\Rightarrow ab + bc + ca = \frac{310}{2}$$

$$\Rightarrow ab + bc + ca = 155$$

16.[D]

$$\frac{(x + 5)}{(x + 1)}$$

$$\frac{x^2 + 4x - 5}{x^2 - 1}$$

$$= \frac{x^2 + 4x - 5}{(x)^2 - (1)} \quad \therefore a^2 - b^2 = (a + b)(a - b)$$

$$= \frac{x^2 + 5x - x - 5}{(x + 1)(x - 1)}$$

$$= \frac{x(x + 5) - 1(x + 5)}{(x + 1)(x - 1)}$$

$$= \frac{(x + 5)(x - 1)}{(x + 1)(x - 1)}$$

$$= \frac{(x + 5)}{(x + 1)}$$

MENTAL ABILITY

NUMBER SERIES

Mental ability is the ability of mind to observe and understand things or patterns in a logical way and reach to a conclusion or judgment based on that logic.

It is the ability to distinguish between important, less important and more important.

Number Series problems deal with numbers. While attempting to solve the question, you have to check the pattern of the series. Series moves with certain mathematical operations like :

- Consecutive odd/even numbers.
- Consecutive prime / composite numbers.
- Squares/cubes of some numbers with/without variation of addition or subtraction of some number.
- Sum/product/difference of preceding number(s) .
- Addition/subtraction/multiplication/division by some number.
- Many more combinations of the relationship given above.

Type of questions asked in the examination:

- Find the missing term(s).
- Find the wrong term(s).

◆ FIND THE MISSING TERM

Ex.1 7, 12, 19, ?, 39

- (A) 29 (B) 28 (C) 26 (D) 24

Sol.(B) The given sequence follows the pattern:

$$+5, +7, +9 \dots \text{i.e., } 7 + 5 = 12, 12 + 7 = 19, \dots$$

$$\therefore \text{Missing number} = 19 + 9 = 28$$

Ex.2 0, 6, 24, 60, 120, 210, ?

- (A) 240 (B) 290 (C) 336 (D) 504

Sol.(C) The given series is

$$1^3 - 1, 2^3 - 2, 3^3 - 3, 4^3 - 4, 5^3 - 5, 6^3 - 6.$$

$$\therefore \text{Next number} = 7^3 - 7 = 343 - 7 = 336$$

Ex.3 4, 6, 12, 14, 28, 30, ?

- (A) 32 (B) 60 (C) 62 (D) 64

Sol.(B) The given sequence is a combination of two series :

I. 4, 12, 28, ? and II. 6, 14, 30.

Now, the pattern followed in each of the above two series is : +8, +16, +32

$$\text{So, missing number} = (28 + 32) = 60$$

Ex.4 1, 3, 3, 6, 7, 9, ?, 12, 21.

- (A) 10 (B) 11 (C) 12 (D) 13

Sol.(D) The given sequence is a combination of two series :

I. 1, 3, 7, ?, 21 and II. 3, 6, 9, 12

The pattern followed in I is +2, +4, +6, +8 ... and the pattern followed in II is +3. Thus, missing number = 7 + 6 = 13.

Ex.5 $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16}, ?$

- (A) $\frac{9}{32}$ (B) $\frac{10}{17}$ (C) $\frac{11}{34}$ (D) $\frac{12}{35}$

Sol.(A) Clearly, the numerators of the fractions in the given sequence form the series 1, 3, 5, 7, in which each term is obtained by adding 2 to the previous term. The denominators of the fractions form the series 2, 4, 8, 16

i.e. $2^1, 2^2, 2^3, 2^4$.

So, the numerator of the next fraction will be $(7 + 2)$ i.e., 9 and the denominator will be 2^5 i.e. 32.

$\therefore$ The next term is $\frac{9}{32}$

Ex.6 94, 166, 258, ?, 4912

- (A) 3610 (B) 2490
(C) 789 (D) 810

Sol.(A) Each number is in two parts. The first part is square of consecutive numbers 3, 4, 5,

$(3)^2$ $(4)^2$ $(5)^2$ $(6)^2$ $(7)^2$
9 4 16 6 25 8 36 10 49 12
4 6 8 10 12

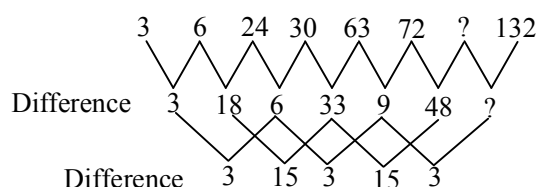
The second part is the sequence of number with difference +2, like 4, 6, 8,

Hence, the required number is 3610.

Ex.7 3, 6, 24, 30, 63, 72, ?, 132

- (A) 128 (B) 122 (C) 120 (D) 124

Sol.(C) The difference between the terms is given below as:



Therefore, alternate difference between the difference is 3 and 15 respectively.

Hence, the next term would be $72 + 48 = 120$.

Ex.8 6, 11, 21, 36, 56, ?

- (A) 42 (B) 51 (C) 81 (D) 91

Sol.(C) The pattern is +5, +10, +15, +20, ...

$\therefore$ Missing number = $56 + 25 = 81$

Ex.9 1, 6, 13, 22, 33, ?

- (A) 44 (B) 45 (C) 46 (D) 47

Sol.(C) The pattern is + 5, + 7, + 9, + 11, ...

$\therefore$ Missing number = $33 + 13 = 46$

Ex.10 1, 9, 17, 33, 49, 73, ?

- (A) 97 (B) 98 (C) 99 (D) 100

Sol.(A) The pattern is +8, +8, +16, +16, +24, ...

$\therefore$ Missing number = $73 + 24 = 97$

Ex.11 3, 7, 15, 31, 63, ?

- (A) 92 (B) 115 (C) 127 (D) 131

Sol.(C) Each number in the series is the preceding number multiplied by 2 and then increased by 1.

Thus, $(3 \times 2) + 1 = 7$, $(7 \times 2) + 1 = 15$,

$(15 \times 2) + 1 = 31$ and so on.

$\therefore$ Missing number = $(63 \times 2) + 1 = 127$

Ex.12 1, 6, 15, ?, 45, 66, 91

- (A) 25 (B) 26 (C) 27 (D) 28

Sol.(D) The pattern is +5, +9, ..., +21, +25

$\therefore$ Missing number = $15 + 13 = 28$

Ex.13 1, 2, 3, 5, 8, ?

- (A) 9 (B) 11 (C) 13 (D) 15

Sol.(C) Each term in the series is the sum of the preceding two terms.

Thus, $1 + 2 = 3$; $2 + 3 = 5$; $3 + 5 = 8$ and so on.

$\therefore$ Missing number = $5 + 8 = 13$

Ex.14 77, 49, 36, 18, ?

- (A) 8 (B) 9 (C) 4 (D) 12

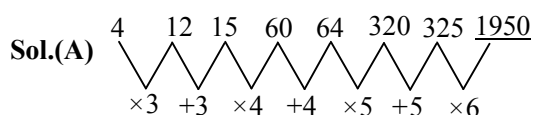
Sol.(A) Next term in the series is the product of the digits of the previous term

$7 \times 7 = 49$, $4 \times 9 = 36$, $3 \times 6 = 18$,

Thus, $1 \times 8 = 8$

Ex.15 4, 12, 15, 60, 64, 320, 325, ?

- (A) 1950 (B) 1850
(C) 1935 (D) 1955



Ex.16 15, -30, 60, -120, ?

- (A) 150 (B) 240 (C) 270 (D) 180

Sol.(B)

$$\begin{array}{ccccccc}
 15 & & -30 & & 60 & & -120 & & 240 \\
 \swarrow & & \searrow & & \swarrow & & \searrow & & \swarrow \\
 & \times(-2) & & \times(-2) & & \times(-2) & & \times(-2) &
 \end{array}$$

◆ **FIND THE WRONG TERM**

Ex.17 2, 5, 9, 11, 14

- (A) 2 (B) 5 (C) 9 (D) 11

Sol.(C) Series : + 3, + 3, + 3,

The next term is got by adding 3 in the preceding term.

$$\therefore 2 + 3 = 5, 5 + 3 = 8$$

Hence, 9 is the wrong term.

Ex.18 10, 100, 1100, 11000, 111000, 1210000.

- (A) 1210000 (B) 11000
(C) 100 (D) 111000

Sol.(D) Given series is :

$$\begin{array}{ccccccc}
 10 & & 100 & & 1100 & & 11000 & & 121000 & & 1210000 \\
 \swarrow & & \searrow & & \swarrow & & \searrow & & \swarrow & & \searrow \\
 & \times 10 & & \times 11 & & \times 10 & & \times 11 & & \times 10 &
 \end{array}$$

Hence, the wrong term is 111000.

Ex.19 2, 6, 11, 17, 23, 32, 41

- (A) 6 (B) 17 (C) 23 (D) 32

Sol.(C) Given series is :

$$\begin{array}{ccccccc}
 & & & & 24 & & \\
 2, & 6, & 11, & 17, & \boxed{23}, & 32, & 41 \\
 \swarrow & \searrow & \swarrow & \searrow & \swarrow & \searrow & \\
 & +4 & +5 & +6 & +7 & +8 & +9
 \end{array}$$

Hence, the wrong term is 23.

Ex.20 61, 52, 63, 95, 46, 18

- (A) 95 (B) 63 (C) 46 (D) 52

Sol.(A) On interchanging the digits of each term, we get a number which is a perfect square of a natural number.

$$4^2 = 16 \Rightarrow 61, 5^2 = 25 \Rightarrow 52, 6^2 = 36 \Rightarrow 63,$$

$$7^2 = 49 \Rightarrow 94,$$

Hence, the wrong term is 95.

Ex.21 126, 62, 30, 15, 6, 2

- (A) 15 (B) 30 (C) 6 (D) 62

Sol.(A)

$$\begin{array}{ccccccc}
 126 & & 62 & & 30 & & 14 & & 6 & & 2 \\
 \swarrow & & \searrow & & \swarrow & & \searrow & & \swarrow & & \searrow \\
 & \div 2-1 & & \div 2-1 & & \div 2-1 & & \div 2-1 & & \div 2-1 &
 \end{array}$$

Hence, the wrong term is 15.

EXERCISE-1

Directions: (Q.1 to Q.15) Find the missing term.

Q.1 9, 7, 14, 11, 33, 28, ?, 133, 931
(A) 64 (B) 46 (C) 140 (D) 123

Q.2 6, 12, 23, 44, 85, ?
(A) 166 (B) 174 (C) 165 (D) 168

Q.3 92, 23, 27, 9, 12, 6, ?
(A) 20 (B) 16 (C) 8 (D) 10

Q.4 0, 5, 8, 17, 24, 37, 48, ?
(A) 65 (B) 67 (C) 56 (D) 71

Q.5 999, 730, 510, 345, 213, ?
(A) 122 (B) 126 (C) 68 (D) 128

Q.6 2, 5, 14, 7, 3, 20, 10, 4, ?
(A) 26 (B) 30 (C) 22 (D) 28

Q.7 3, 7, 35, 47, ?, 119
(A) 99 (B) 98 (C) 64 (D) 89

Q.8 23, 27, 43, 79, 143, ?
(A) 242 (B) 241 (C) 190 (D) 243

Q.9 870, 342, 132, ?, 12, 2
(A) 90 (B) 42 (C) 64 (D) 66

Q.10 9, 19, 35, 75, 143, ?
(A) 287 (B) 285 (C) 196 (D) 295

Q.11 2, 20, 110, 380, ?
(A) 424 (B) 650 (C) 982 (D) 992

Q.12 256, 125, 60, ? 12.5
(A) 26 (B) 40 (C) 28 (D) 25

Q.13 6, 15, 35, 77, 143, 221, 323, ?
(A) 437 (B) 427 (C) 384 (D) 365

Q.14 9, 9, 14, 34, 124, ?
(A) 576 (B) 532 (C) 448 (D) 604

Q.15 125, 215, 343, 511, 729, ?
(A) 754 (B) 683 (C) 845 (D) 999

Directions: (Q.16 to Q.25) Find the wrong term.

Q.16 10, 26, 74, 218, 654, 1946, 5834
(A) 26 (B) 74 (C) 218 (D) 654

Q.17 3, 7, 15, 39, 63, 127, 255, 511
(A) 15 (B) 39 (C) 63 (D) 127

Q.18 1236, 2346, 3456, 4566, 5686
(A) 1236 (B) 3456 (C) 4566 (D) 5686

Q.19 10, 15, 26, 35, 48, 63, 82
(A) 48 (B) 26 (C) 63 (D) 82

Q.20 445, 221, 109, 46, 25, 11, 4
(A) 25 (B) 46 (C) 109 (D) 221

Q.21 2, 6, 10, 20, 30, 42, 56
(A) 6 (B) 10 (C) 20 (D) 30

Q.22 3, 9, 27, 82, 243
(A) 27 (B) 54 (C) 82 (D) 162

Q.23 320, 254, 200, 155, 122, 100, 89
(A) 155 (B) 320 (C) 254 (D) 200

Q.24 15, 34, 71, 134, 223, 350
(A) 71 (B) 134 (C) 223 (D) 350

Q.25 5, 10, 40, 80, 320, 550, 2560
(A) 80 (B) 320 (C) 550 (D) 2560

EXERCISE-2

(Previous Year Questions - NTSE & NSO)

Directions: (Q.1 to Q.52) Find the missing term

- Q.1** 2, 3, 10, 15, 26, ?
(A) 34 (B) 35 (C) 36 (D) 37
- Q.2** 1, 4, 27, 16, 125, 36, ?
(A) 216 (B) 343 (C) 64 (D) 49
- Q.3** 336, 210, 120, ?, 24, 6, 0
(A) 40 (B) 50 (C) 60 (D) 70
- Q.4** 3, 4, 8, 17, 33, ?
(A) 58 (B) 69 (C) 49 (D) 98
- Q.5** 8, 13, 21, 34, 55, ?
(A) 60 (B) 68 (C) 89 (D) 76
- Q.6** 480, 480, 240, 80, 20, ?
(A) 4 (B) 1 (C) 5 (D) 10
- Q.7** 1, 1, 2, 2, 3, 4, 4, 8, 5, 16, ?
(A) 6 (B) 32 (C) 8 (D) 7
- Q.8** 2, 5, 11, 23, 47, ?
(A) 92 (B) 90 (C) 95 (D) 91
- Q.9** 12, 21, 23, 32, 34, 43, 45, ?
(A) 54 (B) 48 (C) 77 (D) 9
- Q.10** 14, 1, 21, 4, 28, 9, ?, ?
(A) 9, 42 (B) 16, 35
(C) 35, 16 (D) 16, 36
- Q.11** 5, 6, 13, 26, 45, ?
(A) 68 (B) 74 (C) 70 (D) 82
- Q.12** 190, 94, 46, 22, 10, 4, ?
(A) 3 (B) 2 (C) 1 (D) 0
- Q.13** 128, 110, 90, 68, ?
(A) 236 (B) 42 (C) 44 (D) 48
- Q.14** 1, 2, 4, 7, ?, 16
(A) 9 (B) 11 (C) 12 (D) 13
- Q.15** 6, 8, 9, 12, 14, 18, ?
(A) 21 (B) 19 (C) 23 (D) 20
- Q.16** 4, 9, 19, 34, 54, ?
(A) 66 (B) 75 (C) 79 (D) 84
- Q.17** 31, 29, 24, 22, 17, ?, ?
(A) 15, 13 (B) 10, 8 (C) 14, 12 (D) 15, 10
- Q.18** 3, 6, 11, 18, ?
(A) 19 (B) 27 (C) 30 (D) 37
- Q.19** 3, 8, 15, 24, ?
(A) 30 (B) 35 (C) 36 (D) 49
- Q.20** 4, 10, 23, 50, 105, ?
(A) 215 (B) 210 (C) 216 (D) 439
- Q.21** 912, 303, 102, 33, ?, 3, 2
(A) 12 (B) 10 (C) 8 (D) 6
- Q.22** 1, 4, 9, ?, 25, 36
(A) 11 (B) 19 (C) 21 (D) 16
- Q.23** 7, 12, 22, 37, ?, 82, 112
(A) 62 (B) 57 (C) 52 (D) 42
- Q.24** 11, 13, 17, 19, ?, 25
(A) 20 (B) 21 (C) 23 (D) 22
- Q.25** 5, 9, 17, 33, ?, 129
(A) 72 (B) 67 (C) 65 (D) 58
- Q.26** 2, 5, 4, 10, 7, 15, 11, 20, ?, ?
(A) 12, 21 (B) 16, 25
(C) 13, 25 (D) 17, 30
- Q.27** 0, 6, 24, 60, 120, ?
(A) 180 (B) 224 (C) 196 (D) 210
- Q.28** 57, 54, 58, 55, 59, 56, 60, ?
(A) 64 (B) 63 (C) 58 (D) 57
- Q.29** 27, 31, 40, 56, 81, 117, ?
(A) 156 (B) 165 (C) 166 (D) 169
- Q.30** 55, 168, 57, 120, 60, 80, 62, 48, 65, 24, ?, ?
(A) 69, 11 (B) 67, 8 (C) 8, 71 (D) 6, 72

- Q.31** 8, 7, 16, 5, 32, 3, 64, 1, 128, ?
(A) 18 (B) 13 (C) -1 (D) 3
- Q.32** 16, 33, 65, 131, ?, 523
(A) 261 (B) 521 (C) 613 (D) 721
- Q.33** 5, 2, 17, 4, ?, 6, 47, 8, 65
(A) 29 (B) 30 (C) 31 (D) 32
- Q.34** 1, 2, 4, 8, ?, 32
(A) 10 (B) 12 (C) 14 (D) 16
- Q.35** 2, 3, 10, 15, 26, ?
(A) 36 (B) 35 (C) 39 (D) 48
- Q.36** 2, 30, 6, 20, 12, 12, ?
(A) 26 (B) 22 (C) 20 (D) 24
- Q.37** 6, 20, 36, 48, 50, ?, 0
(A) 36 (B) 40 (C) 46 (D) 56
- Q.38** 7, 15, 28, 59, 114, ?
(A) 243 (B) 233 (C) 213 (D) 223
- Q.39** 25, 49, 89, 145, 217, ?
(A) 305 (B) 327 (C) 309 (D) 303
- Q.40** 0, 2, 2, 3, 3, 5, 8, 4, 10, ?, 5, 17
(A) 6 (B) 7 (C) 9 (D) 15
- Q.41** 0, 2, 24, 252, ?
(A) 620 (B) 1040 (C) 3120 (D) 5430
- Q.42** 6, 24, 60, 120, ?
(A) 180 (B) 210
(C) 240 (D) 360
- Q.43** 2, 10, 26, ?, 242
(A) 80 (B) 81
(C) 82 (D) 84
- Q.44** 5, 10, 17, 26, 37, 50, ?
(A) 70 (B) 66 (C) 65 (D) 64
- Q.45** 6, 25, 62, 123, ?, 341
(A) 216 (B) 214 (C) 215 (D) 217
- Q.46** 5, 3, 10, 8, 17, 15, ?, 24
(A) 26 (B) 27 (C) 29 (D) 36

- Q.47** 2, 6, 12, 20, 30 ?
(A) 40 (B) 42 (C) 44 (D) 46
- Q.48** 12, 22, 69, 272, 1365, ?
(A) 8196 (B) 8184 (C) 8195 (D) 6830
- Q.49** 65, 48, 64, 49, 63, ?
(A) 53 (B) 52 (C) 51 (D) 50
- Q.50** 7, 23, ?, 79, 119.
(A) 47 (B) 49 (C) 44 (D) 46
- Q.51** 16, 8, 12, ?, 105.
(A) 6 (B) 30 (C) 24 (D) 35
- Q.52** 748, 737, 716, 685, 644, ?
(A) 634 (B) 643 (C) 503 (D) 593

Directions: (Q.53 to Q.56)

In each of the question **53** to **56** some of the numbers are missing in the given series with one term missing shown by question mark (?). This terms is one of the alternatives among the four numbers given under it. Find the right alternative.

- Q.53** 8, 27, 64, ?, 216, 343
(A) 125 (B) 81 (C) 100 (D) 196
- Q.54** 5, 11, 19, ?, 41
(A) 28 (B) 29 (C) 30 (D) 35
- Q.55** 120, ?, 24, 6, 0
(A) 100 (B) 70 (C) 60 (D) 20
- Q.56** 729, 81, 9, 1, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{729}$
(A) $\frac{1}{27}$ (B) $\frac{1}{81}$ (C) $\frac{1}{243}$ (D) $\frac{1}{486}$

Directions: (Q.57 to Q.79) Find the wrong term

- Q.57** 3, 7, 9, 21, 27, 66, 81, 189, 243
(A) 27 (B) 66 (C) 243 (D) 21
- Q.58** 27, 34, 40, 45, 49, 53, 54, 55
(A) 53 (B) 45 (C) 56 (D) 34
- Q.59** 0, 2, 3, 6, 6, 20, 9, 54, 12
(A) 3 (B) 6 (C) 20 (D) 54

- Q.60** 0, 2, 10, 36, 68, 130
(A) 10 (B) 36 (C) 68 (D) 130
- Q.61** 9, 54, 44, 264, 254, 1520, 1514
(A) 1514 (B) 1520 (C) 264 (D) 44
- Q.62** 10, 15, 26, 35, 48, 63, 82
(A) 48 (B) 26 (C) 63 (D) 82
- Q.63** 3, 10, 30, 66, 127, 218
(A) 3 (B) 66 (C) 30 (D) 218
- Q.64** 7, 9, 17, 42, 91, 172, 293
(A) 91 (B) 42 (C) 17 (D) 9
- Q.65** 2, 12, 24, 34, 68, 78, 158, 166
(A) 68 (B) 78 (C) 158 (D) 166
- Q.66** 2, 6, 10, 20, 30, 42, 56
(A) 6 (B) 10 (C) 20 (D) 30
- Q.67** 7, 9, 16, 25, 41, 68, 107, 173
(A) 16 (B) 41 (C) 68 (D) 107
- Q.68** 3, 9, 27, 82, 243
(A) 27 (B) 54 (C) 82 (D) 162
- Q.69** 5, 9, 17, 35, 65, 129
(A) 65 (B) 35 (C) 17 (D) 9
- Q.70** 1, 5, 6, 11, 17, 27, 45, 73
(A) 27 (B) 45 (C) 17 (D) 11
- Q.71** 3, 6, 11, 18, 28, 38, 51, 66
(A) 18 (B) 28 (C) 38 (D) 51
- Q.72** 320, 254, 200, 155, 122, 100, 89
(A) 155 (B) 320 (C) 254 (D) 200
- Q.73** 6, 8, 9, 12, 14, 18, 22, 26, 30
(A) 12 (B) 22 (C) 26 (D) 30
- Q.74** 3, 7, 9, 28, 27, 84, 81, 448, 243
(A) 84 (B) 81 (C) 28 (D) 7
- Q.75** 190, 94, 46, 22, 10, 3
(A) 94 (B) 46 (C) 22 (D) 3
- Q.76** 0, 5, 15, 50, 128
(A) 5 (B) 15 (C) 50 (D) 128
- Q.77** 9, 63, 5, 35, 1, 8
(A) 63 (B) 5 (C) 35 (D) 8

- Q.78** 89, 78, 86, 80, 85, 82, 83
(A) 83 (B) 82 (C) 86 (D) 78
- Q.79** 1, 1, 3, 9, 6, 36, 10, 100, 16, 225
(A) 225 (B) 16 (C) 10 (D) 9
- Q.80** 444, 300, 200, 136, 87, 84, 80
(A) 300 (B) 200 (C) 136 (D) 87
- Q.81** 8, 15, 31, 61, 123, 247, 491
(A) 247 (B) 491 (C) 121 (D) 61
- Q.82** 3, 6, 24, 30, 63, 72, 122, 132
(A) 132 (B) 30 (C) 122 (D) 72
- Q.83** 121, 144, 169, ?, 225, 256.
(A) 196 (B) 296 (C) 220 (D) 222
- Q.84** 5, 10, 20, ?, 80.
(A) 35 (B) 40 (C) 45 (D) 50
- Q.85** 4, 8, 9, 27, 16, ?, 25, 125.
(A) 8 (B) 16 (C) 25 (D) 64
- Q.86** 2, 3, 5, 8, ?, 17.
(A) 6 (B) 12 (C) 13 (D) 15
- Q.87** 4, 9, 25, ?, 121, 169.
(A) 36 (B) 49 (C) 64 (D) 81
- Q.88** 1, 3, 7, 13, 21, ?, 43, 57.
(A) 31 (B) 29 (C) 30 (D) 32
- Q.89** 5, 3, 10, 8, 17, 15, ?, 24.
(A) 25 (B) 23 (C) 26 (D) 27
- Q.90** 97, 77, 59, ?, 29, 17.
(A) 34 (B) 39 (C) 37 (D) 43

(Questions 91 – 98)

Instruction : In each of the question Nos. **91** to **98** a number series is given with one term missing shown by question mark (?). This term is one of the four alternatives given under it. Find the correct alternative.

- Q.91** 1, 2, 3, 15, ?, 56
(A) 31 (B) 40 (C) 37 (D) 45
- Q.92** 100, 50, $33\frac{1}{3}$, 25, 20, ?,
(A) 15 (B) $16\frac{1}{3}$ (C) $17\frac{2}{3}$ (D) $16\frac{2}{3}$

Q.93 17, 16, 8, $\frac{?}{-}$, -83
(A) -1 (B) -8 (C) -19 (D) -26

Q.94 6, 24, 60, 120, $\frac{?}{-}$.
(A) 180 (B) 195 (C) 210 (D) 225

Q.95 49, 64, 56, 57, 63, $\frac{?}{-}$, 70, 43
(A) 64 (B) 50 (C) 52 (D) 166

Q.96 4, 13, 31, 67, $\frac{?}{-}$, 283
(A) 139 (B) 103 (C) 121 (D) 139

Q.97 1, 1, 2, 3, 5, 8, $\frac{?}{-}$, 21
(A) 11 (B) 12 (C) 13 (D) 14

Q.98 3, 24, 81, $\frac{?}{-}$, 375, 648.
(A) 128 (B) 256 (C) 169 (D) 192

Direction (Q.99 to Q.103) Find the missing term in the series given below.

Q.99 2, 12, 30, ?, 90, 120
(A) 48 (B) 56 (C) 63 (D) 72

Q.100 10, 100, 200, 310, ?
(A) 400 (B) 410 (C) 420 (D) 430

Q.101 0, 5, 2, 4.5, 8, 12.5, ?
(A) 16 (B) 17 (C) 16.5 (D) 18

Q.102 109, 74, 46, 25, 11, ?
(A) 3 (B) 0 (C) 11 (D) 4

Q.103 $\frac{2}{3}, \frac{4}{7}, \frac{?}{?}, \frac{11}{21}, \frac{16}{31}$
(A) $\frac{6}{11}$ (B) $\frac{5}{9}$ (C) $\frac{9}{11}$ (D) $\frac{7}{13}$

Direction (Q. NO. 104 to 107) : In each of the following questions write which term in the sequence replaces the question mark.

Q.104 13, 23, 43, 83, 163, ?
(A) 326 (B) 323 (C) 321 (D) 318

Q.105 12, 15, 21, 24, 30, 33, ?, ?
(A) 36, 41 (B) 37, 42
(C) 38, 47 (D) 39, 51

Q.106 16, 40, 100, 250, ?
(A) 575 (B) 625 (C) 425 (D) 525

Q.107 23, 29, 47, 75, ?
(A) 87 (B) 93 (C) 110 (D) 117

Instruction: In each of the following question no. 108 to 117, a number series is given with missing of one term the correct alternative that will continue the same pattern and replace the question mark (?) in the givens

Q.108 2, 3, 5, 9, 17
(A) 34 (B) 31 (C) 32 (D) 33

Q.109 2, 7, 12, 17, ?, 27
(A) 18 (B) 22 (C) 19 (D) 23

Q.110 11, 121, 1331, ?
(A) 14641 (B) 14411
(C) 14141 (D) 14441

Q.111 656, 432, 320, 264, 236, ?
(A) 229 (B) 232 (C) 222 (D) 223

Q.112 3, 19, 97, 391, ?, 2359
(A) 1177 (B) 1084 (C) 1711 (D) 1958

Q.113 2, 7, 24, 77, ?
(A) 1335 (B) 249 (C) 283 (D) 238

Q.114 11, 5, 13, 10, 15, 15, 17, ?, ?
(A) 5, 11 (B) 20, 19
(C) 19, 21 (D) 19, 20

Q.115 1331, 2197, 4913, 6859, ?, 24389
(A) 13824 (B) 9261
(C) 12167 (D) 15625

Q.116 97, 86, 99, 88, 101, ?, ?
(A) 90, 103 (B) 88, 99
(C) 121, 108 (D) 114, 103

Q.117 77, 49, 36, 18, ?
(A) 10 (B) 12 (C) 8 (D) 16

ANSWER KEY

EXERCISE - 1

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	A	C	A	D	D	A	D	B	D	D	C	A	D	D	D	B	D	A	B
Que.	21	22	23	24	25															
Ans.	B	C	D	D	C															

EXERCISE - 2

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	B	B	C	A	C	A	A	C	A	C	C	C	C	B	A	C	D	B	B	C
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	A	D	B	C	C	B	D	D	C	B	C	A	C	D	B	C	C	B	A	D
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	C	B	C	C	B	A	B	B	D	A	B	D	A	B	C	B	B	A	C	B
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	B	A	C	D	C	B	C	C	B	A	B	D	B	A	D	D	D	C	B	D
Que.	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
Ans.	A	C	A	B	D	B	B	A	C	D	A	D	C	C	B	A	C	D	B	D
Que.	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117			
Ans.	D	D	D	B	D	B	C	D	B	A	C	A	D	B	C	A	C			

SOLUTIONS

EXERCISE-1

Sol.1 [C]

$$\begin{array}{ccccccc} 9, & 7, & 14, & 11, & 33, & 28, & 140, & 133, & 931 \\ \underbrace{\quad} & \underbrace{\quad} & \underbrace{\quad} & \underbrace{\quad} & & & & & \\ 2 & 3 & 5 & 7 & & & & & \end{array}$$

difference is prime no.

Sol.2 [A]

$$\begin{aligned} 6 \times 2 + 0 &= 12 \\ 12 \times 2 - 1 &= 23 \\ 23 \times 2 - 2 &= 44 \\ 85 \times 2 - 4 &= 166 \end{aligned}$$

Sol.3 [A]

$$\begin{array}{ccccccc} 92, & 23, & 27, & 9, & 12, & 6, & 8 \\ \underbrace{\quad} & \underbrace{\quad} & \underbrace{\quad} & \underbrace{\quad} & \underbrace{\quad} & \underbrace{\quad} & \\ \div 4 & +4 & :4 & +3 & \div 3 & +3 & \end{array}$$

Sol.4 [A]

$$\begin{aligned} 1^2 - 1 &= 0 \\ 2^2 + 1 &= 5 \\ 3^2 - 1 &= 8 \\ 4^2 + 1 &= 17 \\ 8^2 + 1 &= 65 \end{aligned}$$

Sol.5 [D]

$$\begin{aligned} 999 &= 10^3 - 1 \\ 730 &= 9^3 + 1 \\ 510 &= 8^3 - 2 \\ 345 &= 7^3 + 2 \\ 213 &= 6^3 - 3 \\ 128 &= 5^3 + 3 \end{aligned}$$

Sol.6 [A]

$$\begin{aligned} (2 + 5) \times 2 &= 14 \\ (7 + 13) \times 2 &= 2 \\ (10 + 4) \times 2 &= 28 \end{aligned}$$

Sol.7 [A]

$$\begin{aligned} 2^2 - 1 &= 3 \\ +3 \curvearrowright 3^2 - 2 &= 7 \\ \curvearrowright 6^2 - 1 &= 35 \\ +3 \curvearrowright 7^2 - 2 &= 49 \\ \curvearrowright 10^2 - 1 &= 99 \end{aligned}$$

Sol.8 [D]

$$\begin{array}{ccccccc} 23, & 27, & 43, & 79, & 143, & \dots \\ \underbrace{\quad} & \underbrace{\quad} & \underbrace{\quad} & \underbrace{\quad} & \underbrace{\quad} & \\ 2^2 & 4^2 & 6^2 & 8^2 & 10^2 & \end{array}$$

Sol.9 [B]

$$\begin{aligned} 870 &= 29^2 + 29 \\ 342 &= 18^2 + 18 \\ 132 &= 11^2 + 11 \\ 42 &= 6^2 + 3 \\ 2^0 &= 1^2 + 1 \end{aligned}$$

Sol.10 [D]

$$\begin{aligned} 9 \times 2 + 1 &= 18, & 18 \times 2 - 2 - 3 &= 35 \\ 35 \times 2 + 5 &= 75, & 75 \times 2 - 7 &= 143 \\ 143 \times 2 + 9 &= 295 \end{aligned}$$

Sol.11 [B]

$$\begin{array}{ccccccc} 2, & 20, & 110, & 380, & 650 \\ \underbrace{\quad} & \underbrace{\quad} & \underbrace{\quad} & \underbrace{\quad} & \\ +18 & +90 & +270 & +270 & \\ \times 5 & \times 3 & \times 1 & & \end{array}$$

Sol.12 [C]

$$\begin{aligned} 256, 125, 60 &\text{ ---, } 12.5 \\ \frac{256-6}{2} &= 125 \\ \frac{256-5}{2} = 60 &\Rightarrow \frac{60-4}{2} = 28 \end{aligned}$$

Sol.13 [A]

$$\begin{aligned} 2 \times 3 &= 6 \\ 3 \times 5 &= 15 \\ 5 \times 7 &= 35 \\ 7 \times 11 &= 77 \\ 19 \times 23 &= 437 \end{aligned}$$

Sol.14 [D]

$$\begin{aligned} 9, 9, 14, 34, 124 &\text{ ---} \\ 9 \times 2 - 4 &= 14 \\ 14 \times 3 - 8 &= 34 \\ 34 \times 4 - 12 &= 124 \\ 124 \times 5 - 16 &= 604 \end{aligned}$$

Sol.15 [D]

$$\begin{array}{ccccccc} 125, & 215, & 343, & 511, & 729, & 999 \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 5^3 & 6^3 - 1 & 7^3 & 8^3 - 1 & 9^3 & 10^3 - 1 \end{array}$$

Sol.16 [D]

Each term is 4 less than hence the preceding term.
50, 654 is wrong term, it must be $218 \times 3 - 4 = 650$

Sol.17 [B]

Each term is 1 more than twice the preceding term
50, 39 is wrong term

Sol.18 [D]

unit place is range term place is increased by
from first to tort term.
50, 5686 is wrong term

Sol.19 [A]

Logic is : $3^2 + 1, 4^2 - 1, 5^2 - 1, 6^2 - 1, 7^2 + 1$
so 48 is wrong term

Sol.20 [B]

2, 11, 25, 46, 109, 221, 445

 +7 +14 +28 +56 112 224
 x2 x2 x2 x2 x2

46 is wrong terms

Sol.21 [B]

2, 6, 10, 20, 30, 42, 55

 +4 +6 +8 +10 +12 +14
 10 is wrong term,

Sol.22 [C]

$3^1, 3^2, 3^3, 3^4, 3^5$
82 is wrong term,

Sol.23 [D]

320, 254, 200, 155, 122, 100, 89

 -66 -55 -44 -33 -22 -11

Sol.24 [D]

$1 + 5 = 6, 3 + 4 = 7, 7 + 1 = 8, 2 + 2 + 3 = 7$
 $3 + 5 + 0 = 8$
 pattern after adding digital of number 67878 it
 should be 87876 such type of pattern is called
 palling drone

Sol.25 [C]

5, 10, 40, 80, 320, 550, 2560

 x2 x4 x2 x4 x2 x4
 so, 550 is wrong term

EXERCISE-2**Sol.1 [B]**

Pattern is $n^2 + 1, n^2 - 1$, where $n = 1, 2, 3, 4, 5 \dots$
 So next term is $6^2 - 1 = 35$

Sol.2 [B]

it is combination of two series : -
 1. 27, 125,
 and
 4, 16, 36,
 pattern is n^2, n^3
 So next term is $7^3 = 343$

Sol.3 [C]

Logic is $7^3 - 7, 6^3 - 6, 5^3 - 5, \dots$
 missing term $\Rightarrow 4^3 - 4 = 60$

Sol.4 [A]

3, 4, 8, 17, 33,

 1 4 9 16
 difference is $1^2, 2^2, 3^2, 4^2$
 next term = $33 + 5^2 = 58$

Sol.5 [C]

Third term = first term + second terms
 missing term = $34 + 55 = 89$

Sol.6 [D]

480, 480, 240, 80, 20, 4

 ÷1 ÷2 ÷3 ÷4 ÷5

Sol.7 [A]

1, 1, 2, 2, 3, 4, 4, 8, 5, 16, 6

 +1 +1 +1 +1 +1

Sol.8 [C]

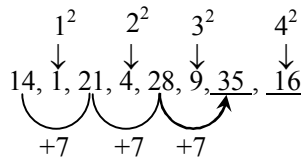
2, 5, 11, 23, 47,
 $2 \times 8 + 1 = 5$
 $5 \times 2 + 1 = 11$
 so next term $\Rightarrow 47 \times 2 + 1 = 95$

Sol.9 [A]

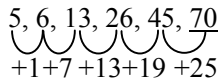
12, 21, 23, 32, 34, 43, 45, 54

 +11 +11 +11
 +11 +11 +11

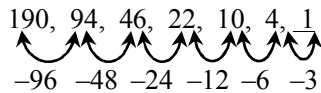
Sol.10 [C]



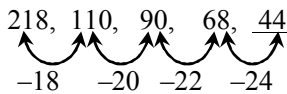
Sol.11 [C]



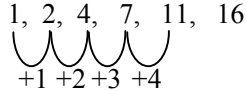
Sol.12 [C]



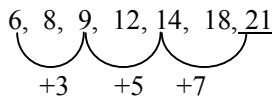
Sol.13 [C]



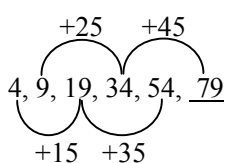
Sol.14 [B]



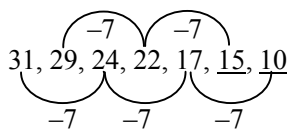
Sol.15 [A]



Sol.16 [C]



Sol.17 [D]



Sol.18 [B]

3, 6, 11, 18 —
logic is $n^2 + 2$, $n = 1, 2, 3, \dots$
missing term = $5^2 + 2 = 27$

Sol.19 [B]

Logic is $n^2 - 1$, $n = 2, 3, 4, 5, \dots$
missing term $\Rightarrow 6^2 - 1 = 35$

Sol.20 [C]

$4 \times 2 + 2 = 10$
 $10 \times 2 + 3 = 23$
 $23 \times 2 + 4 = 50$
 $50 \times 2 + 5 = 105$
 $105 \times 2 + 6 = 216$

Sol.21 [A]

Logic is $\div 3 - 1, \div 3 + 1, \dots$
 $33 \div 3 + 1 = 12$

Sol.22 [D]

1, 4, 9 —, 25, 36
all are perfect squares
 $4^2 = 16$

Sol.23 [B]

7, 12, 22, 37 —, 82, 112
+5 +10 +15 +20
 $37 + 20 = 57$

Sol.24 [C]

11, 13, 17, 19, —, 25
+2 +4 +2 +4
 $\Rightarrow 19 + 4 = 23$

Sol.25 [C]

5, 9, 17, 33, —, 129
+4 +8 +16 +32
 $\Rightarrow 33 + 32 = 65$

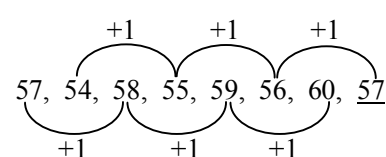
Sol.26 [B]

First series = 2, 4, 7, 11, 16
+2 +3 +4 +5
second series = 2, 10, 15, 20, 25
+2 +3 +4 +5

Sol.27 [D]

Logic is: $n^3 - n$, $n = 1, 2, 3, 4, \dots$
missing term $6^3 - 6 = 210$

Sol.28 [D]



Sol.29 [C]

$$\begin{aligned}27 + 2^2 &= 31 \\31 + 3^2 &= 40 \\40 + 4^2 &= 56 \\56 + 5^2 &= 81 \\81 + 6^2 &= 117 \\117 + 7^2 &= 166\end{aligned}$$

Sol.30 [B]

$$\begin{aligned}\text{I: } & 55, 57, 60, 62, 65, \underline{67} \\& \quad +2 \quad +3 \quad +2 \quad +3 \quad +2 \\ \text{II: } & 168, 120, 80, 48, 24, \underline{8} \\& \quad -48 \quad -40 \quad -32 \quad -24 \quad -16\end{aligned}$$

Sol.31 [C]

$$\begin{aligned}& 8, 7, 16, 5, 32, 3, 64, 1, 128, \underline{1} \\& \quad \times 2 \quad \times 2 \quad \times 2 \quad \times 2\end{aligned}$$

Sol.32 [A]

$$\begin{aligned}\text{Logic is } & \times 2 + 1, \times 2 - 1, \dots \\ \Rightarrow & 131 \times 2 - 1 = 261\end{aligned}$$

Sol.33 [C]

$$\begin{aligned}& 5, 2, 17, 4, \text{---}, 6, 47, 8, 65 \\& \quad +12 \quad +14 \quad +16 \quad +18 \\ \Rightarrow & 17 + 14 = 31\end{aligned}$$

Sol.34 [D]

$$\begin{aligned}& 1, 2, 4, 8, \text{---}, 32 \\& \quad \times 2 \times 2 \times 2 \times 2 \times 2 \\ \Rightarrow & 8 \times 2 = 16\end{aligned}$$

Sol.35 [B]

$$\begin{aligned}& 2, 3, 10, 15, 26, \text{---} \\& \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \\& 1^2+1 \quad 2^2-1 \quad 3^2+1 \quad 4^2-1 \quad 5^2+1 \quad 6^2-1 \\ \Rightarrow & 36 - 1 = 35\end{aligned}$$

Sol.36 [C]

$$\begin{aligned}\text{It is combination of two series :} \\ \text{I. } & 2, 6, 12, \text{---} \\ \text{II. } & 30, 20, 12, \\ \text{pattern in I is } & 1^2 + 1, 2^2 + 2, 3^2 + 3, \dots \\ \text{missing term} & = 4^2 + 4 = 20\end{aligned}$$

Sol.37 [A]

$$\begin{aligned}1 \times 1 \times 6 &= 6 \\2 \times 2 \times 5 &= 20 \\3 \times 3 \times 4 &= 36 \\4 \times 4 \times 3 &= 48 \\5 \times 5 \times 2 &= 50 \\6 \times 6 \times 1 &= 36 \\7 \times 7 \times 0 &= 0\end{aligned}$$

Sol.38 [B]

$$\begin{aligned}\text{Logic is } & \times 2 + 1, \times 2 - 2, \times 2 + 3, \\ \text{missing term} & \Rightarrow 114 \times 2 + 5 = 233\end{aligned}$$

Sol.39 [A]

$$\begin{aligned}& 25, 49, 89, 145, 217, \text{---} \\& \quad +24 \quad +40 \quad +56 \quad +72 \quad +88 \\& \quad \quad +16 \quad +16 \quad +6/6 \quad +16 \\ & 217 + 88 = 305\end{aligned}$$

Sol.40 [D]

It is combination of three series :

$$\begin{aligned}& \quad +3 \quad +5 \quad +7 \\ \text{I: } & 0, 3, 8, \underline{2} \\ \text{II: } & 2, 3, 4, 5 \\ \text{III: } & 2, 5, 10, 17 \\ \text{missing term} & : 8 + 7 = 15\end{aligned}$$

Sol.41 [C]

$$\begin{aligned}\text{Logic is: } & n^2 - n, n = 1, 2, 3, 4, \\ & 1^1 - 1 = 0, \\ & 2^2 - 2 = 2 \\ & 5^5 - 5 = 3125 - 5 = 3120\end{aligned}$$

Sol.42 [B]

$$\begin{aligned}\text{Logic is } & n^3 - n, n = 2, 3, 4, 5 \\ \text{so missing term} & = 6^3 - 6 = 210\end{aligned}$$

Sol.43 [C]

$$\begin{aligned}\text{Logic is: } & 3^n - 1, 3^n + 1, 3^n - 1, 3^n + 1, \dots \\ \text{so missing term} & \Rightarrow 3^4 + 1 = 82\end{aligned}$$

Sol.44 [C]

$$\begin{aligned}& 5, 10, 17, 26, 37, 50, \underline{65} \\& \quad +5 \quad +7 \quad +9 \quad +11 \quad +13 \quad +15\end{aligned}$$

Sol.45 [B]

$$\begin{aligned}\text{Logic is } & n^3 - 2, n = 2, 3, 4, 5, \dots \\ \text{missing term} & \Rightarrow 6^3 - 2 = 214\end{aligned}$$

Sol.46 [A]

5, 3, 10, 8, 17, 15, —, 24
+5 +7 +9

missing term $\Rightarrow 17 + 9 = 26$

Sol.47 [B]

Logic is $n^2 - n$, $n = 2, 3, 4, 5, \dots$

missing term $\Rightarrow 7^2 - 7 = 42$

Sol.48 [B]

12, 22, 69, 272, 1365, 8184
 $\times 2-2 \times 3+3 \times 4-4 \times 5+5 \times 6-6$

Sol.49 [D]

I: 65, 64, 63

II: 48, 49, 50
+1 +1

Sol.50 [A]

Logic is $n^2 - 2$, $n = 3, 5, 7, 9, \dots$

missing term $= 7^2 - 2 = 47$

Sol.51 [B]

16, 8, 12, —, 105
 $\times 0.5 \times 1.5 \times 2.5 \times 3.5$

missing number $\Rightarrow 12 \times 2.5 = 30$

Sol.52 [D]

-1 -1 -1 -1 -1
748, 737, 716, 685, 644, 593
-1 -2 -3 -4 -5

Sol.53 [A]

Logic is n^3 , $n = 2, 3, 4, 5, \dots$

missing number $5^3 = 125$

Sol.54 [B]

5, 11, 19, 29, 41
 $\uparrow \uparrow \uparrow \uparrow \uparrow$
 $2^2+1 \ 3^2+2 \ 4^2+3 \ 5^2+4 \ 6^2+5$

Sol.55 [C]

120, 60, 24, 6, 0
 $\uparrow \uparrow \uparrow \uparrow \uparrow$
 $5^3-5 \ 4^3-4 \ 3^3-3 \ 2^3-2 \ 1^3-1$

Sol.56 [B]

Logic is divide by 9

missing term $\frac{1}{9} \div 9 = \frac{1}{81}$

Sol.57 [B]

It concept of two series

I: 3, 9, 27, 81, 243
 $\times 3 \times 3 \times 3 \times 3$

II: 7, 21, 66, 189,
 $\times 3 \times 3$

$21 \times 3 = 63$

wrong term = 66

Sol.58 [A]

Logic is +7, +6, +5, +4, +1

53 is wrong term

Sol.59 [C]

I: 0, 3, 6, 9, 12
 $+3 +3 +3 +3$

II: 2, 6, 20, 54
 $\times 3 \times 3 \times 3$

So, 20 is wrong term

Sol.60 [B]

logic is $0^3 + 0, 1^3 = 1, 2^3 + 2^3 + 2, 3^3 + 3, \dots$

wrong term is 36

Sol.61 [B]

9, 54, 44, 264, 754, 1520, 1514
 $\times 6 -10 \times 6 -10 \times 6$

$\Rightarrow 254 \times 6 = 1524$

wrong term is 1520

Sol.62 [A]

Sequence is :

$3^2+1, 4^2-1, 5^2+1, 6^2-1, 7^2+1, 8^2-1, \dots$

so, 48 is wrong term and replaced by

Sol.63 [C]

The sequence is :

$1^3+2, 2^3+2, 3^3+2, 4^3+2, \dots$

so, 30 is wrong term

Sol.64 [D]

Difference between terms are : -

$+1^2, +3^2, +5^2, +7^2, +9^2$

so, 9 is wrong term

Sol.65 [C]

Sequence in the given relies terms
 $+10, \times 2, +10, \times 2, +10, \times 2, \dots$
so, 158 is wrong term

Sol.66 [B]

2, 6, 10, 20, 30, 42, 56
 $+4 +6 +8 +10 +12 +14$
10 is wrong term

Sol.67 [C]

$7 + 9 = 16, 9 + 16 = 25, 16 + 25 = 41$
 $25 + 41 = 66$
so 68 is wrong term

Sol.68 [C]

$3^1, 3^2, 3^3, 3^4, 3^5$
82 is wrong term

Sol.69 [B]

$5 \times 2 - 1 = 9$
 $9 \times 2 - 1 = 17$
 $17 \times 2 - 1 = 33$
so, 35 is wrong term

Sol.70 [A]

$1 + 5 = 6, 6 + 5 = 11, 11 + 6 = 17$
 $17 + 11 = 28$
so, 27 is wrong term

Sol.71 [B]

3, 6, 11, 18, 28, 38, 51, 61
 $+3 +5 +7 +9$
28, is wrong term

Sol.72 [D]

320, 254, 200, 155, 122, 100, 89
 $+66 +55 +44 +33 +22 +11$
200 is wrong term

Sol.73 [D]

I: 6, 9, 14, 22, 30
 $+3 +5 +7 +9$
II: 8, 12, 18, 26
 $+4 +6 +8$
22 is wrong term

Sol.74 [A]

I: 3, 9, 27, 81, 243
II: 7, 28, 84, 448
 $+4 +4 +4$
 $28 \times 4 = 112$
so, 84 is wrong term

Sol.75 [D]

3, 10, 22, 46, 94, 190
 $\times 2 +2 \times 2 +2 \times 2 +2 \times 2 +2 \times 2$

Sol.76 [D]

0, 5, 15, 50, 128
 $\times 1 +5 \times 2 +5 \times 3 +5 \times 4 +5$
128 is wrong term

Sol.77 [D]

9, 63, 5, 35, 1, 8
I: 9, 5, 1
63, 35, 8
 $\downarrow \downarrow \downarrow$
 $8^2 - 1 \quad 6^2 - 1 \quad 4^2 - 1$
8 is wrong term

Sol.78 [C]

I: 78, 80, 82
II: 89, 86, 85, 83
86 is wrong term

Sol.79 [B]

I: 1, 3, 6, 10, 16
 $+2 +3 +4 +5$

Sol.80 [D]

440, 300, 200, 136, 87, 84, 80
 $144 \quad 100 \quad 64 \quad 4$
 $\downarrow \downarrow \downarrow \downarrow$
 $12^2 \quad 10^2 \quad 8^2 \quad 2^2$

Sol.81 [A]

8, 15, 31, 61, 123, 247, 491
 $\times 2 - 1 \times 2 + 1 \times 2 - 1 \times 2 + 1 \times 2 - 1 \times 2 + 1$

Sol.82 [A]

$3 \times 1 = 3, \quad 3 \times 2 = 6$
 $6 \times 4 = 24, \quad 6 \times 5 = 30$
 $9 \times 7 = 63, \quad 9 \times 8 = 92$
 $12 \times 10 = 120, \quad 12 \times 11 = 132$

Sol.83 [A]
 $11^2, 12^2, 13^2, 14^2, 15^2, 16^2$
 missing term = 196

Sol.84 [B]
 $5, 10, 20, \text{---}, 80$
 $\times 2 \quad \times 2 \quad \times 2 \quad \times 2$
 $20 \times 2 = 40$

Sol.85 [D]
 $2^2, 2^3, 3^2, 3^3, 4^2, 4^3, 5^2, 5^3, \dots$
 $4^3 = 64$

Sol.86 [B]
 $2, 3, 5, 8, \text{---}, 17$
 $+1 \quad +2 \quad +3 \quad +4 \quad +5$

Sol.87 [B]
 Square of prime number
 $7^2 = 49$

Sol.88 [A]
 $1^2 - 0 = 1, 2^2 - 1 = 3, 3^2 - 2 = 7, 4^2 - 3 = 13, \dots$
 $5^2 - 4 = 21, 6^2 - 5 = 31$

Sol.89 [C]
 I: $5, 10, 17, \text{---}$
 $+5 \quad +7 \quad +9$
 II: $3, 8, 15, 24$
 $+5 \quad +7 \quad +9$
 $17 + 9 = 26$

Sol.90 [D]
 Logic is: $10^2 - 3, 9^2 - 4, 8^2 - 5, 7^2 - 6, \dots$
 missing term = $7^2 - 6 = 43$

Sol.91 [A]
 $1, 2, 6, 15, \text{---}, 56$
 $+1^2 \quad +2^2 \quad +3^2 \quad +3^2 \quad +5^2$
 $15 + 16 = 31$
 places do correction in question, in sequence
 replace 3 by 6

Sol.92 [D]
 $100, 50, 33\frac{1}{3}, 25, 20, \text{---}$
 $100 \div 1 = 100, 100 \div 2 = 50, 100 \div 3 = 33\frac{1}{3}$
 So, $100 \div 6 = 16\frac{2}{3}$

Sol.93 [C]
 $17, 16, 8, \text{---}, -83$
 $-1^3 \quad -2^3 \quad -3^3 \quad -4^3$
 so, $8 - 3^3 = -19$

Sol.94 [B]
 $n^3 - n, \quad n = 2, 3, 4, \dots$
 missing term $6^3 - 6 = 210$

Sol.95 [B]
 I: $49, 56, 63, 70$
 $+7 \quad +7 \quad +7$
 II: $64, 57, \text{---}, 43$
 $-7 \quad -7 \quad -7$
 $57 - 7 = 50$

Sol.96 [A]
 Logic is $\times 5 + 5$
 missing term is $67 \times 2 + 5 = 139$

Sol.97 [C]
 $1 + 1 = 2$
 $2 + 1 = 3$
 $3 + 5 = 8$
 $5 + 8 = 13$
 missing term = 13

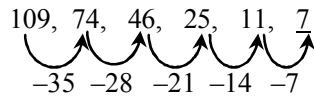
Sol.98 [D]
 $1^3 \times 3 = 3$
 $2^3 \times 3 = 24$
 $3^3 \times 3 = 81$
 $4^3 \times 3 = 192$

Sol.99 [B]
 $2 \times 1 = 2$
 $3 \times 4 = 12$
 $5 \times 6 = 30$
 $7 \times 8 = 56$
 $9 \times 10 = 90$
 $11 \times 12 = 132$

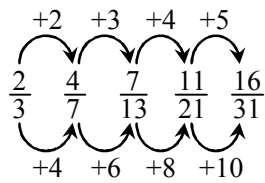
Sol.100 [D]
 $10, 100, 200, 310, \text{---}$
 $+90 \quad +100 \quad +110 \quad +120$
 $\Rightarrow 310 + 120 = 430$

Sol.101 [D]
 $0, 0.5, 2, 4.5, 8, 12.5, \text{---}$
 $+0.5 \quad +1.5 \quad +3.5 \quad +4.5 \quad +5.5$
 $12.5 + 5.5 = 18$

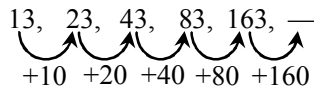
Sol.102 [D]



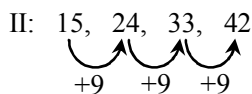
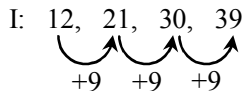
Sol.103 [D]



Sol.104 [C]



Sol.105 [D]



39, 42
do correction in option

Sol.106 [B]

$$\frac{16 \times 5}{2} = 40$$

$$\frac{40 \times 5}{2} = 100$$

$$\frac{100 \times 5}{2} = 250$$

$$\frac{250 \times 5}{2} = 625$$

Sol.107 [C]

$$23 + 2 \times 3 = 29$$

$$29 + 2 \times 9 = 47$$

$$47 + 4 \times 7 = 75$$

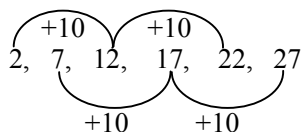
$$75 + 7 \times 5 = 110$$

Sol.108 [D]

$$\times 2^{-1} \text{ is logic}$$

$$17 \times 2 - 1 = 33$$

Sol.109 [B]



Sol.110 [A]

$$11^1, 11^2, 11^3, 11^4$$

$$11^4 = 14641$$

Sol.111 [C]

$$\text{Logic is next term} = \frac{\text{previous term}}{2} + 104$$

$$\frac{656}{2} + 104 = 432$$

$$\frac{432}{2} + 104 = 320$$

$$\frac{236}{2} + 104 = 232$$

Sol.112 [A]

$$3 \times 6 + 1 = 19$$

$$19 \times 5 + 2 = 97$$

$$97 \times 4 + 3 = 391$$

$$391 \times 3 + 4 = 1177$$

Sol.113 [D]

$$2 \times 3 = 1 = 7$$

$$7 \times 2 + 3 = 21$$

$$77 \times 3 + 238$$

Sol.114 [B]

$$\text{I: } 11, 13, 15, 17, 19$$

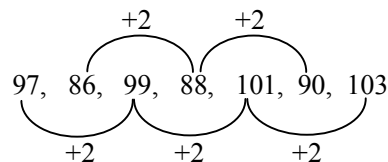
$$\text{II: } 5, 10, 15, 20$$

Sol.115 [C]

$$11^3, 13^3, 17^3, 19^3, 23^3, 29^3$$

$$23^3 = 12167$$

Sol.116 [A]



Sol.117 [C]

$$7 \times 7 = 49$$

$$4 \times 9 = 36$$

$$3 \times 6 = 18$$

$$1 \times 8 = 8$$

ENGLISH

HOW I TAUGHT MY GRANDMOTHER TO READ

(BY SUDHA MURTHY)

Summary

When the writer was about twelve, she lived in a village in north Karnataka with her grandparents. Those days, morning papers came in the afternoon and the weekly magazine came one day late. They would wait eagerly for the bus to arrive because all the magazines and newspapers used to come in that vehicle only. There was a writer named Triveni, who was known for her psychological and easy-to-read novels. She used to write on the daily struggles of the people at that time, which enabled the people to connect with her stories. One of her novels, *Kashi Yatre*, was about an old lady who gave up the money for her trip to Kashi to an orphan girl, so that the girl could marry the man she loved. The writer's grandmother too had a desire to go to Kashi but her desire was never fulfilled. So, she identified herself as the grandmother in that story. The writer's grandmother was illiterate, and so she asked the writer, who was a school-going girl, to read out the latest episode to her. During that time, she would forget all her work and listen with the greatest concentration. Later, she could repeat the entire text by heart. The grandmother was always eager to learn what would happen next. She would also discuss the episode with her friends. The writer was not mature enough to understand why her grandmother took so much interest in *Kashi Yatre*. One day, she went for a wedding with her cousins to the neighbouring village. There, she enjoyed herself thoroughly by enjoying the freedom given by the elders. Although she was supposed to stay there for just a few days, she spent a whole week there. When she came home, she saw that her grandmother, who was otherwise a brave woman, was in tears. The writer, who became worried, asked if everything was fine and if she was okay.

The grandmother nodded but did not speak. The writer did not understand and forgot about it. That night, when the writer was sleeping, her grandmother came and sat next to her. Her affectionate hands touched the writer's forehead and the writer felt that she wanted to talk to her. She asked her what was wrong. The grandmother told her the sad incidents of her childhood, how she had lost her mother, her father was a busy man and had remarried, and at those times, education was not considered essential for girls. She got married at a very young age and had children. After that, she became busy keeping her children happy, because their happiness made her happy. She used to regret not going to school and so she made sure that her children and grandchildren studied well so that they would not feel any regret in the future. Even though the writer could not understand what her grandmother was talking about, she could see the sadness in her face. The worried-looking expression on her grandmother's face made her sad. She leaned forward and held her hand. She asked her if she could help her in any way. The grandmother said "yes" and explained that while the writer was gone to the neighbouring village, she was desperately trying to read the next episodes of *Kashi Yatre*. She felt helpless and said that even though she was well-off she did not feel independent. She went on to request the writer to teach her the Kannada alphabet. The deadline for it would be Saraswati Pooja day during Dassara. The writer, who could see the determination on her grandmother's face, childishly made fun of her desire to learn the Kannada alphabet at an old age. But the grandmother just smiled and said that for learning, there was no age bar. The next day onwards, the writer started her tuition.

Avva, the grandmother, proved to be a wonderful student. Finally the Dassara festival came. Her grandmother called the writer to the pooja place and made her sit down on a stool. They were going to exchange gifts with each other. The grandmother gave her a frock and then touched the writer's feet. The writer was taken aback because elders do not touch the feet of youngsters. Only the feet of God, elders and teachers were to be touched as it was a sign of respect. The writer felt like what the grandmother was doing was not correct. The grandmother explained that she was touching the feet of her teacher, not her granddaughter. The writer had taught her affectionately and had made her independent in such a short period of time. A teacher should be respected regardless of gender and age. The writer returned the namaskar and gave her the gift. It was the complete novel of Kashi Yatre. Her grandmother read the title and the publisher's name immediately, and the writer knew that her student had passed with flying colours.

Literary Devices

Irony The grandmother, who was once dependent on her granddaughter for reading, finally becomes independent and even touches her granddaughter's feet as a teacher. The usual roles of elder and child are reversed.

Imagery Descriptions like the summer night on the terrace under the full moon and the grandmother rubbing her hands over the magazine pages help readers visualise the scenes clearly.

Foreshadowing The grandmother's deep interest in the serial and her regret about not going to school hint early that she will later decide to learn to read.

Symbolism The book Kashi Yatre symbolises knowledge, independence and the grandmother's dream of becoming literate.

Contrast The story contrasts dependence and independence, childhood and old age and ignorance and knowledge to highlight the grandmother's transformation.

Vocabulary

Psychological	Related to the human mind and feelings
Serial	A story published in parts regularly
Ardent	Having a very strong feeling or desire
Savouring	Enjoying something slowly and fully
Protagonist	The main character of a story
Immensely	Extremely
Obstacle	A difficulty that blocks progress
Recite	To repeat something aloud from memory
Scriptures	Holy religious writings
Taken aback	Surprised by something unexpected

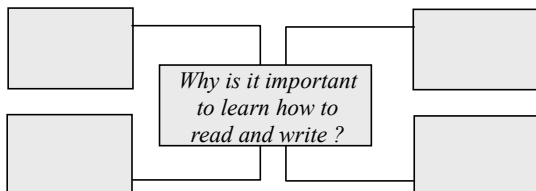


NCERT Folder

NCERT Textbook 'Kaveri' intext and Exercise Questions

➡ Reflect and Respond

I. Complete the given word web.



- Ans.
- To be independent and not depend on others.
 - To gain knowledge and education.
 - To communicate better and understand the world.
 - To build confidence and get better opportunities in life.

II. Read the questions given below and share your answers with your classmates and teacher.

1. Which language(s) do your grandparents or elderly relatives speak?
2. How do they spend their time? How do you spend time with them?
3. What is your favourite experience with them?
4. What is something that the elderly in your family cannot do easily but enjoy watching you do?

Ans. Do it yourself.

III. Read the following passage. Match the highlighted words with their meanings given in the box below.

- | | |
|---|---------------|
| 1. excitedly | 2. believable |
| 3. people living in one particular area | 4. discussion |
| 5. a part of a story | 6. focus |
| 7. main character | 8. directed |

The casting for the (i) **protagonist** of our school's annual play was done after a lot of (ii) **debate** as many good actors had auditioned for the role. We had decided to present an (iii) **episode** from an inspirational story. It was a life story of a group of children who worked with the (iv) **community** to spread literacy. Every day, we reached school early to practise with (v) **concentration**. We waited (vi) **eagerly** for the final presentation. All of us played our roles in a very (vii) **convincing** manner as our theatre teacher had (viii) **guided** us well.

- Ans.
- (i) protagonist → 7. main character
- (ii) debate → 4. discussion
- (iii) episode → 5. a part of a story
- (iv) community → 3. people living in one particular area
- (v) concentration → 6. focus
- (vi) eagerly → 1. excitedly
- (vii) convincing → 2. believable
- (viii) guided → 8. directed

☞ Check Your Understanding

- I. *Complete the cause and effect table given below based on Part I of the story. Share your answers with your classmates and teacher.*

Cause	Effect
1.	(i) The morning papers arrived late in the day and weekly magazines would come one day late.
2. The grandmother, Krishtakka, never went to school.	(ii)
3.	(iii) She was deeply interested in the story and discussed it with her friends.
4. The narrator went to a wedding for a week.	(iv)
5.	(v) The grandmother never went to school.
6. The grandmother regretted not going to school.	(vi)

Ans.

Cause	Effect
1. The transport system in the village was not very good.	(i) The morning papers arrived late in the day and weekly magazines would come one day late.
2. The grandmother, Krishtakka, never went to school.	(ii) She could not read or write and depended on others to read for her.
3. The narrator used to read Kashi Yatre to her grandmother every week	(iii) She was deeply interested in the story and discussed it with her friends.
4. The narrator went to a wedding for a week.	(iv) The grandmother could not read the next episode and felt helpless and sad.
5. In her childhood, girls' education was not considered important	(v) The grandmother never went to school.
6. The grandmother regretted not going to school.	(vi) She decided to learn reading and writing and become independent.

- II. *Do you think the narrator expected to see her grandmother in tears when she returned to the village? If yes, why? If no, why not?*

Ans. No, the narrator did not expect to see her grandmother in tears when she returned to the village. We know this because the narrator said that she was surprised when she saw her grandmother crying. She described her grandmother to be a resilient and strong person as she did not cry even in tough situations.

- III. *How might the narrator help her grandmother to fulfil her desire to learn to read and write?*

Ans. The narrator helped her grandmother by becoming her teacher. She taught her the Kannada alphabet daily, gave homework and encouraged her until she could read the novel independently by Dassara.

Check Your Understanding

I. *State whether the following sentences are true or false. Share your answers with your classmates and teacher.*

- (i) The grandmother wanted to learn the Kannada alphabet to gain independence.
- (ii) The grandmother asked someone in the village to read Kashi Yatre to her while the narrator was away.
- (iii) The narrator was the grandmother's first teacher and taught her how to read Kannada.
- (iv) The grandmother believed that there was no age limit for learning.
- (v) The grandmother touched the narrator's feet as a mark of respect for her as a teacher.
- (vi) The narrator was disappointed with the progress her grandmother made in learning to read.

Ans. (i) True
(ii) False
(iii) True
(iv) True
(v) True
(vi) False

Critical Reflection

I. *Read the extracts given below and answer the questions that follow.*

Extract 1

When I came back to my village, I saw my grandmother in tears. I was surprised, for I had never seen her cry even in the most difficult situations. What had happened? I was worried.

'Avva, is everything all right? Are you okay?'

I used to call her Avva, which means mother in the Kannada spoken in north Karnataka.

She nodded but did not reply. I did not understand and forgot about it. In the night, after dinner, we were sleeping in the open terrace of our house. It was a summer night and there was a full moon. Avva came and sat next to me. Her affectionate hands touched my forehead.

- (i) *Complete the following sentence with the appropriate option.*
The phrase 'never seen her cry in the most 'difficult situations' tells us that the grandmother was _____.

A. strong-willed	B. understanding
C. considerate	D. bold

Ans. A. strong-willed

- (ii) *Complete the following with the correct option from those given in the brackets.*
Grandmother did not reply when the narrator asked if she was alright because she might have been too _____(emotional/tired) to respond.

Ans. emotional

- (iii) *Identify the clue from the extract that indicates a rural setting with traditional customs.*
Ans. “I used to call her Avva, which means mother in the Kannada spoken in north Karnataka.” This line depicts that it is a custom in North Karnataka to call a mother ‘Avva’ as per the regional language there, which is Kannada.
- (iv) *Which lines of the extract establish a tender atmosphere?*
Ans. “It was a summer night and there was a full moon. Avva came and sat next to me. Her affectionate hands touched my forehead.”
- (v) *Which of the following aspect is NOT emphasised in the given extract?*
A. the emotional turmoil of the grandmother
B. the affectionate bond between the narrator and her grandmother
C. the grandmother’s regret over her lack of education
D. the narrator’s concern for her grandmother
Ans. C. the grandmother’s regret over her lack of education

Extract 2

‘I have decided I want to learn the Kannada alphabet from tomorrow onwards. I will work very hard. I will keep Saraswati Puja day during Dassara as the deadline. That day I should be able to read a novel on my own. I want to be independent.’

I saw the determination on her face. Yet I laughed at her.

‘Avva, at this age of sixty-two you want to learn the alphabet? All your hair is grey, your hands are wrinkled, you wear spectacles and you work so much in the kitchen...’

Childishly I made fun of the old lady. But she just smiled.

‘For a good cause if you are determined, you can overcome any obstacle. I will work harder than anybody but I will do it. For learning there is no age bar.’

- (i) *What does the grandmother’s statement, “I want to be independent,” reveal about her character?*
A. She wanted to be literate.
B. She desires self-sufficiency.
C. She wants to prove her intelligence to others.
D. She feels pressured by society to learn.
Ans. B. She desires self-sufficiency.
- (ii) *Fill in the blank with the appropriate option from those given in brackets.*
The grandmother’s determination shows that learning has no _____.
(age limit/gender bias/cultural barriers)
Ans. age limit
- (iii) *Complete the following with a suitable reason.*
The narrator laughs at her grandmother’s decision to learn the alphabet at the age of sixty-two because _____.
Ans. The narrator laughs at her grandmother’s decision to learn the alphabet at the age of sixty-two because she believed that her grandmother was too old to learn the Kannada alphabet.
- (iv) *List any two qualities displayed by the grandmother.*
Ans. The grandmother displayed determination and willingness to learn.

(v) *How can we say that the narrator is making assumptions about her grandmother?*

Ans. We can say that the narrator is making assumptions about her grandmother because she childishly laughs at her and believes that a person at the age of sixty-two cannot learn anything new.

II. Answer the following questions.

Q.1 *Why do you think the grandmother felt embarrassed to ask someone else to read to her while the narrator was away?*

Ans. The grandmother felt embarrassed to ask someone else to read to her because it made her feel weak and dependent on others. She felt helpless that even though her family had more than enough money to live a comfortable life, she was still dependent on others due to her illiteracy.

Q.2 *Why does the narrator initially laugh at her grandmother's determination to learn at the age of sixty-two?*

Ans. The narrator initially laughed at her grandmother's determination to learn at the age of sixty-two. She believed that old people cannot learn new things. She pointed out her grey hair, wrinkled hands, spectacles, and her work in the kitchen, stating she was too old and busy to learn the Kannada alphabet. This shows the prevalence of the misconception that learning stops when we become old. The narrator had never seen an old person learning something, so she childishly laughed at her grandmother's determination and made fun of her.

Q.3 *What significance does the story of Kashi Yatre have in both the grandmother's life and the story?*

Ans. Kashi Yatre is a novel, written by Triveni, which appeared as a serial in the Kannada weekly Karmaveera. It is the story of an old lady and her ardent desire to go to Kashi and worship Lord Vishweshwara and to perform the ultimate punya. However, this desire is met with financial struggles. There is a character of a young orphan girl who falls in love but has no money for the wedding. In the end, the old lady gives away all her savings to the girl without going to Kashi. The grandmother could relate to the plot and the old lady because she herself once wished to go to Kashi, but she couldn't. Therefore, she formed a strong emotional connection with the story and the protagonist. The story had a profound impact on the grandmother's life, and she eagerly waited for the next week of Karmaveera and went to the temple courtyard to discuss it with her friends. Due to the narrator's absence, she realised that she could not read on her own, invoking embarrassment and helplessness in her. In order to read the novel on her own, she determinedly worked hard to learn the Kannada alphabet. If she were not immersed in the story, she would not have felt the need to push herself and learn to read despite her age. The story made her realise that she needed to be literate to be independent, thereby becoming a motivating force for her personal independence.

Q.4 *What does the grandmother's desire to learn the Kannada alphabet reflect about her?*

Ans. The grandmother's desire to learn the Kannada alphabet reflects her determination and willingness to learn new things. We believe in the misconception that old people cannot learn. As we grow old, our brains become rigid and less open to new concepts. However, the grandmother does not let her age stop her from becoming independent, showing her willpower and resilience. The narrator describes how her grandmother did her homework diligently and read and wrote repeatedly to grasp the concept firmly. This shows that the grandmother is a hard-working person.

Q.5 *What lessons can we infer from the grandmother's action of touching the narrator's feet?*

Ans. When the Dassara festival arrived, it was time for the grandmother to try reading the novel, Kashi Yatre on her own. Before she did so, she bent down and touched the narrator's feet. At that moment, the grandmother saw the narrator not as her granddaughter but as her teacher who taught her the Kannada alphabet. The namaskar was a gesture of gratitude and respect to the narrator, who showed faith in her grandmother and helped her be literate and independent. The lessons that we can infer from the grandmother's action of touching the narrator's feet are that age is not a barrier to who can learn and who can teach and all teachers should be respected by their students regardless of their ages.

Q.6 *What does the following line tell us about the broader theme of the story?*

'For a good cause if you are determined, you can overcome any obstacle.'

Ans. 'For a good cause if you are determined, you can overcome any obstacle.' The story, "How I Taught My Grandmother to Read", is centred around the theme of lifelong learning. It shows that learning does not stop at a certain age. At an old age, our minds become less flexible and open to new concepts, making it hard for older people to learn things like a language. However, it is not completely impossible. In the story, we see how the grandmother worked hard and determinedly, reading and writing each alphabet in the Kannada language repeatedly and doing her homework diligently. She overcame the difficulties of old age and learnt the Kannada alphabet, giving her the ability to read the Kashi Yatre novel on her own. This aligns with the given line. The grandmother wished to be independent and literate, which is a good cause. Due to her diligent, determined, and resilient nature, she was able to overcome the obstacles of old age and learnt how to read the Kannada alphabet successfully.

Q.7 *How effectively does the story highlight the value of education in supporting personal independence?*

Ans. The story "How I Taught My Grandmother to Read" highlights the role of education in personal independence. The story shows the narrator's grandmother, who is attached to a story but cannot read it on her own. The grandmother had never received her education and she was married off at a young age. She was too busy with her children and grandchildren to think about her own needs and independence. Because of this, she is unable to read her favorite story on her own, making her feel embarrassed and helpless. This changes when the narrator teaches her the Kannada alphabet and she is able to read the novel at the day of Dassara. Therefore, education does not just give us knowledge and help us in our careers, but it also empowers us and fosters personal independence. It gives us the ability to sustain a good lifestyle without relying on someone else for the basic needs. Because of education, we will be able to read and write without any difficulties and external support, helping us to stand on our own feet.

Vocabulary and Structures in Context

The expression 'hide and seek' is used in the text. This is called a binomial.

Binomials are expressions consisting of two words, usually linked by a conjunction like 'and' or 'or', that are commonly used together in a fixed order. For example: odds and ends, right or wrong.

- I. **Match the binomials in Column 1 with their meanings in Column 2. You may refer to a dictionary.**

Column 1	Column 2
1. sink or swim	(i) put different things together to get a range of possibilities
2. on and off	(ii) complete part of or belong to
3. mix and match	(iii) choose only the best (things, people, etc.)
4. all or nothing	(iv) at sometime in the future
5. part and parcel	(v) sometimes, occasionally
6. pick and choose	(vi) increase or develop very quickly
7. sooner or later	(vii) succeed or fail without help
8. leaps and bounds	(viii) something to be done completely or not at all

Now, use any five of the above binomials in sentences of your own.

- Ans. 1-(vii), 2-(v), 3-(i), 4-(viii), 5-(ii), 6-(iii), 7-(iv), 8-(vi)
- You'll sink or swim based on your own efforts.
 - I've been exercising on and off for years.
 - You can mix and match colors to create your own outfit.
 - She has an all or nothing attitude toward her goals.
 - Rain is part and parcel of living in this coastal city.
 - You can pick and choose your favorite desserts from the menu.
 - He'll realize his mistake sooner or later.
 - Her English skills have improved by leaps and bounds.

- II. **Read the following words from the text given in the box below.**

unhappy	unusual	unfortunately
irrespective	independent	

These words are made by adding suitable prefixes ('un', 'ir', and 'in') to give an opposite or negative meaning to the words.

Now, make words by adding the suitable prefixes given in the box to the words from the text in Column 1. Write the prefixed words in Column 2. One example has been done for you.

un	im	dis
in	mis	extra

Column 1	Column 2
1. popular	(i) unpopular
2. belief	(ii)
3. important	(iii)
4. respect	(iv)
5. correct	(v)
6. continue	(vi)
7. understand	(vii)
8. ordinary	(viii)
9. interesting	(ix)
10. possible	(x)

- Ans.** (i) unpopular (ii) disbelief (iii) unimportant
 (iv) disrespect (v) incorrect (vi) discontinue
 (vii) misunderstand (viii) extraordinary (ix) uninteresting
 (x) impossible

III. Identify any five words with prefixes from the story and make sentences using each.

Ans. The given words with prefixes from the story are

1. **Unhappy:** The grandmother felt unhappy when she could not read the magazine.
2. **Independent:** She worked hard to become independent.
3. **Unusual:** It was unusual for an elder to touch a child's feet.
4. **Irrespective:** A teacher deserves respect irrespective of age.
5. **Unfortunately:** Unfortunately, she never went to school in her childhood.

IV. In the sentence, “I knew, then, that my student had passed with flying colours”, the phrase “passed with flying colours” is an idiom. It means outstanding performance. Now, match the idioms related to ‘learning’ given in Column 1 with their meanings in Column 2. You may refer to a dictionary.

Column 1	Column 2
1. to hit the books	(i) to memorise something
2. to draw a blank	(ii) to think very hard
3. to learn the ropes	(iii) to study seriously
4. to rack one's brain	(iv) to study or work late into the night
5. to learn by heart	(v) to be unable to remember
6. burn the midnight oil	(vi) to understand how to do an activity

Now, use these idioms in sentences of your own.

Ans. 1-(iii), 2-(v), 3-(vi), 4-(ii), 5-(i), 6-(iv)

1. **Hit the books** I need to hit the books because my exams are near.
2. **Draw a blank** I tried to remember the answer but drew a blank.
3. **Learn the ropes** It took me a week to learn the ropes at my new school.
4. **Rack my brain** I racked my brain to solve the difficult maths problem.
5. **Learn by heart** We had to learn the poem by heart for the test.
6. **Burn the midnight oil** She burnt the midnight oil to complete her project.

V. Read the following sentences from the text and underline the verbs.

1. *Secretly, I bought Kashi Yatre which had been published as a novel by that time.*
2. *I knew, then, that my student had passed with flying colours.*

In sentence 1, verb ‘bought’ is in simple past tense.

In sentence 2, verb ‘had passed’ is in past perfect tense.

We use past perfect tense to indicate two completed actions — one occurring earlier (later past) and the other more recently (recent past). The more recent action is expressed in the simple past, while the earlier action takes the past perfect tense. Examples

- By the time I reached the party, everyone had finished eating.
- When she returned home, her brother had finished the project.

Fill in the blanks with simple past and past perfect tense form of the verbs given in brackets.

- A. When the delegates _____ (arrive) at the conference, the keynote speaker _____ (already begin) the session.
- B. After the students _____ (learn) how to identify fake news online, they _____ (start) verifying information before sharing it.
- C. Before Kiran _____ (start) using digital payment platforms, she _____ (ensure) her understanding of online fraud prevention.
- D. By the time Varun _____ (recognise) the importance of budgeting, he _____ (exhaust) most of his savings.
- E. When Raghu _____ (log in) to the cybersecurity webinar, the instructor _____ (already discuss) the importance of strong passwords.

Ans. A. arrived, had already begun B. learned, started C. started, had ensured
D. recognized, had exhausted E. logged in, had already discussed

Fill in the blanks with the correct form of verbs given in brackets.

Last year, my parents and I A. _____ (take) a financial planning course. When we B. _____ (review) our expenses, we realised we C. _____ (spend) too much on unnecessary purchases. After my parents D. _____ (discuss) ways to save, I E. _____ (open) a savings account. By the time we F. _____ (set) our budget, the course G. _____ (already introduce) investment strategies. We H. _____ (hurry) to take notes, but many participants I. _____ (complete) their financial plans. Despite that, we J. _____ (enjoy) learning how to manage money wisely.

Ans. A. took B. reviewed C. had spent
D. had discussed E. opened F. set
G. had already introduced H. hurried I. had completed
J. enjoyed



Listen and Respond

- I. **You will listen to a speaker talk about digital literacy. As you listen, complete the following sentences with one to three exact words that you hear. (Transcript for teacher on page 259)**

TRANSCRIPT

Good morning, everyone. Today, I would like to discuss an essential skill in our daily lives—digital literacy. Earlier, literacy meant just reading and writing. Now, it includes the ability to use digital tools effectively and responsibly. Digital literacy means accessing, creating, and sharing information using technology. It's not just about using gadgets but also understanding digital platforms, identifying reliable sources, and protecting personal data.

So, why do you think digital literacy is important?

First, it empowers individuals—students learn to research critically, professionals handle digital transactions, and senior citizens stay connected. Second, it promotes safe internet use by helping us recognise misinformation, scams, and cyber threats. Third, it prepares us for a digital future, where skills in online services, job applications, and governance are essential. Being digitally literate is no longer optional—it's a necessity.

This is the reason the Digital Saksharta Abhiyan (DISHA) or National Digital Literacy Mission (NDLM) Scheme has been formulated to impart IT training to people across the country.

1. Literacy these days includes the ability to use _____.
2. Digital literacy is also about protecting _____.
3. Digital literacy enables the elderly to _____.
4. Digital literacy encourages using the internet safely by helping us recognise _____ and _____.
5. The National Digital Literacy Mission helps people across the country by imparting _____.

- Ans.**
1. digital tools effectively and responsibly
 2. personal data
 3. stay connected
 4. misinformation, scams, and cyber threats
 5. IT training



Speaking Activity

Turncoat is a type of solo debate where the speaker argues for and against a topic, switching sides after a certain period of time.

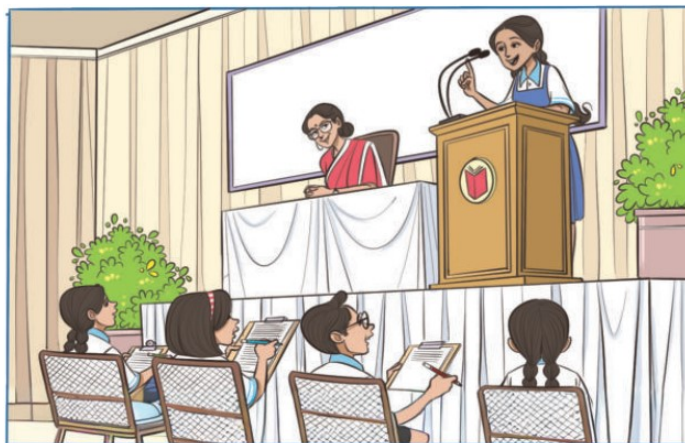
- I. Choose your topic and speak 'for' and 'against' for not more than one minute each.**

Topic 1: It is important to learn a new language apart from your mother tongue.

Topic 2: Learning can happen only when you are young.

Use the guidelines given below.

- Begin with speaking 'for' the topic for one minute.
- Your teacher will signal that it is time to switch sides.
- Then speak 'against' the topic for one minute.



You may use the following sentence prompts.

'For'	'Against'
• To begin with, I would like to speak 'for' the topic...	• On the contrary/On the other hand...
• My first argument in favour of...	• There are two sides to this topic...
• Most importantly I want to mention that...	• There's no doubt that...
• In addition to that.../ Moreover, I feel...	• If I could make a point here...
• I strongly feel...	• I am pretty sure that...
• I have a reason to believe...	• Well, I am not sure whether you can really...
• There's no doubt that...	• So, to put it in a few words...

Ans. Topic 1: It is important to learn a new language apart from your mother tongue.

Good day, everyone. To begin with this topic, I would like to speak 'for' the topic, "It is important to learn a new language apart from your mother tongue."

My first argument in favour of the topic is that learning a new language boosts career prospects. In India, knowing English alongside Hindi or regional languages opens doors to multinational jobs. A person wishing to work or live abroad will need to know the regional languages of the particular country before moving there. Studies show that multilingual people earn 20% more on average. Second, it fosters cultural understanding. Languages carry history, cultural facts, beauty, and values. This builds empathy and reduces cultural barriers. I strongly feel that we cannot establish equality all over the world if we cannot understand other cultures. I have a reason to believe that learning languages involves learning the meaning behind popular sayings, thereby helping understand the culture. Most importantly, I want to mention that it sharpens the brain. Learning new languages stimulates the rigid muscles in our brain and encourages problem-solving when we try to express ourselves properly in a foreign language.

I would like to conclude with the point that learning a new language equips us for success in both personal and professional spheres.

Now, I will be putting forward the discussion 'against' the topic.

There is no doubt that knowing multiple languages has significant value, especially in today's world of networking and globalisation. However, mandating a new one isn't essential and can harm progress.

Primarily, it overloads students. Students already juggle so many subjects. Adding foreign languages like French and German will create unnecessary pressure on them. CBSE data reveals high dropout rates in language classes due to poor results. Next, technology bridges gaps. Apps like Google Translate make communication instant. I am pretty sure that it would be more beneficial to spend time on STEM or local skills. Moreover, it risks cultural erosion. Prioritising English over Punjabi in Chandigarh weakens regional identity. Institutions should focus on the mother tongue language foremost.

So, to put in a few words, I would like to reference the saying "jack of all, master of none". If we aim at learning new languages but not strengthening our core in a few languages, we will not be able to articulate our comprehension, listening, reading, and speaking abilities in even one language.

Topic 2: Learning can happen only when you are young.

Good day. My first argument in favour of the topic is that learning happens best when you are young.

Young brains are fresh and ready. They can take in new things quickly. Children remember words, numbers, and skills easily. As people grow older, their brains get full and slow down. It becomes hard to learn new things. In addition to that, young people have more time. They do not have big jobs or family duties. They can study all day without worry. Older people are busy with work and home. They get tired and forget what they learned. Moreover, children love to learn. They are curious and ask many questions. This makes learning fun and fast. Older people sometimes lose this interest. They think learning is too much work. I strongly feel that habits start young. Good study habits form early. Older people find it tough to change old ways.

In short, young people learn faster, deeper, and longer.

Now, I will be putting forward the discussion ‘against’ the topic. I’m not really sure that we can limit learning and impose any sort of barrier.

I am pretty sure that grown-up brains can still learn. They do not stop working. Older people learn from life every day. They use what they know to understand new things better. There’s no doubt that adults have more focus. Young children play and get distracted. Older people pay full attention because they see why learning matters. They learn skills for jobs or family. Moreover, life gives second chances for every opportunity, including learning. Many adults go back to school or take classes. They learn driving, cooking, or computers when they need them. Additionally, older people have patience. They practise slowly and remember well. Young people rush and forget quickly. Adults make learning part of daily life. It is important to remember that everyone learns differently. Some need time to grow before they learn best. Forcing all learning at a young age does not work for everyone. To top this off, today we have books, classes, and helpers for all ages. The Internet and online education have made learning more flexible and accessible. Older people can learn anytime they want.

So, learning happens well at any time. Age is not a limit. Learning never ends.



Writing Task

- I. As a socially responsible citizen, you strongly believe that students can contribute significantly in promoting adult literacy. Write a letter to the Editor of a local newspaper emphasising the importance of student participation in adult literacy camps conducted by various organisations. Discuss the benefits of such initiatives for both students and society. Suggest effective ways to spread awareness and encourage more students to volunteer for this noble cause.

A letter to the editor is a formal letter in which the writer gives suggestions and creates awareness on an issue of public interest or an issue already raised in an article/write-up/published letter.

POINTS TO REMEMBER

Format

- **Name, designation, address**
- **Date**
- **Subject: 4–5 words in title case**
- **Salutation: Sir/Madam**
- **Body of the letter: In at least three paragraphs**
- **Paragraph 1:** Statement of problem: Raise the issue by citing some latest news items, survey report or personal experience, etc.
 - This is with reference to the article about _____ dated _____ in your newspaper.
 - As a concerned citizen I would like to draw your attention towards...
- **Paragraph 2:** Analyse the issue in terms of its causes and consequences.
 - The issue at hand affects a large section of society and...
 - It is imperative that...
 - It would be beneficial if...
 - Such initiatives nurture a sense of...
 - By engaging in these programmes students...

- **Paragraph 3:** Offer suggestions/solutions. Request the relevant authorities to take the action and request the editor to publish your views.
 - A possible solution to this issue could be...
 - One effective way to address this might be...
 - Authorities could consider implementing...
 - I trust this matter will be considered seriously for the benefit of all.
 - I am hopeful that necessary steps will be taken to...
 - I hope this letter gets published in the columns of your esteemed daily.
- **Complimentary close: Yours truly,**
- **Language**
 - Formal, clear, and specific
 - Short and to the point sentences
 - Purpose stated clearly and concisely
 - Relevant details to be given

Ans. A-23, Green Park
New Delhi – 110016
8 March 20XX

The Editor
The Times of India
New Delhi – 11XXXX

Subject : Importance of Promoting Adult Literacy

Sir/Madam

This is with reference to recent reports highlighting the problem of adult illiteracy in our country. As a concerned citizen, I would like to draw your attention towards the urgent need to promote adult literacy programmes.

A large section of society still lacks basic reading and writing skills, which affects employment opportunities and overall development.

Adult literacy campaigns can bring many benefits to both individuals and society. Educated adults become more confident, independent and capable of making informed decisions. Students who volunteer in such initiatives also develop empathy, responsibility and practical knowledge.

I request the authorities and educational institutions to organize awareness drives and encourage students to participate in teaching programmes. Workshops, community classes and online learning platforms can be introduced to reach more people.

Yours truly
ABC



Learning Beyond the Text

- I. India has 22 major languages according to the Eighth Schedule of the Indian Constitution. Each language has its own unique identity. Talk to your classmates and teacher and complete the given table with words from any five Indian languages.

Language	Grandmother	Grandfather	Mother	Father	Sister	Brother
1. Kannada	Avva					
2. Hindi						
3. Tamil						
4. Bengali						
5. Marathi						

Ans.

Language	Grandmother	Grandfather	Mother	Father	Sister	Brother
1. Kannada	Avva	Ajja	Amma	Appa	Akka(elder sister)	Anna
2. Hindi	Dadi/Nani	Dada/Nana	Maa	Papa	Behen	Bhai
3. Tamil	Paati	Thaatha	Amma	Appa	Akka	Anna/Thambi
4. Bengali	Thakuma/Didima	Thakurda/Dadu	Maa	Baba	Bon	Bhai
5. Marathi	Aaji	Aajoba	Aai	Baba	Tai	Bhau



Self Assessment Test

Extract Based Questions

Read the following extract carefully and answer the questions that follow.

She told me that she wanted to learn to read before the Saraswati Puja during Dassara. I laughed at her childish ambition because she was sixty-two years old. But she smiled and said that there is no age limit for learning. From the next day, I began teaching her. She was a dedicated student who completed all her homework sincerely. She read, repeated, wrote and practised determination and every day with great enthusiasm.

1. *What does the phrase 'there is no age limit for learning' convey?*

- (i) learning is only for children
- (ii) determination to study at any age
- (iii) confidence and motivation
- (iv) regret about old age
- (v) belief in lifelong learning

A ii, iii, v B Only i
C iii and iv D Only v

II. *How does the description of the grandmother's daily practice contribute to the theme of determination in the story?*

III. *Select the option that correctly completes the following.*

The narrator laughed at her grandmother because she thought the idea was

- A unrealistic for her age.
- B very easy to achieve

IV. *What emotion is the grandmother most likely conveying through her smile and hard work?*

Short Answer Type Questions

Q.1 *How did the grandmother spend her time after listening to the weekly episode of the novel?*

Q.2 *How did the narrator feel when her grandmother shared her life story in the middle of the night?*

Q.3 *What qualities made the grandmother a wonderful student?*

Q.4 *Why was the Saraswati Puja chosen as the deadline for learning to read?*

Q.5 *What lesson did the narrator learn from her grandmother's determination to study?*

Q.6 *The grandmother had some great qualities. Elaborate the ones that impress you the most?*

Long Answer Type Questions

Q.1 *Explain how the novel 'Kashi Yatre' influenced the grandmother emotionally and personally.*

Q.2 *Narrate how the grandmother expressed her desire to learn reading and how the narrator initially reacted.*

Q.3 *How does the story highlight the importance of education and independence in one's life?*

Self Assessment Solutions

Extract Based Questions

1. A
- II. The grandmother practised reading and writing every day with sincerity and dedication. Her regular efforts show strong determination, proving that consistent hard work and perseverance help achieve even difficult goals.
- III. A unrealistic for her age.
- IV. She conveys determination and confidence, showing strong belief in herself and her ability to learn despite her age.

Short Answer Type Questions

1. After listening to the weekly episode of the novel, the grandmother would go to the temple courtyard to meet her friends. There, she discussed the latest part of the story with great excitement. This showed her deep interest in the novel and her habit of sharing stories with others.
2. The narrator felt surprised and confused when her grandmother began sharing her life story in the middle of the night. However, she also felt concerned and emotional after seeing her grandmother's worried face and tears, which made her realise that something serious was troubling her deeply.
3. The grandmother was a sincere, hardworking and highly determined student. She completed all her homework regularly and practised reading and writing every day. Her strong willpower, patience and dedication helped her learn quickly and achieve her goal of becoming independent through education.

4. The grandmother chose Saraswati Puja as her deadline because it is dedicated to Goddess Saraswati, the symbol of knowledge and learning. Achieving her goal on this auspicious day would make her success more meaningful and special. The deadline also motivated her to practise sincerely and remain disciplined throughout her learning journey.
5. The narrator learned that determination and hard work can help anyone achieve their goals, regardless of age. Her grandmother's strong willpower taught her that learning has no age limit and that education brings independence, confidence and self-respect. This experience inspired the narrator deeply.

6. The grandmother's determination and strong willpower impress me the most. Despite being sixty-two, she decided to learn reading and worked hard every day. Her humility and respect for education also stand out. She showed that learning has no age limit and inspired everyone through her dedication and courage.

Long Answer Type Questions

1. The novel 'Kashi Yatre' had a deep emotional and personal impact on the grandmother. She strongly identified with the old woman in the story who longed to visit Kashi but faced difficulties. Like the protagonist, the grandmother had also never visited Kashi and shared the same desire. This emotional connection made the story very meaningful to her.

The novel also inspired her values and actions. The old lady in the story sacrifices her savings to help an orphan girl, showing kindness and selflessness. This touched the grandmother deeply and increased her interest in the story. Her eagerness to follow every episode made her realise the importance of reading. When she could not read the story herself, she felt helpless and decided to learn the alphabet, changing her life completely.

2. One night, the grandmother sat beside the narrator and shared her feelings. She explained that when the magazine arrived during the narrator's absence, she could not read the story herself. This made her feel helpless and dependent. She then firmly decided to learn the Kannada alphabet and set Saraswati Puja as her deadline to read a novel independently.

The narrator initially reacted with laughter and sarcasm. She found it surprising that a sixty-two-year-old woman wanted to learn to read and write. However, the grandmother remained calm and confident. She smiled and said that there is no age limit for learning. Her strong determination soon changed the narrator's attitude, and she happily agreed to teach her grandmother with patience and care.

3. The story clearly highlights that education brings confidence, dignity and independence. The grandmother lived a comfortable life but felt helpless because she could not read. When she could not read her favourite novel during her granddaughter's absence, she realised how dependent she was on others. This incident made her understand the true importance of education.

Her decision to learn reading at the age of sixty-two shows that education can change a person's life at any stage. With determination and regular practice, she became independent and confident. The story teaches that education is not only about knowledge but also about self-respect and freedom. It encourages readers to value learning and believe that it is never too late to become educated.