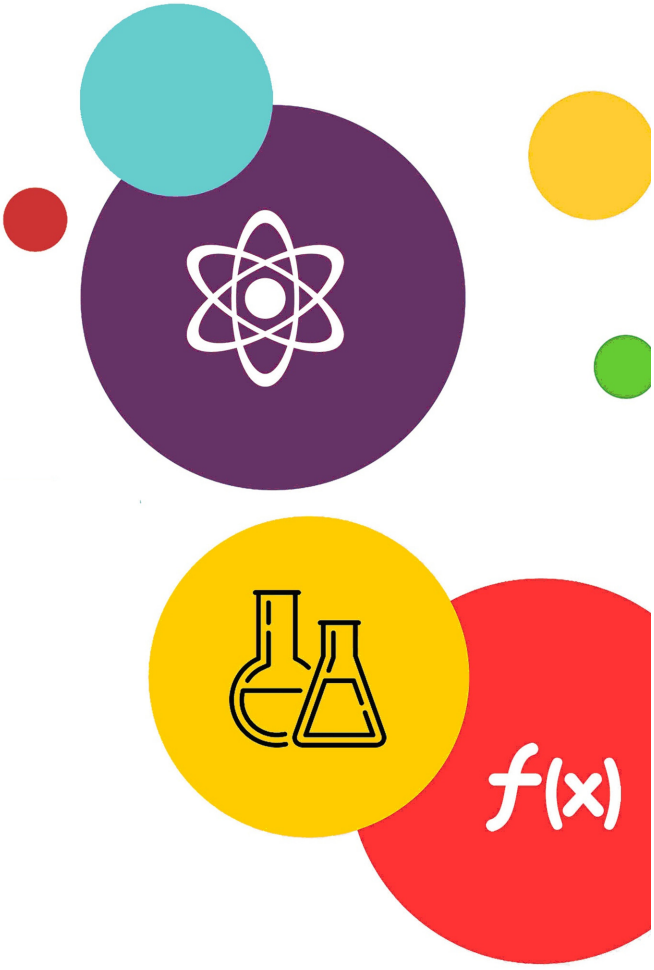
STUDY MATERIAL

JEE

FOR MAIN & ADVANCED



CHEMISTRY

Class 11

CP PUBLICATION

CHEMISTRY

Study Material for JEE Main & Advanced preparation

Prepared by Career Point Kota Experts

CAREER POINT

CONTENTS OF THE PACKAGE AT A GLANCE

CHEMISTRY

Class 11

Physical Chemisty (I)

- Atomic Structure
- Basic Concepts of Chemistry
- Redox & Volumetric Analysis
- Gaseous State
- Chemical Energetics
- Chemical Equilibrium
- Acid Base & Ionic Equilibrium

Inorganic Chemistry (I)

- Periodic Table
- Chemical Bonding
- Hydrogen & Its compounds
- s-block elements

Organic Chemistry (I)

- Classification & Nomenclature
- Isomerism
- General Organic Chemistry
- Hydrocarbons
- Aromatic Chemistry
- Environmental Chemistry

Note to the Students

Career Point offers this must have Study Package in Physics to meet the complete curriculum needs of engineering aspirants. The set comprises of 3 books. The set caters to the different requirements of students in classes XI. It offers complete and systematic coverage of **JEE Main** and **JEE Advanced** syllabi and aims to provide firm foundation in learning and develop competitive edge in preparation of the JEE and other engineering entrance examinations.

COMPONENTS OF EACH CHAPTER

These books are designed with an engaging and preparation-focused pedagogy and offer a perfect balance of conceptual learning and problem solving skills.

Theory & Concepts

Each chapter consists of high quality theory that covers all the topics, sub-topics and concepts of JEE syllabus.

Atomic Structure

1. INTRODUCTION

(a) The word atom was first introduced by Ostwald (1803 - 1807) in scientific world.

(b) According to him matter is ultimately made up of extremely small indivisible particles called atoms.

(c) It takes part in chemical reactions.

(d) Atom is neither created nor destroyed

2. DALTON'S ATOMIC THEORY

Dalton proposed the atomic theory on the basis of the law of conservation of mass and law of definite proportions. He also proposed the law of multiple proportion as a logical consequence of this theory. The salient features of this theory are-

(a) Each element is composed by extremely small particles called atoms.

(b) Atoms of a particular element are all alike but differ with the atoms of other elements.

(c) Atom of each element is an ultimate particle, and has a characteristic mass but is structureless.

(d) Atom is indestructible i.e. it can neither be destroyed nor created by simple chemical reactions.

(e) Atom of an element takes part in chemical reaction to form molecule.

(f) In a given compound, the relative number and kind of atom are same.

(g) Atoms of different elements combine in fixed ratio of small whole numbers to form compound atoms (now called molecules).

✦ **Merits and Demerits of Dalton's theory :**

A. Merits :

(a) Dalton's theory explains the law of conservation of mass and some other laws of chemical combination.

(b) Atoms of elements take part in chemical reaction is true till today.

B. Demerits :

(a) There is no mention of atomic weights of elements.

(b) He could not explain that why do atoms of same element combined with each other.

(c) The law of definite proportion fails if different isotopes are used.

3. EARLIER EFFORTS TO REVEAL STRUCTURE OF ATOM

✦ **Evidence for the Electrical nature of matter :**

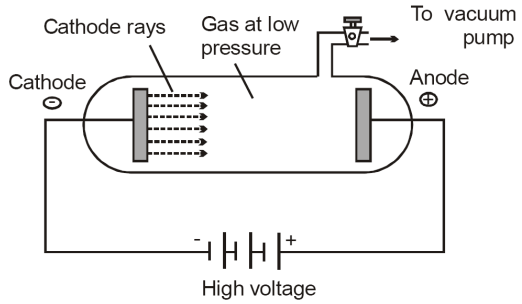
(a) In 1833 Michael faraday gave the first important clue about the electrical nature of atoms

(b) He observed that when electricity is passed through an electrolyte (in the molten state or dissolved state), it undergoes chemical changes.

(c) This phenomenon is called electrolysis

(d) Later on in 1874, stoney pointed out that like matter, electricity is composed of small discrete units of electricity. He proposed the name electron for these discrete units of electricity

✦ **Cathode Rays - discovery of electron :**



(a) The electron was discovered as a result of the studies of the passage of electricity through gases at extremely low pressures known as discharge tube experiments.

(b) In 1859, Julics Plucker started the study of conduction of electricity through gases at low pressure (10^{-4} atm.) in a discharge tube.

(c) When a high voltage of the order of 10,000 volts or more was impressed across the electrodes, some sort of invisible rays moved from the negative electrode to the positive electrodes these rays are called as cathode rays

(d) Further investigations were made by W.crookes, J.perrin, J.J. Thomson and others.

(e) Cathode rays have the following properties.

(i) Path of travelling is straight from the cathode with a very high velocity.As it produces shadow of an object placed in its path

Important Points

This part contains important concepts & formulas of chapter at one place in short manner, So that student can revise all these in short time.

Points to Remember

1. The wave character is of no significance in case of large objects like cricket ball, a car, a train etc.
2. The most important applications of de-Broglie concept is in the construction of electron microscope and the study of surface structure of solids by electron diffraction.
3. Smaller the wavelength of the electron wave, more is the resolving power of the electron microscope
4. Uncertainty in measurement is not due to lack of any experimental technique but due to nature of subatomic particle itself
5. Shapes of orbitals are functional representation of mathematical solutions of Schrodinger equations. They do not represent any picture of electric charge or matter.

Solved Examples (JEE Main/Advanced)

To understand the application of concepts, there is a solved example section. It contains large variety of all types of solved examples with explaination to ensure understanding the application of concepts.

SOLVED EXAMPLES

Ex.1 The ratio of the wave lengths of last lines of Balmer and Lyman series is -
(A) 4 : 1 (B) 27 : 5
(C) 3 : 1 (D) 9 : 4

Sol.(A) The wave length of a spectral line may be given by the following expression

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For Lyman series $n_1 = 1$, For Balmer series $n_1 = 2$
For the last line in both the series $n_2 = \infty$
For Lyman series

$$\frac{1}{\lambda_L} = R\left(1 - \frac{1}{\infty}\right) = R\,(1-0) = R$$

$$\lambda_L = \frac{1}{R}$$

For Balmer series

$$\frac{1}{\lambda_B} = R\left(\frac{1}{4} - \frac{1}{\infty}\right) = \frac{R}{4}$$

$$\lambda_B = \frac{4}{R}$$

$$\frac{\lambda_B}{\lambda_L} = \frac{4}{R} \times \frac{R}{1} = \frac{4}{1}$$

Ex.2 Consider an electron which is brought close to the nucleus of the atom from an infinite distance, the energy of the electron-nucleus system -
(A) increases (B) decreases
(C) remains same (D) none of these

Ex.3 The energy of first Bohr's orbit of H-atom is -864 Arbitrary units. The energy of third Bohr's orbit of H atom is -
(A) 96 Arbitrary units
(B) 192 Arbitrary units
(C) 288 Arbitrary units
(D) 384 Arbitrary units

Sol.(A) The energy of first Bohr's orbit of H-atom

$$-\frac{2\pi^2 me^4}{h^2} = -864$$

The energy of third Bohr's orbit of H atom

$$= -\frac{2\pi^2 me^4}{h^2} \times \frac{1}{3^2} = -864 \times \frac{1}{9}$$

= –96 Arbitrary units
Energy required to separate the electron
$= E_\infty - E_n$
= 0 – (–96)
= 96 Arbitrary units

Ex.4 In an electronic transition, the wavelength of a spectral line is inversely related to -
(A) The nuclear charge of the atom
(B) The difference in energy levels
(C) The velocity of electron
(D) The number of orbitals involved in transition

Sol.(A) $$\frac{1}{\lambda} = \frac{2\pi^2 me^4 Z^2}{ch^3(4\pi\varepsilon_0)^2} = \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\lambda \propto \frac{1}{Z^2}$$

Ex.5 The ratio of time periods in first and second orbits of hydrogen atom is -
(A) 1 : 4 (B) 1 : 8 (C) 1 : 2 (D) 2 : 1

$2\pi r$

Practice Exercises

Exercise Level - 1 : It contains objective questions with single correct choice to ensure sufficient practice to accutrately appply formule and concepts.

EXERCISE (Level-1)

Question based on **Sub-Atomic particles and Dalton's atomic theory**

Q.1 Proton is -
(A) Nucleus of deuterium
(B) Ionised hydrogen molecule
(C) Ionised hydrogen atom
(D) An α-particle

Q.2 Which is not deflected by magnetic field -
(A) Neutron (B) Positron
(C) Proton (D) Electron

Q.3 According to Dalton's atomic theory, an atom can -
(A) Be created
(B) Be destroyed
(C) take part in a chemical reaction
(D) None of these

Q.4 Arrange α-particle(α), electron (e^-), proton(p) and neutron (n) in increasing order of their e/m value (specific charge, consider magnitude only not sign) -
(A) $\alpha < e^- < p < n$ (B) $n < \alpha < p < e^-$
(C) $n < p < \alpha < e^-$ (D) $e^- < p < n < \alpha$

Question based on **Rutherford's Experiment**

Q.5 Rutherford's alpha particle scattering experiment eventually led to the conclusion that -
(A) mass and energy are related
(B) electrons occupy space around the nucleus
(C) neutrons are burried deep in the nucleus
(D) the point of impact with matter can be precisely determined

Q.6 Which of the following conclusion could not be derived from Rutherford's α-particle scattering experiment ?
(A) Most of the space in the atom is empty
(B) The radius of the atom is about 10^{-10} m while that of nucleus is 10^{-15} m.
(C) Electrons move in a circular path of fixed energy called orbits
(D) Electrons and the nucleus are held together by electrostatic forces of attraction

Question based on **Electromagnetic waves, hydrogen spectra & concept of quantization**

Q.7 The line spectra of two elements are not identical because -
(A) the elements do not have the same number of neutrons
(B) they have different mass number
(C) their outermost electrons are at different energy levels
(D) they have different valencies

Q.8 A certain radio station broadcasts on a frequency of 980 kHz (kilohertz). What is the wavelength of electromagnetic radiation broadcast by the radio station ?
(A) 306 m (B) 3.06 m
(C) 30.6 m (D) 3060 m

Q.9 Calculate the wavelength of the spectral line when the electron in the hydrogen atom undergoes a transition from fourth energy level to second energy level ?
(A) 4.86 nm (B) 486 nm
(C) 48.6 nm (D) 4860 nm

Q.10 The wave number of the first line of Balmer series of hydrogen is 15200 cm^{-1}. The wave number of the corresponding line of Li^{2+} ion is-
(A) 15200cm^{-1} (B) 60800 cm^{-1}
(C) 76000 cm^{-1} (D) 136800 cm^{-1}

Q.11 The frequency of one of the lines in Paschen series of a hydrogen atom is 2.34×10^{14}Hz. The higher orbit, n_2, which produces this transitions is -
(A) three (B) four
(C) six (D) five

Q.12 In hydrogen spectrum, the series of lines appearing in ultra violet region of electromagnetic spectrum are called -
(A) Lyman lines (B) Balmer lines
(C) Pfund lines (D) Brackett lines

Q.13 Which of the following series of lines in the atomic spectrum of hydrogen appear in the visible region ?
(A) Lyman (B) Paschen
(C) Brackett (D) Balmer

Q.14 Which of the following is not correct according to Planck's quantum theory ?
(A) Energy is emitted or absorbed discontinuously
(B) Energy of a quantum is directly proportional to its frequency
(C) A photon is also a quantum of light
(D) Energy less than a quantum can also be emitted or absorbed

Q.15 To which electronic transition between Bohr orbits in hydrogen, the second line in the Balmer series belongs ?
(A) $3 \rightarrow 2$ (B) $4 \rightarrow 2$
(C) $5 \rightarrow 2$ (D) $6 \rightarrow 2$

Exercise Level - 2 : It contains single objective type questions with moderate difficulty level to enhance the conceptual and application level of the student.

EXERCISE (Level-2)

Q.1 What is the maximum number of electrons in an atom that can have the quantum numbers $n = 4$, $m_\ell = +1$?
(A) 4 (B) 15 (C) 3 (D) 6

Q.2 Arrange the orbitals of H-atom in the increasing order of their energy -
$3p_x$, 2s, $4d_{xy}$, 3s, $4p_z$, $3p_y$, 4s
(A) $2s < 3s = 3p_x = 3p_y < 4s = 4p_z = 4d_{xy}$
(B) $2s < 3s < 3p_x = 3p_y < 4s = 4p_z = 4d_{xy}$
(C) $2s < 3s < 3p_x = 3p_y < 4s = 4p_z = 4d_{xy}$
(D) $2s < 3s < 3p_x = 3p_y < 4s < 4p_z < 4d_{xy}$

Q.3 If the I.P. of Li^{+2} is 122.4 eV. Find out 6^{th} I.P. of carbon -
(A) 122.4 × 4eV (B) 122.4 × 2eV
(C) 122.4 × 3eV (D) 122.4 × 5eV

Q.4 The energy difference between two electronic states is 46 .12 kcal /mole. What will be the frequency of the light emitted when an electron drops from the higher to the lower energy state (Planck' constant = 9.52×10^{-14} kcal sec $mole^{-1}$)
(A) 4.84×10^{15} cycles sec^{-1}
(B) 4.84×10^{-5} cycles sec^{-1}
(C) 4.84×10^{-12} cycles sec^{-1}
(D) 4.84×10^{14} cycles sec^{-1}

Q.5 If the kinetic energy of an electron is increased 4 times, the wavelength of the de Broglie wave associated with it would become :
(A) 4 times (B) 2 times
(C) 1/2 times (D) 1/4 times

Q.6 Multiple of fine structure of spectral lines is due to-
(A) Presence of main energy levels
(B) Presence of sub- levels
(C)Presence of electronic configuration
(D) Is not a characteristics of the atom.

Q.7 Wave mechanical mode of the atom depends upon-
(A) de-Broglie concept of dual nature of electron
(B) Heisenberg uncertainty principle
(C) Schrodinger uncertainty principle
(D) All

Q.8 Calculate total no. of e^- having m = 0 in Cr atom -
(A) 12 (B) 13 (C) 5 (D) 24

Q.9 Which of the following subshell can accommodate as many as 10 electrons -
(A) 2d (B) 3d (C) $3d_{xy}$ (D) $3d_{z^2}$

Q.10 How many spherical nodes are present in a 4s orbital in hydrogen atom -
(A) 0 (B) 1 (C) 2 (D) 3

Q.11 Assuming the velocity to be same which subatomic particle possesses smallest de-Broglie wavelength -
(A) An electron
(B) A proton
(C) An α-particle
(D) All have same wavelength

Q.12 I.P. of hydrogen atom is equal to 13.6 eV. What is the energy required for the process :
$He^+ + \text{energy} \longrightarrow He^{+2} + e^-$
(A) 2 × 13.6 eV (B) 1 × 13.6 eV
(C) 4 × 13.6 eV (D) None of these

Q.13 If elements with principal quantum number $n > 4$ is not allowed in nature, the number of possible elements would be -
(A) 60 (B) 32 (C) 64 (D) 50

Q.14 If the value of $(n + \ell)$ is not > 3, then the maximum number of electrons in all the orbitals would be -
(A) 12 (B) 10 (C) 2 (D) 6

Q.15 It is not possible to explain the Pauli's exclusion principle with the help of this atom -
(A) B (B) Be (C) C (D) H

Q.16 How fast is an electron moving if it has a wavelength equal to the distance it travels in one second -
(A) $\sqrt{\frac{h}{m}}$ (B) $\sqrt{\frac{m}{h}}$ (C) $\sqrt{\frac{h}{p}}$ (D) $\sqrt{\frac{h}{2(KE)}}$

Q.17 An atom has a mass of 0.02 kg & uncertainity in its velocity is 9.218×10^{-6} m/s then uncertainty in position is
($h = 6.626 \times 10^{-34}$ J - s)
(A) 2.86×10^{-28} m (B) 2.86×10^{-32} cm
(C) 1.5×10^{-27} m (D) 3.9×10^{-10} m

Q.18 Energy of H–atom in the ground state is –13.6 eV, Hence energy in the second excited state is –
(A) –6.8 eV (B) –3.4 eV
(C) –1.51 eV (D) –4.3 eV

Q.19 Uncertainty in position of a particle of 25 g in space is 10^{-5} m. Hence uncertainty in velocity (ms^{-1}) is (Planck's constant $h = 6.6 \times 10^{-34}$ Js)
(A) 2.1×10^{-28} (B) 2.1×10^{-34}
(C) 0.5×10^{-34} (D) 5.0×20^{-24}

Q.20 The orbital angular momentum for an electron revolving in an orbit is given by $\sqrt{\ell(\ell+1)}.\frac{h}{2\pi}$. This momentum for an s-electron will be given by
(A) $\frac{h}{2\pi}$ (B) $\sqrt{2}.\frac{h}{2\pi}$
(C) $+\frac{1}{2}.\frac{h}{2\pi}$ (D) zero

Exercise Level - 3 : It contains previous years question of JEE Main from Year 2005 to 2024.

EXERCISE (Level-3)

Old Examination Questions [JEE Main]

Q.1 In a multi-electron atom, which of the following orbitals described by the three quantum members will have the same energy in the absence of magnetic and electric fields ? **[AIEEE- 2005]**

(a) n = 1, ℓ = 0, m = 0 (b) n = 2, ℓ = 0, m = 0

(c) n = 2, ℓ = 1, m = 1 (d) n = 3, ℓ = 2, m = 1

(e) n = 3, ℓ = 2, m = 0

(A) (b) and (c) (B) (a) and (b)

(C) (d) and (e) (D) (c) and (d)

Q.2 Of the following sets which one does NOT contain isoelectronic species ? **[AIEEE- 2005]**

(A) CN^-, N_2, C_2^{2-}

(B) PO_4^{3-}, SO_4^{2-}, ClO_4^-

(C) BO_3^{3-}, CO_3^{2-}, NO_3^-

(D) SO_3^{2-}, CO_3^{2-}, NO_3^-

Q.3 According to Bohr's theory, the angular momentum of an electron in 5th orbit is - **[AIEEE 2006]**

(A) 1.0 h/π (B) 10 h/π

(C) 2.5 h/π (D) 25 h/π

Q.4 Uncertainty in the position of an electron (mass = 9.1×10^{-31} kg) moving with a velocity 300 m/s, accurate upto 0.001 %, will be ($h = 6.63 \times 10^{-34}$ Js) **[AIEEE 2006]**

(A) 5.76×10^{-2} m (B) 1.92×10^{-2} m

(C) 3.84×10^{-2} m (D) 19.2×10^{-2} m

Q.5 Which of the following sets of quantum numbers represents the highest energy of an atom ? **[AIEEE 2007]**

(A) n = 3, ℓ = 1, m = 1, s = +½

(B) n = 3, ℓ = 2, m = 1, s = +½

(C) n = 4, ℓ = 0, m = 0, s = +½

(D) n = 3, ℓ = 0, m = 0, s = +½

Q.6 The ionization enthalpy of hydrogen atom is 1.312×10^6 J mol^{-1}. The energy required to excite the electron in the atom from n = 1 to n = 2 is **[AIEEE 2008]**

(A) 6.56×10^5 J mol^{-1}

(B) 7.56×10^5 J mol^{-1}

(C) 9.84×10^5 J mol^{-1}

(D) 8.51×10^5 J mol^{-1}

Q.7 In an atom, an electron is moving with a speed of 600 m/s with an accuracy of 0.005%. Certainity with which the position of the electron can be located is ($h = 6.6 \times 10^{-34}$ kg m^{2}s^{-1}, mass of electron, $e_m = 9.1 \times 10^{-31}$ kg) **[AIEEE 2009]**

(A) 1.52×10^{-4} m (B) 5.10×10^{-3} m

(C) 1.92×10^{-3} m (D) 3.84×10^{-3} m

Q.8 Calculate the wavelength (in nanometer) associated with a proton moving at 1.0×10^3 m s^{-1} (Mass of proton = 1.67×10^{-27} kg and h = 6.63×10^{-34} Js) - **[AIEEE 2009]**

(A) 0.032 nm (B) 0.40 nm

(C) 2.5 nm (D) 14.0 nm

Q.9 A gas absorbs a photon of 355 nm and emits at two wavelengths. If one of the emissions is at 680 nm, the other is at : **[AIEEE 2011]**

(A) 1035 nm (B) 325 nm

(C) 743 nm (D) 518 nm

Q.10 The frequency of light emitted for the transition n = 4 to n = 2 of He^+ is equal to the transition in H atom corresponding to which of the following ? **[AIEEE 2011]**

(A) n = 2 to n = 1 (B) n = 3 to n = 2

(C) n = 4 to n = 3 (D) n = 3 to n = 1

Q.11 The electrons identified by quantum numbers n and ℓ **[AIEEE-2012]**

(a) n = 4, ℓ = 1 (b) n = 4, ℓ = 0

(c) n = 3, , ℓ = 2 (d) n = 3, ℓ = 1

can be placed in order of increasing energy as -

(A) (d) < (b) < (c) < (a) (B) (b) < (d) < (a) < (c)

(C) (a) < (c) < (b) < (d) (D) (c) < (d) < (b) < (a)

Q.12 The following sets of quantum numbers represents four electrons in an atom :

(i) n = 4, ℓ = 1 (ii) n = 4, ℓ = 0

(iii) n = 3, ℓ = 2 (vi) n = 3, ℓ = 1

The sequence representing increasing order of energy , is : **[AIEEE Online-2012]**

(A) (i) < (iii) < (ii) < (iv) (B) (ii) < (iv) < (i) < (iii)

(C) (iv) < (ii) < (iii) < (i) (D) (iii) < (i) < (iv) < (ii)

Q.13 The limiting line in Balmer series will have a frequency of :

(Rydberg constant, $R_\infty = 3.29 \times 10^{15}$ cycles/s) **[AIEEE Online-2012]**

(A) 3.65×10^{14} s^{-1} (B) 8.22×10^{14} s^{-1}

(C) 3.29×10^{15} s^{-1} (D) 5.26×10^{13} s^{-1}

Q.14 If the kinetic energy of an electron is increased four times, the wavelength of the de-Broglie wave associated with it would become : **[AIEEE Online-2012]**

(A) Two times (B) Half

(C) One fourth (D) Four times

Q.15 Which pair of elements with the given atomic numbers is expected to have similar properties ? **[AIEEE Online-2012]**

(A) 11, 12 (B) 40, 72

(C) 20, 36 (D) 10, 28

Exercise Level - 4 : It contains all variety of questions as per level of JEE Advanced such as MCQ, Column match, Passage based & Numerical type etc.

EXERCISE (Level-4)

Part-A : Multiple correct answer type questions

Q.1 Which of the following properties is/are proportional to the energy of the electromagnetic radiation ?
(A) Frequency (B) Wave number
(C) Wavelength (D) Number of photons

Q.2 Which of the following statements are incorrect?
(A) There are five unpaired electrons in (n –1)d suborbit in Fe^{3+}
(B) Fe^{3+}, Mn^{+} and Cr all having 24 electrons will have same value of magnetic moment
(C) Copper (I) chloride is coloured salt
(D) Every coloured ion is paramagnetic

Q.3 Which is not the correct orbital notation if the wave function is –

$$\psi = \frac{1}{81\sqrt{6\pi}}\left(\frac{1}{a_0}\right)^{3/2} \sigma^2 e^{-\sigma/3} (3\cos^2\theta - 1);$$

Here $\sigma = r/a_0$ and $a_0 = \frac{h^2 \in_0}{\pi m e^2}$

(A) 4s (B) $2P_x$ (C) $3P_y$ (D) $3d_{z^2}$

Q.4 Which of the following orbitals have no spherical nodes ?
(A) 1s (B) 2s
(C) 2p (D) 3p

Q.5 In which of the following sets of orbitals, electrons have equal orbital angular momentum ?
(A) 1s and 2s (B) 2s and 2p
(C) 2p and 3p (D) 3p and 3d

Q.6 Which of the following sets of quantum number are correct ?

(A) $n = 3, \ell = 2, m = +1, s = +\frac{1}{2}$

(B) $n = 3, \ell = 3, m = +3, s = +\frac{1}{2}$

(C) $n = 4, \ell = 0, m = 0, s = -\frac{1}{2}$

(D) $n = 5, \ell = 2, m = +4, s = -\frac{1}{2}$

Q.7 Rutherford's experiment established that :
(A) Inside the atom there is a heavy positive centre
(B) Nucleus contains protons and neutrons
(C) Most of the space in the atoms is empty
(D) Size of the nucleus is very small

Q.8 Which of the following statements are incorrect ?
(A) For designating orbitals three quantum numbers are needed
(B) The second ionization energy of helium is 4 times, the first ionization of hydrogen
(C) The third ionization energy of lithium is 9 times, the first ionization of hydrogen
(D) Radius of third orbit of Li^{2+} is 3 times the radius of third orbit of hydrogen atom

Q.9 Which of the following statements (regarding an atom of H) are correct ?
(A) Kinetic energy of the electron is maximum in the first orbit
(B) Potential energy of the electron is maximum in the first orbit
(C) Radius of the second orbit is four times the radius of the first orbit
(D) Various energy levels are equally spaced

Q.10 Which of the following transition in H–atom would result in emission of radiations of same frequency ?
(A) $4s \rightarrow 3p$ (B) $4d \rightarrow 3p$
(C) $5s \rightarrow 4s$ (D) $3s \rightarrow 2p$

Q.11 The radial distribution functions [P(r)] is used to determine the most probable radius, which is used to find the electron in a given orbital

$\frac{dP(r)}{dr}$ for 1s-orbital of hydrogen like atom having atomic number Z, is

$$\frac{dP}{dr} = \frac{4Z^3}{a_0^3}\left(2r - \frac{2Zr^2}{a_0}\right)e^{-2Zr/a_0}$$

Then which of the following statements is/are connect ?
(A) At the point of maximum value of radial distribution function $\frac{dP(r)}{dr} = 0$; One antinode is present
(B) Most probable radius of Li^{2+} is $\frac{a_0}{3}$ pm
(C) Most probable radius of He^{+} is $\frac{a_0}{2}$ pm
(D) Most probable radius of hydrogen atom is a_0 pm

Exercise Level - 5 : It contains previous years question of JEE Advanced from Year 2005 to 2024.

EXERCISE (Level-5)

Old Examination Questions [JEE Advanced]

Q.1 The number of radial nodal surface in 3s and 2p **[IIT-2005]**

(A) 2, 0 (B) 2, 1 (C) 1, 0 (D) 0, 2

Q.2 **Give answer :** **[IIT-2005]**

(a) For first orbit of hydrogen atom, calculate the velocity of electron ($r = a_0 = 0.529$ Å)

(b) Calculate the de-broglie wavelength of electron in first Bohr orbit

(c) Calculate the orbital angular momentum of 2p orbital in terms of $h/2\pi$ units

Q.3 According to Bohr's theory, **[IIT-2006]**

E_n = Total energy ;

K_n = Kinetic energy

V_n = Potential energy

r_n = Radius of n^{th} orbit

Match the following :

Column-I		Column-II	
(A)	V_n/K_n = ?	(P)	0
(B)	If radius of nth orbital is r_n, $r_n \propto E_n^x$, x = ?	(Q)	–1
(C)	Angular momentum in lowest orbital	(R)	–2
(D)	$\frac{1}{r^n} \propto Z^y$, y = ?	(S)	1

Q.4 Match the entries in column-I with the correctly related quantum no. (s) in column-II **[IIT-2008]**

Column-I			Column-II
(A)	orbital angular momentum of the electron in a hydrogen like atomic orbital	(P)	Principal quantum number
(B)	A hydrogen like one electron wave function obeying Pauli's number principle	(Q)	Azimuthal quantum number
(C)	Shape, size and orientation of hydrogen like atomic orbital	(R)	magnetic quantum number
(D)	Probability density of electron at the nucleus in hydrogen like atom	(S)	Electron spin quantum number

Passage based objective questions

Passage :1 (Ques. 5 to 7)

The hydrogen like species Li^{2+} is in a spherically symmetric state S_1 with one radial node. Upon absorbing light the ion undergoes transition to a state S_2. The state S_2 has one radial node and its energy is equal to the ground state energy of the hydrogen atom. **[IIT-2010]**

Q.5 The state S_1 is -

(A) 1s (B) 2s (C) 2p (D) 3s

Q.6 Energy of the state S_1 in units of the hydrogen atom ground state energy is -

(A) 0.75 (B) 1.50 (C) 2.25 (D) 4.50

Q.7 The orbital angular momentum quantum number of the state S_2 is -

(A) 0 (B) 1 (C) 2 (D) 3

Q.8 The maximum number of electrons that can have principal quantum number, n = 3 and spin quantum number, $m_s = -1/2$, is. **[IIT-2011]**

Q.9 The work function (ϕ) of some metals is listed below. The number of metals which will show photoelectric effect when light of 300 nm wavelength falls on the metal is. **[IIT-2011]**

Metal	Li	Na	K	Mg	Cu	Ag	Fe	Pt	W
ϕ (eV)	2.4	2.3	2.2	3.7	4.8	4.3	4.7	6.3	4.75

Q.10 The kinetic energy of an electron in the second Bohr orbit of a hydrogen atom is [a_0 is Bohr radius] **[IIT-2012]**

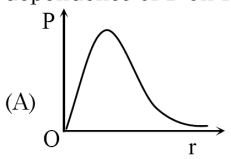
(A) $\frac{h^2}{4\pi^2 ma_0^2}$ (B) $\frac{h^2}{16\pi^2 ma_0^2}$

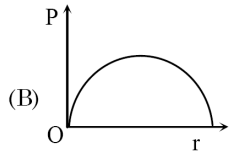
(C) $\frac{h^2}{32\pi^2 ma_0^2}$ (D) $\frac{h^2}{64\pi^2 ma_0^2}$

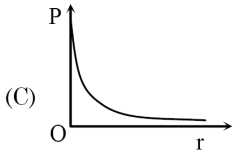
Q.11 In an atom, the total number of electrons having quantum numbers n = 4, $|m_l| = 1$ and $m_s = -1/2$ is **[JEE-Advance-2014]**

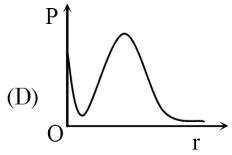
Q.12 Not considering the electronic spin, the degeneracy of the second excited state (n = 3) of H atom is 9, while the degeneracy of the second excited state of H^- is – **[JEE-Advance-2015]**

Q.13 P is the probability of finding the 1s electron of hydrogen atom in a spherical shell of infinitesimal thickness, dr, at a distance r from the nucleus. The volume of this shell is $4\pi r^2 dr$, The qualitative sketch of the dependence of P on r is- **[JEE-Advance-2016]**

(A) 

(B) 

(C) 

(D) 

Exercise Level - 6 : Advanced level a bit complex questions for students for solid rock prepertion for Top Rankers.

EXERCISE (Level-6)

Review Exercise

Q.1 An electron is accelerated from a very low velocity (~ zero speed) by the application of a potential difference of V volts. If the de Broglie wavelength should change (i.e., decrease) by 1.0% what percent increase in V causes it-

Q.2 An electron first accelerated through 100 volts suffers successively two retardations (i) through 19 volts and then (ii) through 32 volts. Its de Broglie wavelengths in the three situations are respectively λ_1, λ_2 and λ_3. Calculate $\frac{\lambda_3 - \lambda_2}{\lambda_1}$.

Q.3 Photons having energy equivalent to binding energy of 2nd state of Li^+ ion are used at metal surface of work function 10.6 eV. If the ejected electrons are further accelerated through the potential difference of 5 V then the minimum value of de-Broglie wavelength associated with the electron is -

Q.4 A hydrogen atom in its ground state absorbs a photon and goes into the first excited state. It then absorbs a second photon which just ionizes it. What is the ratio of the wavelengths of the first photon and the second photon ?

Q.5 A hydrogen like atom with atomic number 'Z' is in higher excited state of quantum number 'n'. This excited state atom can make a transition to the first excited state by successively emitting two photons of energies 10 eV and 17 eV respectively. Alternatively, the atom from the same excited state can make a transition to the 2nd excited state by emitting two photons of energies 4.25 eV and 5.95 eV respectively. The 'n' and 'Z' are-

Q.6 The Schrodinger wave equation for hydrogen atom is

$$\Psi(\text{radial}) = \frac{1}{16\sqrt{4}}\left(\frac{Z}{a_0}\right)^{3/2}$$
$$[(\sigma - 1)(\sigma^2 - 8\sigma + 12)]\, e^{-\sigma/2}$$

where a_0 and Z are the constant in which answer can be expressed and $\sigma = \frac{2Zr}{a_0}$

minimum and maximum position of radial nodes from nucleus are.....respectively.

Q.7 For a hypothetical hydrogen like atom, the potential energy of the system is given by U(r) $= \frac{-Ke^2}{r^3}$, where r is the distance between the two particles. If Bohr's model of quantization of angular momentum is applicable then velocity of particle is given by:

Q.8 An element undergoes a reaction as shown:
$X + 2e^- \rightarrow X^{2-}$, energy released = 30.87 eV/atom. If the energy released, is used to dissociate 4 gms of H_2 molecules, equally into H^+ and H*, where H* is excited state of H atoms where the electron travels in orbit whose circumference equal to four times its de Broglie's wavelength. Determine the least moles of X that would be required: Given: I.E. of H = 13.6 eV/atom, bond energy of H_2 = 4.526 eV/molecule

Q.9 In a measurement of quantum efficiency of photosynthesis in green plants, it was found that 10 quanta of red light of wavelength 6850 Å were needed to release one molecule of O_2. The average energy storage in this process is 112 kcal/mol O_2 evolved.
What is the energy conversion efficiency in this experiment ? Given: 1 cal = 4.18 J; $N_A = 6 \times 10^{23}$; $h = 6.63 \times 10^{-34}$ J. s

Q.10 Find the value of wave number $(\overline{V})$ in terms of Rydberg's constant, when transition of electron takes place between two levels of He^+ ion whose sum is 4 and difference is 2.

Answer key !

Ankswer key is provided at the end of the exercis!e sheets.

ANSWER KEY

EXERCISE (Level-1)

1. (C)	**2.** (A)	**3.** (C)	**4.** (B)	**5.** (B)	**6.** (C)	**7.** (C)
8. (A)	**9.** (B)	**10.** (D)	**11.** (D)	**12.** (A)	**13.** (D)	**14.** (D)
15. (B)	**16.** (C)	**17.** (D)	**18.** (B)	**19.** (C)	**20.** (C)	**21.** (A)
22. (D)	**23.** (D)	**24.** (C)	**25.** (C)	**26.** (A)	**27.** (D)	**28.** (A)
29. (D)	**30.** (C)	**31.** (A)	**32.** (C)	**33.** (C)	**34.** (A)	**35.** (C)
36. (C)	**37.** (C)	**38.** (B)	**39.** (A)	**40.** (A)		

Revision Plan

We emphasis that every student should prepare his/her own revision plan. For this purpose there is Revision Plan Section in each chapter which student should prepare while going thorugh the study material. This will be useful at the time of final revision before final exam for quick & effective revision.

Revision Plan

Prepare Your Revision plan today!

After attempting Exercise Sheet, please fill below table as per the instruction given.

A. Write Question Number (QN) which you are unable to solve at your own in **column A**.

B. After discussing the Questions written in **column A** with faculty, strike off them in the manner so that you can see at the time question number during Revision, to solve such questions again.

C. Write down the Question Number you feel are important or good in the **column B**.

EXERCISE	COLUMN A	COLUMN B
	Questions unable to solve in first attempt	Good or Important questions
Topic wise practice questions		
Level-1		
Level-2		
Level-3		
Level-4		
Level-5		
Level-6		

Revision Strategy:

Whenever you wish to revision this chapter, follow the following steps-

Step-1: Review your theory notes.

Step-2: Solve Questions of column A

Online Solutions

Self explanatory and detailed soltuion of all excercises above are available on Career Point website www.careerpoint.ac.in

ATOMIC STRUCTURE

EXERCISE (Level-1)

Answer Key & Solution

Question Number	Solution	Question Number	Solution	Question Number	Solution	Question Number	Solution
1	Click Here	11	Click Here	21	Click Here	31	Click Here
2	Click Here	12	Click Here	22	Click Here	32	Click Here
3	Click Here	13	Click Here	23	Click Here	33	Click Here
4	Click Here	14	Click Here	24	Click Here	34	Click Here
5	Click Here	15	Click Here	25	Click Here	35	Click Here
6	Click Here	16	Click Here	26	Click Here	36	Click Here
7	Click Here	17	Click Here	27	Click Here	37	Click Here
8	Click Here	18	Click Here	28	Click Here	38	Click Here
9	Click Here	19	Click Here	29	Click Here	39	Click Here
10	Click Here	20	Click Here	30	Click Here	40	Click Here

Sol.1 **[C]**

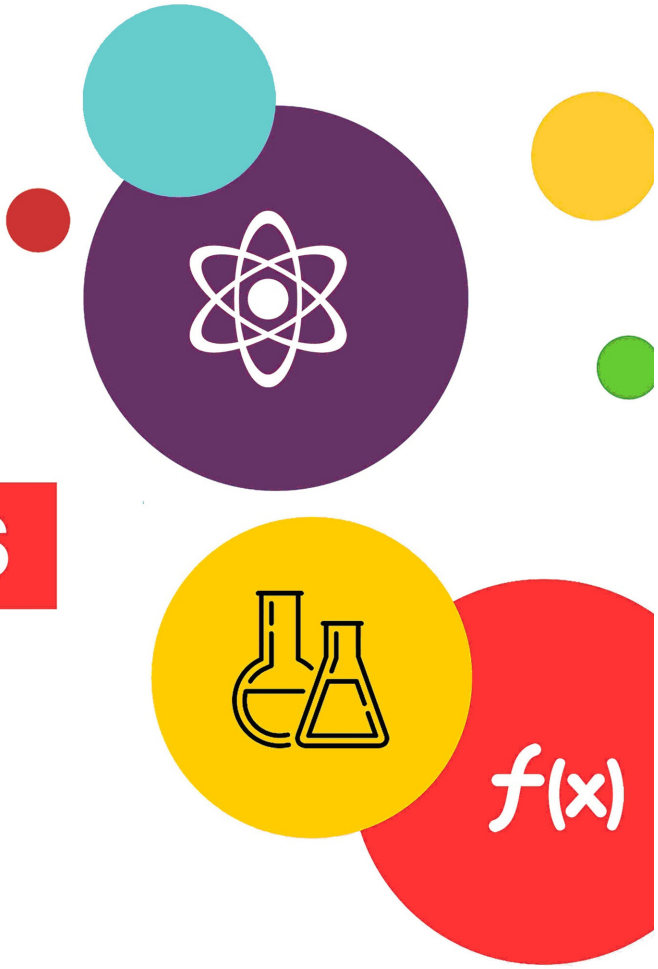
Proton is ionised hydrogen atom l.e., H^+.

Top

STUDY MATERIAL

JEE

FOR MAIN & ADVANCED



MATHEMATICS

Class 11

CP PUBLICATION

MATHEMATICS

Study Material for JEE Main & Advanced preparation

Prepared by Career Point Kota Experts

CAREER POINT

CONTENTS OF THE PACKAGE AT A GLANCE

MATHEMATICS

Class 11

Trigonometry

- Trigonometric Ratios
- Trigonometric Equations
- Properties of Triangle
- Radii of Circle

Algebra (Part-I)

- Elementary Mathematics & Logarithm
- Quadratic Equation
- Progressions
- Binomial Theorem
- Permutation & Combination
- Statistics
- Set & Relation

Coordinate Geometry

- Point & Straight Line
- Circle
- Parabola
- Ellipse
- Hyperbola

Note to the Students

Career Point offers this must have Study Package in Physics to meet the complete curriculum needs of engineering aspirants. The set comprises of 3 books. The set caters to the different requirements of students in classes XI. It offers complete and systematic coverage of **JEE Main** and **JEE Advanced** syllabi and aims to provide firm foundation in learning and develop competitive edge in preparation of the JEE and other engineering entrance examinations.

COMPONENTS OF EACH CHAPTER

These books are designed with an engaging and preparation-focused pedagogy and offer a perfect balance of conceptual learning and problem solving skills.

Theory & Concepts

Each chapter consists of high quality theory that covers all the topics, sub-topics and concepts of JEE syllabus.

Quadratic Equation

1. POLYNOMIAL

An algebraic expression of the form $a_0 + a_1x + a_2x^2 + a_3x^3 + ...a_{n-1}x^{n-1} + a_nx^n$ where $a_0, a_1, a_2....a_n$ are constants ($a_n \neq 0$) and n is a positive integer is called n degree polynomial in x.

✦ Real Polynomial

Let $a_0, a_1, a_2.....a_n$ be real numbers and x is a real variable.
Then $f(x) = a_0 + a_1x + a_2x^2 +.....+ a_nx^n$ is called real polynomial of real variable x with real coefficients.
eg. $3x^3 - 4x^2 + 5x - 4$, $x^2 - 2x + 1$ etc. are real polynomials.

✦ Complex Polynomial

If $a_0,a_1,a_2...a_n$ be complex numbers and x is a complex variable.
Then $f(x) = a_0 + a_1x + a_2x^2 + ...a_nx^n$ is called a complex polynomial of complex variable x with complex coefficients.
eg. $3x^2 - (2 + 4i)x + (5i - 4)$, $x^3 - 5ix^2 + (1 + 2i)x + 4$ etc. are complex polynomials.

✦ Degree of Polynomial

Highest Power of variable x in a polynomial is called as a degree of polynomial
e.g. $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + ...a_{n-1}x^{n-1} + a_nx^n$ is n degree polynomial.
$f(x) = 4x^3 + 3x^2 - 7x + 5$ is 3 degree polynomial
$f(x) = 3x - 4$ is single degree polynomial or Linear polynomial
$f(x) = bx$ is odd Linear polynomial
$f(x) = x + \frac{1}{x}$ is not a polynomial in x.

NOTE As a special case a constant is also called Polynomial of degree zero.

✦ Quadratic Polynomial

A polynomial of degree two of the form $ax^2 + bx + c(a \neq 0)$ is called a quadratic polynomial or quadratic expression in x.
e.g $3x^2 + 7x + 5$, $x^2 - 7x + 3$
General form : $f(x) = ax^2 + bx + c$ where $a, b, c \in R$ and $a \neq 0$

2. QUADRATIC EQUATION

A quadratic Polynomial f(x) when equated to zero is called Quadratic Equation.
e.g $3x^2 + 7x + 5 = 0$, $-9x^2 + 7x + 5 = 0$, $x^2 + 2x = 0$, $2x^2 = 0$
General form : $ax^2 + bx + c = 0$ where, $a, b, c \in R$ and $a \neq 0$, the numbers a,b,c are called the coefficients of the equation.

3. ROOTS OF THE QUADRATIC EQUATION

The values of x which satisfy the quadratic equation is called as Roots (also called solutions or zeros) of the Quadratic Equation.

✦ General methods of solving a Quadratic equation

(i) Factorization Method

Let $ax^2 + bx + c = a(x - \alpha)(x - \beta) = 0$
Then $x = \alpha$ and $x = \beta$ will satisfy the given equation
Hence factorize the equation and equating each to zero gives roots of equation.
e.g. $3x^2 - 2x - 1 = 0 \equiv (x - 1)(3x + 1) = 0 \Rightarrow x = 1, -\frac{1}{3}$

(ii) Hindu Method (Sri Dharacharya Method)

By completing the perfect square as

$ax^2 + bx + c = 0 \Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

Adding and substracting $\left(\frac{b}{2a}\right)^2$

$\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}\right] = 0$ which gives,

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Hence the Quadratic equation $ax^2 + bx + c = 0$ ($a \neq 0$) has two roots, given by

$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

NOTE Every quadratic equation has two and only two roots.

Example Based on

Roots of the Quadratic Equation

Example. 1

The roots of the equation $x^2 - 2x - 8 = 0$ are-
(A) $-4, 2$ (B) $4, -2$
(C) $4, 2$ (D) $-4, -2$
Solution.(B)
Quadratic Equation $x^2 - 2x - 8 = 0$
After factorization
$(x - 4)(x + 2) = 0 \Rightarrow x = 4, -2$

Example. 2

The roots of the equation $x^2 - 4x + 1 = 0$ are -
(A) $2 \pm \sqrt{3}$ (B) $2, 4$
(C) $-2 \pm \sqrt{3}$ (D) $\sqrt{3} \pm 2$

Important Points

This part contains important concepts & formulas of chapter at one place in short manner, So that student can revise all these in short time.

Some Important Points

(i) If a polynomial f(x) is divided by x – h, the remainder is f(h) (Remainder Theorem)

(ii) $2[a^2 + b^2 + c^2 + bc - ca - ab] = (b - c)^2 + (c - a)^2 + (a - b)^2$ which is always positive.

(iii) Every equation of n^{th} degree ($n \geq 1$) has less than or equal to n real roots & exactly n complex roots and if the equation has more than n roots, it is an identity.

(iv) If α is a root of the equation f (x) = 0 then the polynomial f (x) is exactly divisible by $(x - \alpha)$ or $(x - \alpha)$ is a factor of f(x)

Solved Examples (JEE Main/Advanced)

To understand the application of concepts, there is a solved example section. It contains large variety of all types of solved examples with explaination to ensure understanding the application of concepts.

SOLVED EXAMPLES

Ex.1 $\frac{8x^2+16x-51}{(2x-3)(x+4)} > 3$ if x is such that -

(A) $x < -4$ (B) $-3 < x < \frac{3}{2}$

(C) $x > \frac{5}{2}$ (D) All three correct

Sol. Consider $\frac{8x^2+16x-51}{(2x-3)(x+4)} - 3 > 0$

$\Rightarrow \frac{2x^2+x-15}{2x^2+5x-12} > 0 \Rightarrow \frac{(2x-5)(x+3)}{(2x-3)(x+4)} > 0$

Hence both Nr and Dr are positive if $x < -4$ or $x > \frac{5}{2}$ and both negative if $-3 < x < \frac{3}{2}$

Hence all the statements are true. **Ans.[D]**

Ex.2 Find all real values of x, which satisfy $x^2 - 3x + 2 > 0$ and $x^2 - 3x - 4 \leq 0$

Sol. If $a < b$, $(x - a)(x - b) > 0 \Rightarrow x < a$; $x > b$ and $(x - a)(x - b) \leq 0 \Rightarrow a \leq x \leq b$

Now $x^2 - 3x + 2 > 0 \Rightarrow (x - 1)(x - 2) > 0$

$\Rightarrow x < 1$; $x > 2$ ... (i)

and $x^2 - 3x - 4 \leq 0 \Rightarrow (x + 1)(x - 4) \leq 0$

$\Rightarrow -1 \leq x \leq 4$ ... (ii)

Combining (i) and (ii) we get real values of x, which satisfy $x^2 - 3x + 2 > 0$ and $x^2 - 3x - 4 \leq 0$ and are given by $-1 \leq x < 1$; $2 < x \leq 4$. **Ans.**

Ex.3 If $a, b, c \in R$, $a \neq 0$ and the quadratic equation $ax^2 + bx + c = 0$ has no real root, then show that $(a + b + c)\, c > 0$.

Sol. Let $f(x) = ax^2 + bx + c$

Since equation $ax^2 + bx + c = 0$ i.e. equation $f(x) = 0$ has no real root, therefore $f(x)$ will have same sign for all real values of x.

$\Rightarrow f(0)$ and f(1) will have same sign.

$\Rightarrow f(1).f(0) > 0 \Rightarrow (a + b + c)\, c > 0$

Subtracting $3\alpha - a = 0 \Rightarrow \alpha = a/3$

Hence $\frac{a^2}{9} - 11\frac{a}{3} + a = 0$, $a = 0$ or $a = 24$

since $a \neq 0$, $a = 24$

$\therefore$ the common factor of $\begin{cases} x^2 - 11x + 24 \\ x^2 - 14x + 48 \end{cases}$

is clearly $x - 8$ **Ans.[C]**

Ex.6 If one root of the equation $x^2 - (p + 1)x + p^2 + p - 8 = 0$ is greater than 2 and the other root is smaller than 2, then p is such that

(A) $-\frac{1}{3} < p < 3$ (B) $-2 < p < 3$

(C) $2 < p < 3$ (D) None of these

Sol. $f(2) = 4 - 2(p + 1) + p^2 + p - 8 < 0$

$\Rightarrow (p - 3)(p + 2) < 0 \Rightarrow -2 < p < 3$

Ans.[B]

Ex.7 The number of real roots of the equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ is

(A) 2 (B) 1

(C) infinite (D) None of these

Sol. Let $e^{\sin x} = y$ then given equation reduces to

$y - \frac{1}{y} - 4 = 0$

$y^2 - 4y - 1 = 0$

$y = \frac{4 \pm \sqrt{20}}{2} = 2 \pm \sqrt{5} = 4.23, -0.23$

But $y = e^{\sin x}$ is never negative.

So $y = e^{\sin x} = 4.23$

$\Rightarrow \sin x = \log 4.23 > 1$

which is not possible. Hence the equation has no real root. **Ans.[D]**

Ex.8 If x is real then the value of the expression $\frac{x^2+14x+9}{x^2+2x+3}$ lies between -

Practice Exercises

Exercise Level - 1 : It contains objective questions with single correct choice to ensure sufficient practice to accutrately appply formule and concepts.

EXERCISE (Level-1)

Question based on **Quadratic Equation & Nature of roots**

Q.1 The roots of the equation $x^2 - 2\sqrt{2}\,x + 1 = 0$ are-
(A) real and different
(B) imaginary and different
(C) real and equal
(D) rational and different

Q.2 The roots of the equation
$(b + c)x^2 - (a + b + c)x + a = 0$
$(a, b, c \in Q,\ b + c \neq a)$ are-
(A) irrational and different
(B) rational and different
(C) imaginary and different
(D) real and equal

Q.3 If the roots of the equation $ax^2 + x + b = 0$ be real and different, then the roots of the equation $x^2 - 4\sqrt{ab}\,x + 1 = 0$ will be-
(A) rational (B) irrational
(C) real (D) imaginary

Q.4 The number of real solution of the equation $\left(\frac{9}{10}\right)^x = -3 + x - x^2$ is-
(A) 1 (B) 2 (C) 0 (D) 3

Q.5 If a < c < b then the roots of the equation
$(a - b)^2x^2 + 2(a + b - 2c)x + 1 = 0$ are-
(A) imaginary
(B) real
(C) one real and one imaginary
(D) equal and imaginary

Q.6 If a, b, c are three distinct positive real numbers then the number of real roots of $ax^2 + 2b|x| - c = 0$ is
(A) 4 (B) 2
(C) 0 (D) None of these

Q.7 The number of real solutions of
$x - \frac{1}{x^2 - 4} = 2 - \frac{1}{x^2 - 4}$ is-
(A) 0 (B) 1 (C) 2 (D) infinite

Q.8 If $x = 2 + 2^{1/3} + 2^{2/3}$, then the value of $x^3 - 6x^2 + 6x$ is-
(A) –2 (B) 3 (C) 4 (D) 2

Q.9 If b and c are odd integers, then the equation $x^2 + bx + c = 0$ has-
(A) two odd roots
(B) two integer roots, one odd and one even
(C) no integer roots
(D) None of these

Q.10 The roots of the quadratic equation
$(a + b - 2c)x^2 - (2a - b - c)x + (a - 2b + c) = 0$ are-
(A) $a + b + c$ & $a - b + c$
(B) $\frac{1}{2}$ and $a - 2b + c$
(C) $a - 2b + c$ & $\frac{1}{a + b - 2c}$
(D) None of these

Q.11 Sum of roots of the equation
$(x + 3)^2 - 4|x + 3| + 3 = 0$ is-
(A) 4 (B) 12 (C) – 12 (D) – 4

Q.12 If roots of the equation
$x^2(1 + m^2) + 2mcx + c^2 - a^2 = 0$ are equal, then value of c is-
(A) $a\sqrt{(1 + m^2)}$ (B) $a\sqrt{(1 - m^2)}$
(C) $m\sqrt{(1 + a^2)}$ (D) $m\sqrt{(1 - a^2)}$

Question based on **Sum and Product of the roots**

Q.13 If α, β are roots of the equation $x^2 + px - q = 0$ & γ, δ are roots of $x^2 + px + r = 0$, then the value of $(\alpha - \gamma)(\alpha - \delta)$ is-
(A) $p + r$ (B) $p - r$
(C) $q - r$ (D) $q + r$

Q.14 If α, β are roots of the equation $2x^2 - 35x + 2 = 0$, then the value of $(2\alpha - 35)^3 . (2\beta - 35)^3$ is equal to-
(A) 1 (B) 8
(C) 64 (D) None of these

Q.15 The value of 'a' for which the sum of the squares of the roots of $2x^2 - 2(a - 2)x - a - 1 = 0$ is least is
(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) –1

Q.16 If α, β are roots of $Ax^2 + Bx + C = 0$ and α^2, β^2 are roots of $x^2 + px + q = 0$ then p is equal to-
(A) $\frac{B^2 - 4AC}{A^2}$ (B) $\frac{2AC - B^2}{A^2}$
(C) $\frac{B^2 - 2AC}{A^2}$ (D) $\frac{4AC - B^2}{A^2}$

Q.17 If α, β are roots of the equation $ax^2 + bx + c = 0$ and $\alpha - \beta = \alpha\beta$, then
(A) $b^2 - 4ac = c^2$
(B) $b^2 - 4ac = a^2$
(C) $a(b^2 + 4ac) = 2c$
(D) $b^2 + 4ac = a$

Q.18 If $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$ then the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is
(A) $\frac{19}{3}$ (B) $\frac{25}{3}$
(C) $-\frac{19}{3}$ (D) None of these

Exercise Level - 2 : It contains single objective type questions with moderate difficulty level to enhance the conceptual and application level of the student.

EXERCISE (Level-2)

Single correct answer type questions

Q.1 If the difference of the roots is equal to the product of the roots of the equation $2x^2 - (a+1)x + (a-1) = 0$ then the value of a is-
(A) 2 (B) 3 (C) 4 (D) 5

Q.2 If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of the square of their reciprocal, then-
(A) c^2b, a^2c, b^2a are in A.P.
(B) c^2b, a^2c, b^2a are in G.P.
(C) $\frac{b}{c}, \frac{a}{b}, \frac{c}{a}$ are in H.P.
(D) $\frac{b}{c}, \frac{a}{b}, \frac{c}{a}$ are in G.P.

Q.3 If x is real, then the values of the expression $\frac{(x+m)^2 - 4mn}{2(x-n)}$ are not -
(A) greater than $(m + n)$
(B) greater than $(m + 2n)$
(C) between $2m$ and $2n$
(D) between m and $m + n$

Q.4 If roots of the equation $ax^2 + bx + c = 0$ are $\frac{\alpha}{\alpha-1}$ and $\frac{\alpha+1}{\alpha}$, then $(a + b + c)^2$ equals-
(A) $2b^2 - ac$ (B) $b^2 - ac$
(C) $b^2 - 4ac$ (D) $4b^2 - 2ac$

Q.5 If the roots of the equation $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ are equal in magnitude but opposite in sign, then their product is -
(A) $\frac{1}{2}(a^2 + b^2)$ (B) $\frac{-1}{2}(a^2 + b^2)$
(C) $\frac{1}{2}ab$ (D) $\frac{-1}{2}ab$

Q.6 If $(\lambda^2 + \lambda - 2)x^2 + (\lambda + 2)x < 1$ for all $x \in R$, then λ belong to interval.
(A) $\left[-2, \frac{2}{5}\right)$ (B) $(-2, 1)$ (C) $\left(\frac{2}{5}, 1\right)$ (D) $\left(-2, \frac{2}{5}\right)$

Q.7 The roots of the equation $\log_2 (x^2 - 4x + 5) = (x - 2)$ are -
(A) 4, 5 (B) 2, –3 (C) 2, 3 (D) 3, 5

Q.8 If $ax^2 + bx + 6 = 0$ does not have two distinct real roots, where $a \in R$, $b \in R$, then the least value of $3a + b$ is-
(A) –2 (B) –1 (C) 4 (D) 1

Q.9 If the roots of the equation $x^2 - 5x + 16 = 0$ are α, β and the roots of the equation $x^2 + px + q = 0$ are $(\alpha^2 + \beta^2)$ and $\frac{\alpha\beta}{2}$, then-
(A) $p = 1$ and $q = 56$ (B) $p = 1$ and $q = -56$
(C) $p = -1$ and $q = 56$ (D) $p = -1$ and $q = -56$

Q.10 The value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other, is-
(A) $-\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $-\frac{2}{3}$ (D) $\frac{1}{3}$

Q.11 If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$ then its roots are-
(A) 0, 1 (B) – 1, 1 (C) 0, – 1 (D) – 1, 2

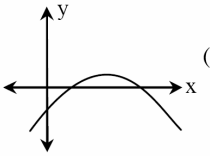
Q.12 The number of values of a for which $(a^2 - 3a + 2)x^2 + (a^2 - 5a + 6)x + a^2 - 4 = 0$ is an identity in x is -
(A) 0 (B) 2 (C) 1 (D) 3

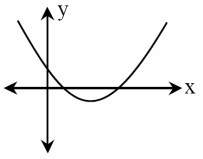
Q.13 Let α, β be the roots of the equation $ax^2 + 2bx + c = 0$ and γ, δ be the roots of the equation $px^2 + 2qx + r = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then-
(A) $q^2 ac = b^2 pr$ (B) $qac = bpr$
(C) $c^2 pq = r^2 ab$ (D) $p^2 ab = a^2 qr$

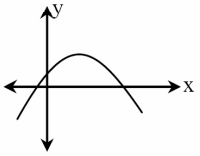
Q.14 The number of real solutions of the equation $(15 + 4\sqrt{14})^t + (15 - 4\sqrt{14})^t = 30$ are, where $t = x^2 - 2|x|$
(A) 0 (B) 2 (C) 4 (D) 6

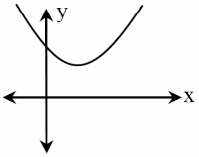
Q.15 If α, β, γ are the roots of the equation $x^3 - x - 1 = 0$, then the value of $\sum\left(\frac{1+\alpha}{1-\alpha}\right)$ is -
(A) –3 (B) –5
(C) –7 (D) None of these

Q.16 Graph of the function $f(x) = Ax^2 - Bx + C$, where $A = (\sec\theta - \cos\theta)(\operatorname{cosec}\theta - \sin\theta)(\tan\theta + \cot\theta)$, $B = (\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 - (\tan^2\theta + \cot^2\theta)$ & $C = 12$, is represented by

(A) 

(B) 

(C) 

(D) 

Q.17 If the roots of the quadratic equation $x^2 + 6x + b = 0$ are real and distinct and they differ by atmost 4 then the least value of b is-
(A) 5 (B) 6 (C) 7 (D) 8

Q.18 If p and q are distinct reals, then $2\{(x-p)(x-q) + (p-x)(p-q) + (q-x)(q-p)\} = (p-q)^2 + (x-p)^2 + (x-q)^2$ is satisfied by-
(A) no value of x
(B) exactly one value of x
(C) exactly two value of x
(D) infinite values of x

Exercise Level - 3 : It contains previous years question of JEE Main from Year 2005 to 2024.

EXERCISE (Level-3)

Old Examination Questions [JEE Main]

Q.1 The value of a for which the sum of the squares of the roots of the equation $x^2-(a-2)x-a-1=0$ assume the least value is - **[AIEEE-2005]**
(A) 1 (B) 0 (C) 3 (D) 2

Q.2 If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals – **[AIEEE-2005]**
(A) –2 (B) 3 (C) 2 (D) 1

Q.3 In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$, $a \neq 0$ then - **[AIEEE-2005]**
(A) $a = b + c$ (B) $c = a + b$
(C) $b = c$ (D) $b = a + c$

Q.4 If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval **[AIEEE-2005]**
(A) (5, 6] (B) (6, ∞) (C) (–∞, 4) (D) [4, 5]

Q.5 If the roots of the quadratic equation $x^2 + px + q = 0$ are tan 30° and tan 15°, respectively then the value of $2 + q - p$ is - **[AIEEE-2006]**
(A) 3 (B) 0 (C) 1 (D) 2

Q.6 All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than –2 but less than 4, lie in the interval – **[AIEEE-2006]**
(A) $m > 3$ (B) $-1 < m < 3$
(C) $1 < m < 4$ (D) $-2 < m < 0$

Q.7 If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is - **[AIEEE-2006]**
(A) 41 (B) 1 (C) $\frac{17}{7}$ (D) $\frac{1}{4}$

Q.8 If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is- **[AIEEE-2007]**
(A) (–3, 3) (B) (–3, ∞) (C) (3, ∞) (D) (– ∞, – 3)

Q.9 The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is **[AIEEE-2008]**
(A) 4 (B) 3 (C) 2 (D) 1

Q.10 If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is- **[AIEEE-2009]**
(A) greater than $4ab$ (B) less than $4ab$
(C) greater than $-4ab$ (D) less than $-4ab$

Q.11 If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009}$ = **[AIEEE-2010]**
(A) – 2 (B) – 1 (C) 1 (D) 2

Q.12 If $a, b, c, d \in R$ and 1 is a root of the equation $ax^2+bx+c=0$, then the curve $y=4ax^2+3bx+2c$, $a \neq 0$, intersects x-axis at : **[AIEEE Online - 2012]**
(A) no point
(B) exactly two distinct point
(C) exactly one point
(D) two distinct points whose coordinates are always rational numbers

Q.13 The value of k for which the equation $(k - 2)x^2 + 8x + k + 4 = 0$ has both roots real, distinct and negative is **[AIEEE Online - 2012]**
(A) 4 (B) 3 (C) 6 (D) 1

Q.14 If the sum of the square of the roots of the equation $x^2 - (\sin\alpha - 2)x - (1 + \sin\alpha) = 0$ is least, then α is equal to : **[AIEEE Online - 2012]**
(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

Q.15 If the equations $x^2+2x+3=0$ and $ax^2+bx+c=0$, $a, b, c \in R$, have a common root, then $a : b : c$ is **[JEE Main - 2013]**
(A) 1 : 3 : 2 (B) 3 : 1 : 2
(C) 1 : 2 : 3 (D) 3 : 2 : 1

Q.16 If α and β are roots of the equation $x^2 + px + \frac{3p}{4} = 0$, such that $|\alpha - \beta| = \sqrt{10}$, then p belongs to the set **[JEE Main Online - 2013]**
(A) {2, –5} (B) {–3, 2} (C) {–2, 5} (D) {3, –5}

Q.17 The least integral value α of x such that $\frac{x-5}{x^2+5x-14} > 0$, satisfies : **[JEE Main Online - 2013]**
(A) $\alpha^2 + 3\alpha - 4 = 0$ (B) $\alpha^2 - 5\alpha + 4 = 0$
(C) $\alpha^2 - 7\alpha + 6 = 0$ (D) $\alpha^2 + 5\alpha - 6 = 0$

Q.18 If p and q are non-zero real numbers and $\alpha^3 + \beta^3 = -p$, $\alpha\beta = q$, then a quadratic equation whose roots are $\frac{\alpha^2}{\beta}, \frac{\beta^2}{\alpha}$ is: **[JEE Main Online - 2013]**
(A) $px^2 - qx + p^2 = 0$ (B) $qx^2 + px + q^2 = 0$
(C) $px^2 + qx + p^2 = 0$ (D) $qx^2 - px + q^2 = 0$

Q.19 The values of 'a' for which one root of the equation $x^2 - (a + 1)x + a^2 + a - 8 = 0$ exceeds 2 and the other is lesser than 2, are given by : **[JEE Main Online - 2013]**
(A) $3 < a < 10$ (B) $a \geq 10$
(C) $-2 < a < 3$ (D) $a \leq -2$

Exercise Level - 4 : It contains all variety of questions as per level of JEE Advanced such as MCQ, Column match, Passage based & Numerical type etc.

EXERCISE (Level-4)

Part-A : Multiple correct answer type questions

Q.1 The roots of the equation, $(x^2 + 1)^2 = x(3x^2 + 4x + 3)$, are given by-

(A) $2 - \sqrt{3}$ (B) $\frac{-1+i\sqrt{3}}{2}$

(C) $2 + \sqrt{3}$ (D) $\frac{-1-i\sqrt{3}}{2}$

Q.2 The set of values of p for which $(p-2)x^2 + 7x + p^2 - 4p = 0$ has roots of opposite signs are-

(A) $0 < p < 2$ (B) $2 < p < 4$

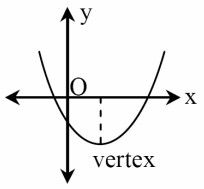
(C) $p < 0$ (D) $0 < p < 4$

Q.3 Let $x^2 - px + q = 0$, where $p \in R$, $q \in R$ have the roots α, β such that $\alpha + 2\beta = 0$ then-

(A) $2p^2 + q = 0$ (B) $2q^2 + p = 0$

(C) $q < 0$ (D) None of these

Q.4 Graph of $y = ax^2 + bx + c = 0$ is given adjacently. What conclusions can be drawn from this graph-



(A) $a > 0$ (B) $b < 0$

(C) $c < 0$ (D) $b^2 - 4ac > 0$

Q.5 Equation $2x^2 - 2(2a+1)x + a(a+1) = 0$ has one root less than 'a' and other root greater than 'a', if

(A) $0 < a < 1$ (B) $-1 < a < 0$

(C) $a > 0$ (D) $a < -1$

Q.6 If $\left(\sqrt{(49+20\sqrt{6})}\right)^{\sqrt{a\sqrt{a\sqrt{a.....\infty}}}} + \left(5-2\sqrt{6}\right)^{x^2+x-3-\sqrt{x\sqrt{x\sqrt{x.....\infty}}}} = 10$

where $a = x^2 - 3$, then x is-

(A) $-\sqrt{2}$ (B) $\sqrt{2}$ (C) -2 (D) 2

Q.7 The correct statement is/are-

(A) If x_1 and x_2 are roots of the equation $2x^2 - 6x - b = 0$; $(b > 0)$ then $\frac{x_1}{x_2} + \frac{x_2}{x_1} < -2$

(B) Equation $ax^2 + bx + c = 0$ has real roots if $a < 0$ and $c > 0$

(C) If $P(x) = ax^2 + bx + c$ & $Q(x) = -ax^2 + bx + c$, where $ac \neq 0$ then $P(x).Q(x)$ has at least two real roots.

(D) If both the roots of the equation $(3a+1)x^2 - (2a+3b)x + 3 = 0$ are infinite then $a = 0$ and $b \in R$.

Q.8 If $\alpha_1 < \alpha_2 < \alpha_3 < \alpha_4 < \alpha_5 < \alpha_6$, then the equation $(x-\alpha_1)(x-\alpha_3)(x-\alpha_5) + 3(x-\alpha_2)(x-\alpha_4)(x-\alpha_6) = 0$ has

(A) three real roots

(B) no real root in $(-\infty, \alpha_1)$

(C) one real root in (α_1, α_2)

(D) no real root in (α_5, α_6)

Q.9 If equations $(a+2)x^2 + bx + c = 0$ & $2x^2 + 3x + 4 = 0$ have a common root where $a, b, c \in N$, then-

(A) $b^2 - 4ac < 0$

(B) minimum value of $a + b + c$ is 16

(C) $b^2 < 4ac + 8c$

(D) minimum value of $a + b + c = 7$

Q.10 If one of the roots of $x^2 - bx + c = 0$; $b, c \in Q$ is $\sqrt{7-4\sqrt{3}}$ then-

(A) $\log_b c = 0$ (B) $b + c = 5$

(C) $\log_c b = 0$ (D) $bc = -4$

Q.11 If α, β and γ are the roots of the equation $x^3 - 3x + 1 = 0$ then

(A) $\Pi(\alpha + 1) = -3$ (B) $\Sigma(\alpha + 1) = 0$

(C) $\Sigma(\alpha + 1)(\beta + 1) = -3$ (D) $\Sigma\alpha^2 = 6$

Q.12 If the expression $ax^4 + bx^3 - x^2 + 2x + 3$ leaves remainder $4x + 3$ when divided by $(x^2 + x - 2)$ then

(A) $a^2 + b^2 = 5$ (B) $2b - a = 3$

(C) $4a + b = 9$ (D) $a - b = 1$

Q.13 The integral value(s) of 'a' for which the roots of $ax(x+2) + a - (x+2) = 0$ are rational is given by

(A) 2 (B) 5 (C) 12 (D) 20

Q.14 If both roots of the equation $4x^2 - 2x + k = 0$ are completely in $(-1, 1)$, then k may be equal to -

(A) -1 (B) 0 (C) 2 (D) -3

Q.15 If $(\ell n\ \alpha)x^2 - 2x\ \ell n(e\alpha) + \ell n(e^4\alpha) \geq 0\ \forall\ x \in R$ (where $\alpha > 1$), then value of α can be

(A) $\sqrt{e}$ (B) e (C) $\frac{3}{2}$ (D) 2

Part-B : Assertion Reason type Questions

The following questions 16 to 19 consists of two statements each, printed as Statement (1) & Statement (2). While answering these questions you are to choose any one of the following four responses.

(A) If both Statement (1) and Statement (2) are true & the Statement (2) is correct explanation of the Statement (1).

(B) If both Statement (1) and Statement (2) are true but Statement (2) is not correct explanation of the Statement (1).

(C) If Statement (1) is true but the Statement (2) is false.

(D) If Statement (1) is false but Statement (2) is true

Q.16 **Statement (1) :** If a and b are integers and roots of $x^2 + ax + b = 0$ are rational then they must be integers.

Statement (2): If the coefficient of x^2 in a quadratic equation is unity then its roots must be integers

Exercise Level - 5 : It contains previous years question of JEE Advanced from Year 2005 to 2024.

EXERCISE (Level-5)

Old Examination Questions [JEE Advanced]

Q.1 If roots of $x^2 - 10cx - 11d = 0$ are a, b and the roots of $x^2 - 10ax - 11b = 0$ are c, d, then the value of $a + b + c + d$ is equal to (a, b, c, d are different numbers) ..... **[IIT-2006]**

Q.2 Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}$, 2β be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is - **[IIT-2007]**

(A) $\frac{2}{9}(p-q)(2q-p)$

(B) $\frac{2}{9}(q-p)(2p-q)$

(C) $\frac{2}{9}(q-2p)(2q-p)$

(D) $\frac{2}{9}(2p-q)(2q-p)$

Q.3 The smallest value of k, for which both the roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real and have values at least 4, is **[IIT-2009]**

(A) 1 (B) 2

(C) 0 (D) 4

Q.4 Let p & q be real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -q$. If α and β are non zero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is - **[IIT-2010]**

(A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$

(B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

(C) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$

(D) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

Q.5 Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is **[IIT 2011]**

(A) 1 (B) 2

(C) 3 (D) 4

Q.6 A value of b for which the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$, have one root in common is **[IIT 2011]**

(A) $-\sqrt{2}$ (B) $-i\sqrt{3}$

(C) $i\sqrt{5}$ (D) $\sqrt{2}$

Q.7 Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S ? **[JEE Advanced -2015]**

(A) $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$ (B) $\left(-\frac{1}{\sqrt{5}}, 0\right)$

(C) $\left(0, \frac{1}{\sqrt{5}}\right)$ (D) $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

Q.8 Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x\sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x\tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals **[JEE Advanced -2016]**

(A) $2(\sec\theta - \tan\theta)$ (B) $2\sec\theta$

(C) $-2\tan\theta$ (D) 0

Passage (Q.9 to 10)

Let p, q be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For $n = 0, 1, 2, \ldots\ldots$, let $a_n = p\alpha^n + q\beta^n$.

FACT : If a and b are rational numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$. **[JEE Advanced -2017]**

Q.9 If $a_4 = 28$, then $p + 2q =$

(A) 14 (B) 12

(C) 7 (D) 21

Q.10 $a_{12} =$

(A) $a_{11} - a_{10}$ (B) $a_{11} + a_{10}$

(C) $a_{11} + 2a_{10}$ (D) $2a_{11} + a_{10}$

Exercise Level - 6 : Advanced level a bit complex questions for students for solid rock prepertion for Top Rankers.

EXERCISE (Level-6)

Review Exercise

Q.1 Let a, b, c be real numbers with $a \neq 0$ and α, β be the roots of the equation $ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β. **[IIT-2001]**

(A) $\alpha\beta^2$ (B) $\alpha^2\beta$
(C) both (A) and (B) (D) None of these

Q.2 If one root of the equation $x^2 + px + q = 0$ is square of the other then for any p & q, it will satisfy the relation- **[IIT Sc.-2004]**

(A) $p^3 - q\,(3p - 1) + q^2 = 0$
(B) $p^3 - q\,(3p + 1) + q^2 = 0$
(C) $p^3 + q\,(3p - 1) + q^2 = 0$
(D) $p^3 + q\,(3p + 1) + q^2 = 0$

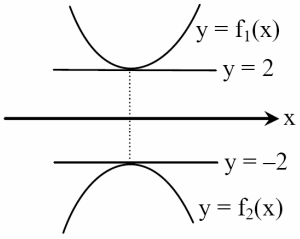
Passage (Q. 3 to 5)

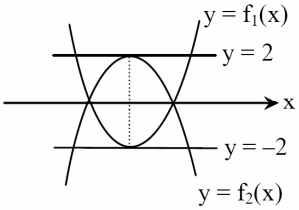
Let $f_1(x) = a_1x^2 + b_1x + c_1$, $f_2(x) = a_2x^2 + b_2x + c_2$ be quadratic functions with real coefficients. Sum of roots of $f_1(x) = 0$ is equal to sum of roots of $f_2(x) = 0$. Range of $y = f_1(x)$ can be $[2, \infty)$ or $[-2, \infty)$. Range of $y = f_2(x)$ can be $(-\infty, -2]$ or $(-\infty, 2]$

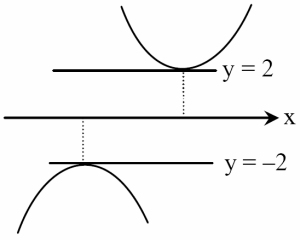
Q.3 If arithmetic mean of roots of $f_1(x)$ $f_2(x) = 0$ is equal to 1, then

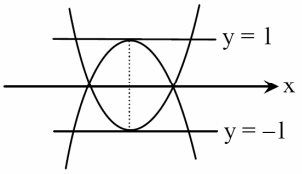
(A) $b_1 + 2a_1 = 0$, $b_2 + 2a_2 \neq 0$
(B) $b_1 + 2a_1 \neq 0$, $b_2 + 2a_2 = 0$
(C) $b_1 + 2a_1 = 0$, $b_2 + 2a_2 = 0$
(D) $a_1b_2 + a_2b_1 = 4a_1a_2$

Q.4 Which of the following can be possible graphs of $y = f_1(x)$ and $y = f_2(x)$

(i)


(ii)


(iii)


(iv)


(A) (i), (ii) (B) (i), (ii), (iii)
(C) (i), (ii), (iv) (D) (i), (ii), (iii), (iv)

Q.5 If $y = f_2(x)$ passes through (1, –2) and $f_1(x) = 0$ has a negative root then

(A) $a_2c_2 < 0$ (B) $a_1c_1 < 0$
(C) $b_1c_1 < 0$ (D) $b_2c_2 > 0$

Q.6 Solution of

$$(2+\sqrt{3})^{x^2-2x+1} + (2-\sqrt{3})^{x^2-2x-1} = \frac{4}{2-\sqrt{3}} \text{ are}$$

(A) $1 \pm \sqrt{3}$, 1 (B) $1 \pm \sqrt{2}$, 1
(C) $1 \pm \sqrt{3}$, 2 (D) $1 \pm \sqrt{2}$, 2

Q.7 For $a \leq 0$, determine all real roots of the equation $x^2 - 2a\,|x - a| - 3a^2 = 0$ **[IIT-1986]**

Q.8 The real values of λ for which the equation, $4x^3 + 3x^2 - 6x + \lambda = 0$ has two distinct real roots in [0, 1] lie in the interval-

(A) $(0, \infty)$ (B) $(3, \infty)$
(C) $\left(-5, \frac{7}{4}\right)$ (D) $\left[0, \frac{7}{4}\right)$

Q.9 If a, b, c, d and p are distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \leq 0$ then show that a, b, c, d are in G.P. **[IIT-1987]**

Q.10 Solve $|x^2 + 4x + 3| + 2x + 5 = 0$. **[IIT-1988]**

Q.11 Show that the equation $x^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}} = \sqrt{2}$ has exactly three solutions. **[IIT-1989]**

Q.12 Let a, b, c be real numbers, $a \neq 0$. If α is a root of $a^2x^2 + bx + c = 0$. β is the root of $a^2x^2 - bx - c = 0$ & $0 < \alpha < \beta$, then show that the equation $a^2x^2 + 2bx + 2c = 0$ has a root γ that always satisfies $\alpha < \gamma < \beta$. **[IIT-1989]**

Q.13 If α & β are the roots of $x^2 + px + q = 0$ and α^4, β^4 are the roots of $x^2 - rx + s = 0$, then show that the equation $x^2 - 4qx + 4q^2 - r = 0$ has always two real roots. **[IIT-1989]**

Answer key

Ankswer key is provided at the end of the exercise sheets.

ANSWER KEY

EXERCISE (Level-1)

1. (A)	**2.** (B)	**3.** (D)	**4.** (C)	**5.** (A)	**6.** (B)	**7.** (A)	**8.** (D)	**9.** (C)	**10.** (D)
11. (C)	**12.** (A)	**13.** (D)	**14.** (C)	**15.** (B)	**16.** (B)	**17.** (A)	**18.** (A)	**19.** (C)	**20.** (B)
21. (D)	**22.** (A)	**23.** (D)	**24.** (B)	**25.** (B)	**26.** (D)	**27.** (B)	**28.** (D)	**29.** (C)	**30.** (C)
31. (A)	**32.** (C)	**33.** (A)	**34.** (A)	**35.** (C)	**36.** (C)	**37.** (B)	**38.** (A)		

Revision Plan

We emphasis that every student should prepare his/her own revision plan. For this purpose there is Revision Plan Section in each chapter which student should prepare while going thorugh the study material. This will be useful at the time of final revision before final exam for quick & effective revision.

Revision Plan

Prepare Your Revision plan today!

After attempting Exercise Sheet, please fill below table as per the instruction given.

A. Write Question Number (QN) which you are unable to solve at your own in **column A**.

B. After discussing the Questions written in **column A** with faculty, strike off them in the manner so that you can see at the time question number during Revision, to solve such questions again.

C. Write down the Question Number you feel are important or good in the **column B**.

EXERCISE	COLUMN A	COLUMN B
	Questions unable to solve in first attempt	Good or Important questions
Topic wise practice questions		
Level-1		
Level-2		
Level-3		
Level-4		
Level-5		
Level-6		

Revision Strategy:

Whenever you wish to revision this chapter, follow the following steps-

Step-1: Review your theory notes.

Step-2: Solve Questions of column A

Online Solutions

Self explanatory and detailed soltuion of all excercises above are available on Career Point website www.careerpoint.ac.in

QUADRATIC EQUATION

EXERCISE (Level-1)

Answer Key & Solution

Question Number	Solution
1	Click Here
2	Click Here
3	Click Here
4	Click Here
5	Click Here
6	Click Here
7	Click Here
8	Click Here
9	Click Here
10	Click Here

Question Number	Solution
11	Click Here
12	Click Here
13	Click Here
14	Click Here
15	Click Here
16	Click Here
17	Click Here
18	Click Here
19	Click Here
20	Click Here

Question Number	Solution
21	Click Here
22	Click Here
23	Click Here
24	Click Here
25	Click Here
26	Click Here
27	Click Here
28	Click Here
29	Click Here
30	Click Here

Question Number	Solution
31	Click Here
32	Click Here
33	Click Here
34	Click Here
35	Click Here
36	Click Here
37	Click Here
38	Click Here

Sol.1 **[A]**

$$\text{Roots} = \frac{2\sqrt{2} \pm \sqrt{8-4}}{2}$$

$$\sqrt{2} \pm 1$$

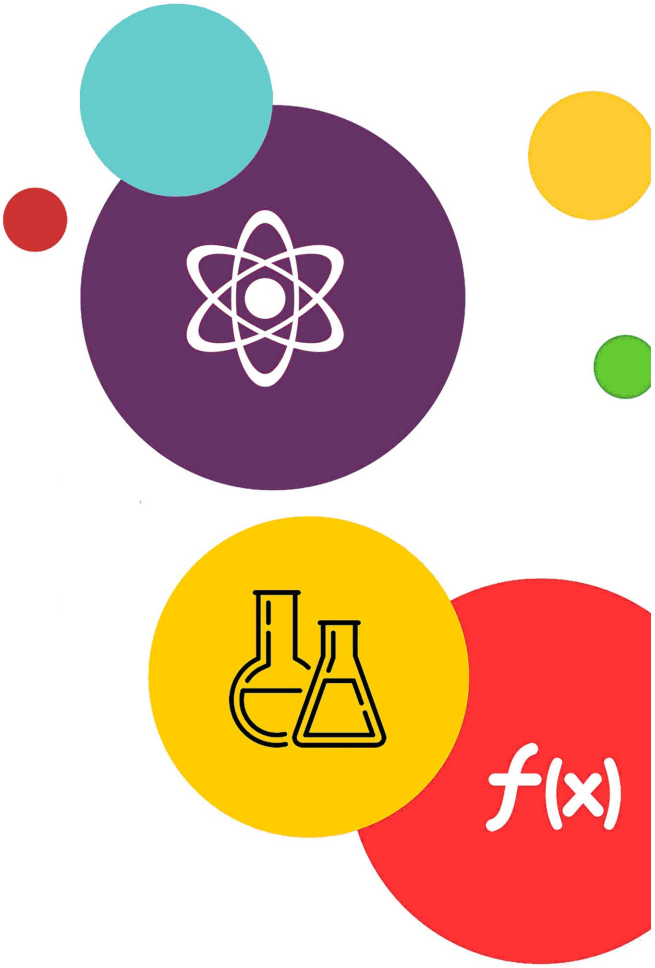
Roots are real and different

Top

STUDY MATERIAL

JEE

FOR MAIN & ADVANCED



PHYSICS

Class 11

CP PUBLICATION

PHYSICS

Study Material for JEE Main & Advanced preparation

Prepared by Career Point Kota Experts

CAREER POINT

CONTENTS OF THE PACKAGE AT A GLANCE

PHYSICS

Class 11

Mechanics (Part-I)

- Essential Mathematics & Vector
- Unit & Dimension
- Motion in One dimension
- Projectile motion
- Laws of motion
- Friction

Mechanics (Part-II)

- Circular Motion
- Work, Power, Energy
- Conservation Laws
- Rotational motion
- Gravitation
- S.H.M.
- Properties of matter
- Fluid Mechanics

Heat & Wave

- Calorimetry
- K.T.G.
- Thermodynamics
- Heat Transfer
- Thermal expansion
- Transverse wave
- Sound wave
- Doppler's Effect

Note to the Students

Career Point offers this must have Study Package in Physics to meet the complete curriculum needs of engineering aspirants. The set comprises of 3 books. The set caters to the different requirements of students in classes XI. It offers complete and systematic coverage of **JEE Main** and **JEE Advanced** syllabi and aims to provide firm foundation in learning and develop competitive edge in preparation of the JEE and other engineering entrance examinations.

COMPONENTS OF EACH CHAPTER

These books are designed with an engaging and preparation-focused pedagogy and offer a perfect balance of conceptual learning and problem solving skills.

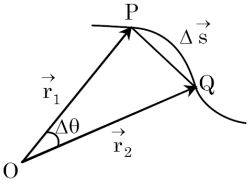
Theory & Concepts

Each chapter consists of high quality theory that covers all the topics, sub-topics and concepts of JEE syllabus.

Rotational Motion

1. ANGULAR DISPLACEMENT

(a) When a particle moves in a curved path, the angle subtended by its position vector about a fixed point is known as angular displacement.

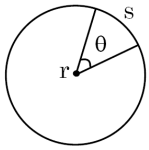


(b) unit : radian.

(c) dimension : $M^0L^0T^0$.

(d) It is a vector quantity whose direction is given by right hand screw rule.

(e) If θ be the angular displacement of a particle moving in a circular path of radius r and s be the length of arc, $s = r\theta$



NOTE $360^\circ = 2\pi$ radian

2. ANGULAR VELOCITY

(a) The angular displacement per unit time is defined as angular velocity. If a particle moves from P to Q in time Δt, $\omega = \frac{\Delta\theta}{\Delta t}$, where $\Delta\theta$ is the angular displacement during the time interval Δt.

If $\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$

= Instantaneous angular velocity.

$\omega = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$

= average angular velocity.

(b) Unit : Radian /sec

(c) Dimension : $[M^0L^0T^{-1}]$, which is same as that of frequency.

(d) It is a vector quantity, whose direction is normal to the rotational plane & its direction is given by right hand screw rule.

(e) If w be the angular velocity, v be the linear velocity & r be the radius of path, we have the following relation.

$\vec{v} = \vec{\omega} \times \vec{r}$ or $v = r\omega$.

(f) If f be the frequency, $\omega = 2\pi f$

If T be the time period, $\omega = \frac{2\pi}{T}$

(g) The angular velocity of a rotating rigid body can be either positive or negative, depending on whether body is rotating in the direction of increasing θ (anticlockwise) or decreasing θ (clockwise).

(h) The magnitude of an angular velocity is called the angular speed which is also represented by ω,

✦ Equation of Linear Motion and Rotational Motion :

Linear Motion	Rotational Motion
(a) If acceleration is 0, v = constant and s = vt.	(a) If acceleration is 0, ω = constant and $\theta = \omega t$.
(b) If acceleration a = constant, (i) $s = \frac{(u+v)}{2}t$ (ii) $a = \frac{v-u}{t}$ (iii) $v = u + at$ (iv) $s = ut + (1/2)at^2$ (v) $v^2 = u^2 + 2as$ (vi) $S_{nth} = u + \frac{1}{2}a(2n-1)$	(b) If acceleration a = constant then (i) $\theta = \frac{(\omega_1+\omega_2)}{2}t$ (ii) $\alpha = \frac{\omega_2-\omega_1}{t}$ (iii) $\omega_2 = \omega_1 + \alpha t$ (iv) $\omega_2 = \omega_1 t + \frac{1}{2}\alpha t^2$ (v) $\omega_2^2 = \omega_1^2 + 2\alpha\theta$. (vi) $\theta_{nth} = \omega_1 + (2n-1)\frac{\alpha}{2}$
(c) If acceleration is not constant, the above equation will not be applicable. In this case (i) $v = \frac{dx}{dt}$ (ii) $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$ (iii) $vdv = ads$	(c) If acceleration is not constant, the above equation will not be applicable. In this case (i) $\omega = \frac{d\theta}{dt}$ (ii) $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ (iii) $\omega d\omega = \alpha d\theta$

Example Based on

Angular velocity

Example. 1

A wheel of mass 6 kg is rotating at 300 rpm. Its angular velocity will be –

(A) 31.4 rad/sec (B) 3.14 rad/sec

(C) 0.314 rad/sec (D) 0.0314 rad/sec

Solution. (A)

Here, $\omega = \frac{2\pi n}{t} = \frac{2\times3.14\times300}{60} = 31.4$ rad/sec

Important Points

This part contains important concepts & formulas of chapter at one place in short manner, So that student can revise all these in short time.

Important points

(a) if a force is parallel to a given axis then component of its torque along the given axis will be zero.

(b) torque can be calculated by using component of force

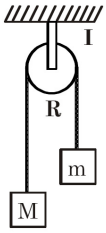
(c) if line of action of force passes through the axis of rotation then, force can not produce any torque about the given axis.

Solved Examples (JEE Main/Advanced)

To understand the application of concepts, there is a solved example section. It contains large variety of all types of solved examples with explaination to ensure understanding the application of concepts.

SOLVED EXAMPLES

Ex.1 The pulley shown in figure has a moment of inertia I about its axis and its radius is R. Find the magnitude of the acceleration of the two blocks. Assume that the string is light and does not slip on the pulley.



Sol. Suppose the tension in the left string is T_1 and that in the right string is T_2. Suppose the block of mass M goes down with an acceleration a and the other block moves up with the same acceleration. This is also the tangential acceleration of the rim of the wheel as the string does not slip over the rim. The angular acceleration of the wheel is, therefore, $\alpha = a/R$. The equations of motion for the mass M, the mass m and the pulley are as follows.

$$Mg - T_1 = Ma \quad \text{....(i)}$$

$$T_2 - mg = ma \quad \text{.... (ii)}$$

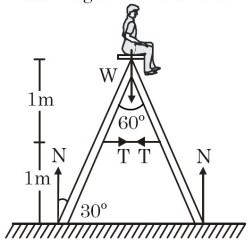
$$T_1R - T_2R = I\alpha = I\,a/R \quad \text{.... (iii)}$$

Putting T_1 and T_2 from (i) and (ii) into (iii),

$$[M(g-a) - m(g+a)]\,R = I\frac{a}{R}$$

which gives $$a = \frac{(M-m)gR^2}{I+(M+m)R^2}$$

Ex.2 The ladder shown in figure has negligible mass and rests on a frictionless floor. The crossbar connects the two legs of the ladder at the middle. The angle between the two legs is 60°. The fat person sitting on the ladder has a mass of 80 kg. Find the contact force exerted by the floor on each leg and the tension in the crossbar.

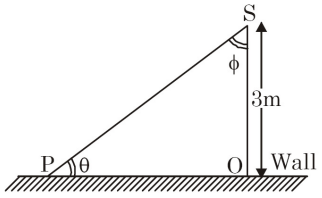


N (2m) tan 30°= T(1m)

or $T = N\frac{2}{\sqrt{3}} = (392\text{ N}) \times \frac{2}{\sqrt{3}} = 450\text{ N}.$

Ex.3 A spot light S rotates in a horizontal plane with a constant angular velocity of 0.1 rad/s. The spot of light P moves along, the wall at a distance 3m. What is the velocity of the spot P when $\theta = 45°$?

Sol. If x is the distance of point P from O then from fig.



$\tan\phi = (x/h)$

or $x = h\tan\phi$

or $\frac{dx}{dt} = h(\sec^2\phi)\frac{d\phi}{dt}$

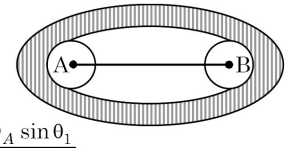
i.e. $v = h\sec^2\phi\,\omega$ [as $(dx/dt) = v$ and $(d\phi/dt) = \omega$]

Here $h = 3m, \phi = 180 - (45+90) = 45°$

and $\omega = 0.1$ rad/sec.

so $v = 3 \times (\sqrt{2})^2 \times 0.1 = 0.6$ m/s.

Ex.4 Two particles A and B are moving as shown in fig. At this moment of time the angular speed of A with respect to B is -



(A) $\frac{v_A + v_B}{r}$

(B) $\frac{v_A - v_B}{r}$

(C) $\frac{v_B\sin\theta_2 - v_A\sin\theta_1}{r}$, in anticlockwise direction

(D) $\frac{v_B\sin\theta_2 + v_A\sin\theta_1}{r}$, in anticlockwise direction

Sol.(C) Resolving the velocities along x and y axes, we have,

Practice Exercises

Exercise Level - 1 : It contains objective questions with single correct choice to ensure sufficient practice to accutrately appply formule and concepts.

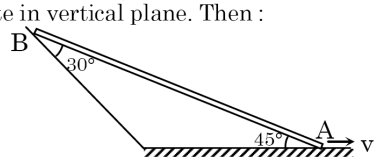
EXERCISE (Level-1)

Questions based on Rotational Kinematics

Q.1 If the angular velocity vector of a spinning body points out of the page then, when viewed from above the page, the body is spinning :
(A) clockwise about an axis that is perpendicular to the page
(B) counterclockwise about an axis that is perpendicular to the page
(C) about an axis that is parallel to the page
(D) about an axis that is changing orientation

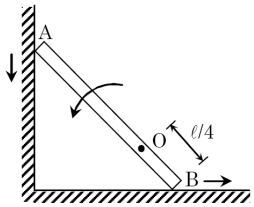
Q.2 A wheel starts from rest and spins with a constant angular acceleration. As time goes on the acceleration vector for a point on the rim:
(A) decreases in magnitude and becomes more nearly tangent to the rim
(B) decreases in magnitude and becomes more nearly radial
(C) increases in magnitude and becomes more nearly tangent to the rim
(D) increases in magnitude and becomes more nearly radial

Q.3 The end A of a uniform rod is pulled with a constant velocity v. In consequence, the rod rotate in vertical plane. Then :



(A) $v_A = \sqrt{\frac{2}{3}}\, v$ (B) $v_C = \frac{v}{2}\left(\sqrt{\frac{2}{3}}+1\right)$
(C) $a_A = 0$ (D) $a_B = 0$

***Q.4** A rod of length ℓ rotates in the vertical plane under the constraint of horizontal & vertical surface. The path of the point O on it is :



(A) circle (B) an ellipse
(C) hyperbola (D) spiral

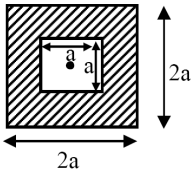
Questions based on Moment of Inertia

Q.5 A solid uniform sphere of radius R and mass M has a rotational inertia about a diameter that is given by $(2/5)MR^2$. A light string of length 3R is attached to the surface and used to suspend the sphere from the ceiling. Its rotational inertia about the point of attachment at the ceiling is :
(A) $(2/5)\ MR^2$ (B) $16\ MR^2$
(C) $(47/5)\ MR^2$ (D) $(82/5)\ MR^2$

Q.6 Two identical rods are joined to form a 'X'. The smaller angle between the rod is θ. The moment of inertia of the system about an axis passing through the point of intersection of the rods and perpendicular to their plane is proportional to -
(A) θ (B) $\sin^2\theta$
(C) $\cos^2\theta$ (D) independent of θ

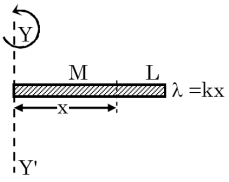
Q.7 A rigid body can be hinged about any point on the x – axis. When it is hinged such that the hinge is at x, the moment of inertia is given by $I = 2x^2 - 12x + 27$. The x coordinate of centre of mass is equal to
(A) 3 (B) 5 (C) 2 (D) 4

Q.8 A square of side 'a' is cut from a square of side '2a' as shown in the figure. Mass of this square with hole is M. Then its moment of inertia about an axis passing through its CM and perpendicular to its plane will be –



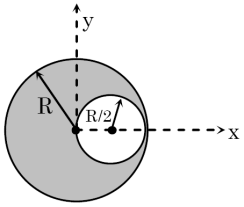
(A) $\frac{Ma^2}{6}$ (B) $\frac{2Ma^2}{6}$ (C) $\frac{4Ma^2}{6}$ (D) $\frac{5Ma^2}{6}$

Q.9 Mass M is distribution over the rod of length L. If linear mass density (λ) linearly increases with length as $\lambda = kx$. The M.I. of rod about one end perpendicular to rod i.e. (YY')



(A) $\frac{ML^2}{3}$ (B) $\frac{ML^2}{12}$ (C) $\frac{2}{3}ML^2$ (D) $\frac{KL^4}{4}$

Q.10 A circular hole of radius R/2 is cut from a circular disc of radius R. The disc lies in xy plane and its centre coincides with the origin. If the remaining mass of the disc is M, then determine its moment of inertia.



(A) $\frac{13MR^2}{24}$ (B) $\frac{5MR^2}{24}$ (C) $\frac{7MR^2}{24}$ (D) $\frac{11MR^2}{24}$

Exercise Level - 2 : It contains single objective type questions with moderate difficulty level to enhance the conceptual and application level of the student.

EXERCISE (Level-2)

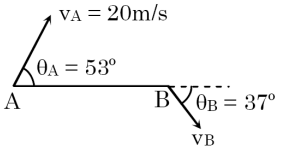
Single correct answer type questions

Q.1 String is wrapped around the periphery of a 5.0 cm radius cylinder, free to rotate on its axis. The string is pulled straight out at a constant rate of 10 cm/s and does not slip on the cylinder. As each small segment of string leaves the cylinder, its acceleration changes by:

(A) 0 (B) 0.010 m/s^2

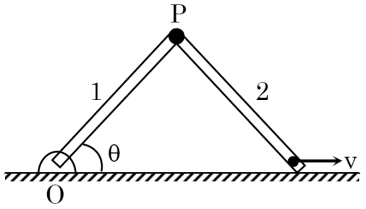
(C) 0.020 m/s^2 (D) 0.20 m/s^2

Q.2 The two ends A and B of a uniform of length ℓ = 1 m and mass m are moving with velocities V_A and V_B as shown. The length AP, where P is point on the rod with velocity 12 m/s, is 8 K cm. Find the value of K.



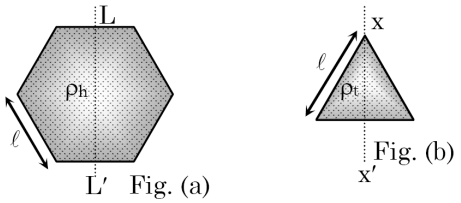
(A) 8 (B) 4 (C) 10 (D) None

Q.3 Two identical rods 1 and 2 each of length ℓ are smoothly pinned at P. The rod 1 is smoothly hinged at O and the free end of the other rod 2 is pulled with constant velocity v. What is the ratio of angular speeds of rod 1 and rod 2.



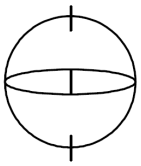
(A) 1 : 1 (B) 1 : 2 (C) 1 : $\sqrt{2}$ (D) $\sqrt{2}$: 1

Q.4 Moment of inertia of a uniform hexagonal plate about an axis LL' is 'I' as shown in the figure (a). The moment of inertia (about axis XX') of an equilateral uniform triangular plate of thickness double that of the hexagonal plate is (Ratio of specific gravity $\frac{\rho_t}{\rho_h} = 3$)



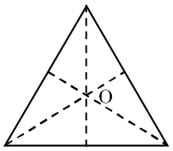
(A) $\frac{I}{5}$ (B) $\frac{I}{10}$ (C) I (D) zero

Q.5 A solid sphere of mass M and radius R has a ring of same mass and radius R wound round it as shown ? Find the moment of inertia of this system about a diameter perpendicular to the plane of ring?



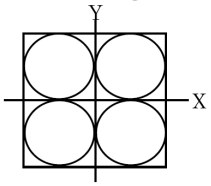
(A) $\frac{8}{5}MR^2$ (B) $\frac{6}{5}MR^2$ (C) $\frac{7}{5}MR^2$ (D) $\frac{4}{5}MR^2$

Q.6 A rod of mass M kg and length L meter is bent in the form of an equilateral triangle as shown in the figure. The moment of inertia of triangle about a vertical axis to perpendicular to the plane of triangle and passing through the centre (in units of kg-m^2) is -



(A) $\frac{ML^2}{12}$ (B) $\frac{ML^2}{54}$ (C) $\frac{ML^2}{162}$ (D) $\frac{ML^2}{108}$

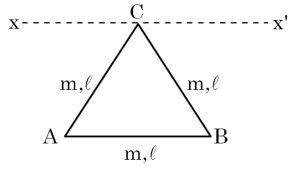
Q.7 Four holes of radius R are cut from a thin square plate of side 4R and mass M. The moment of inertia of the remaining portion about z-axis is –



(A) $\frac{\pi}{12}MR^2$ (B) $\left(\frac{4}{3}-\frac{\pi}{4}\right)MR^2$

(C) $\left(\frac{4}{3}-\frac{\pi}{6}\right)MR^2$ (D) $\left(\frac{8}{3}-\frac{10\pi}{16}\right)MR^2$

Q.8 Three Identical thin rods, each of mass m and length ℓ, are joined to form an equilateral triangular frame. Find the moment of inertia of the frame about an axis xx' parallel to its one side and passing through opposite vertex.



(A) $\frac{m\ell^2}{4}$ (B) $\frac{5m\ell^2}{4}$ (C) $\frac{m\ell^2}{3}$ (D) $\frac{7m\ell^2}{4}$

Exercise Level - 3 : It contains previous years question of JEE Main from Year 2005 to 2024.

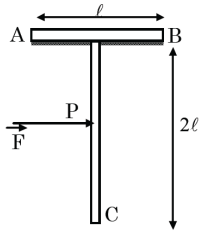
EXERCISE (Level-3)

Old Examination Questions [JEE Main]

Q.1 The moment of inertia of uniform semicircular disc of mass M and radius r about a line perpendicular to the plane of the disc through the centre is - **[AIEEE-2005]**

(A) $\frac{1}{4}Mr^2$ (B) $\frac{2}{5}Mr^2$ (C) Mr^2 (D) $\frac{1}{2}Mr^2$

Q.2 A 'T' shaped object with dimensions shown in the figure, is lying on a smooth floor. A force '$\vec{F}$' is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C. **[AIEEE-2005]**



(A) $\frac{2}{3}\ell$ (B) $\frac{3}{2}\ell$ (C) $\frac{4}{3}\ell$ (D) ℓ

Q.3 A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity ω′ = **[AIEEE 2006]**

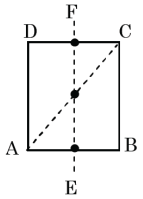
(A) $\frac{\omega m}{(m+M)}$ (B) $\frac{\omega m}{(m+2M)}$

(C) $\frac{\omega(m+2M)}{m}$ (D) $\frac{\omega(m-2M)}{(m+2M)}$

Q.4 Four point masses, each of value m, are placed at the corners of a square ABCD of side ℓ. The moment of inertia of this system about an axis passing through A and parallel to BD is – **[AIEEE 2006]**

(A) 3 mℓ² (B) mℓ² (C) 2 mℓ² (D) $\sqrt{3}$ mℓ²

Q.5 For the given uniform square lamina ABCD, whose centre is O, **[AIEEE 2007]**



(A) $\sqrt{2}\, I_{AC} = I_{EF}$ (B) $I_{AD} = 3I_{EF}$

(C) $I_{AC} = I_{EF}$ (D) $I_{AC} = \sqrt{2}\, I_{EF}$

Q.6 A circular disc of radius R is removed from a bigger circular disc of radius 2R such that the circumferences of the discs coincide. The distance of centre of mass of the new disc is from the centre of the bigger disc is αR. The value of α is - **[AIEEE 2007]**

(A) 1/3 (B) 1/2 (C) 1/6 (D) 1/4

Q.7 A round uniform body of radius R, mass M and moment of inertia 'I', rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is **[AIEEE 2007]**

(A) $\frac{g\sin\theta}{1+I/MR^2}$ (B) $\frac{g\sin\theta}{1+MR^2/I}$

(C) $\frac{g\sin\theta}{1-I/MR^2}$ (D) $\frac{g\sin\theta}{1-MR^2/I}$

Q.8 Angular momentum of the particle rotating with a central force is constant due to - **[AIEEE 2007]**

(A) Constant Force
(B) Constant linear momentum
(C) Zero Torque
(D) Constant Torque

Q.9 Consider a uniform square plate of side 'a' and mass 'm'. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is - **[AIEEE 2008]**

(A) $\frac{1}{12}ma^2$ (B) $\frac{7}{12}ma^2$ (C) $\frac{2}{3}ma^2$ (D) $\frac{5}{6}ma^2$

Q.10 A thin uniform rod of length ℓ and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of - **[AIEEE 2009]**

(A) $\frac{1}{6}\frac{\ell\omega}{g}$ (B) $\frac{1}{2}\frac{\ell^2\omega^2}{g}$ (C) $\frac{1}{6}\frac{\ell^2\omega^2}{g}$ (D) None

Q.11. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is : **[AIEEE-2011]**

(A) $\frac{3}{2}$g (B) g (C) $\frac{2}{3}$g (D) $\frac{g}{3}$

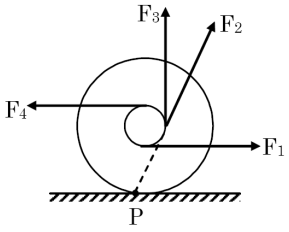
Q.12 A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc : **[AIEEE-2011]**

Exercise Level - 4 : It contains all variety of questions as per level of JEE Advanced such as MCQ, Column match, Passage based & Numerical type etc.

EXERCISE (Level-4)

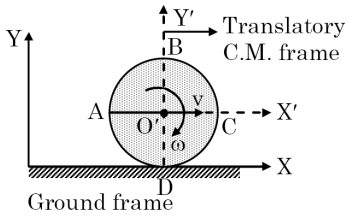
Part-A : Multiple correct answer type questions

Q.1 A spool of wire rests on a horizontal surface as shown in figure. As the wire is pulled, the spool does not slip at contact point P. On separate trails, each one of the forces F_1, F_2, F_3 and F_4 is applied to the spool. For each of these forces the spool



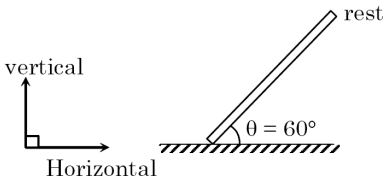
(A) will rotate anticlockwise if F_1 is applied
(B) will not rotate if F_2 is applied
(C) will rotate anticlockwise if F_3 is applied
(D) will rotate clockwise if F_4 is applied

Q.2 A thin rigid uniform circular disc rolls without slipping on a horizontal rigid surface (or the ground). At a certain instant, its position w.r.t. ground frame is as shown in the figure.



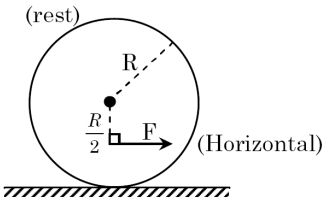
(A) Sector ABC has greater kinetic energy than sector ADC w.r.t. ground frame
(B) Sector BO′C has greater kinetic energy than sector CO′D w.r.t. ground frame
(C) Sector BO′C has the same kinetic energy as sector AO′B w.r.t. ground frame
(D) All the sectors AO′B, BO′C, CO′D and AO′D have same kinetic energy w.r.t. the centre of mass frame

Q.3 A uniform rod released on a smooth horizontal surface from rest as shown in figure. Rod makes an angle $\theta = 60°$ with horizontal initially.



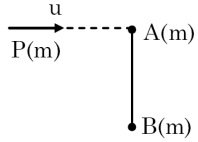
(A) Acceleration of centre of mass of rod just after releases in $\frac{3g}{7}$
(B) Acceleration of centre of mass of rod just after releases in $\frac{3g}{5}$
(C) Acceleration of point of contact just after release is $\frac{3\sqrt{3}g}{7}$
(D) Acceleration of point of contact just after release is $\frac{3\sqrt{3}g}{5}$

Q.4 In figure there is a disc (uniform) of mass m and radius R & a force F is applied in horizontal direction as shown in figure. Which of the following options is/are correct



(A) If friction is sufficient so that disc does not slip then angular acceleration of disc is perpendicular to the plane of paper inward.
(B) If friction is sufficient so that disc does not slip then angular acceleration of disc is perpendicular to the plane of paper outward.
(C) If surface is frictionless then angular acceleration of disc will be perpendicular to the plane of paper inward
(D) If surface is frictionless then angular acceleration of disc will be perpendicular to the plane of paper outward

Q.5 Two particles A and B, of mass m each, are joined by a rigid massless rod of length ℓ. A particle P of mass m, moving with a speed u normal to AB, strikes A and sticks to it. The centre of mass of the 'A+B+P' system is C.



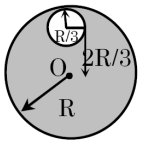
(A) The velocity of C before impact is u/3
(B) The velocity of C after impact is u/3
(C) The velocity of 'A + P' immediately after impact is u/2
(D) The velocity of B immediately after impact is zero

Exercise Level - 5 : It contains previous years question of JEE Advanced from Year 2005 to 2024.

EXERCISE (Level-5)

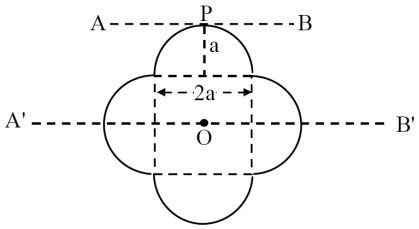
Old Examination Questions [JEE Advanced]

Q.1 Mass and radius of a circular disc is 9M & R respectively. Moment of inertia of the disc about an axis passing through point O after removal of a disc of Radius R/3 as shown in the figure is- **[IIT – 2005]**



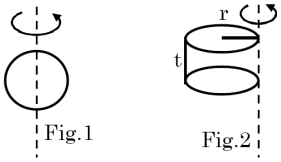
(A) 37/9 MR^2 (B) 9 MR^2
(C) 40 /9 MR^2 (D) 4 MR^2

Q.2 A symmetric lamina of mass M consists of a square shape with a semicircular section over each of the edge of the square as shown in figure. The side of the square is 2a. The moment of inertia of the lamina about an axis through its centre of mass and perpendicular to the plane is 1.6 Ma^2. The moment of inertia of the lamina about the tangent AB in the plane of the lamina is- **[IIT – 2006]**



(A) 4.8 Ma^2 (B) 3.2 M a^2
(C) 6.4 M a^2 (D) 1.6 M a^2

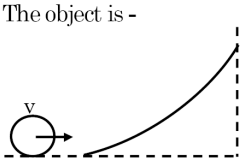
Q.3 Moment of inertia of solid sphere of mass m and radius R about axis passing through center of mass is I as shown in figure 1. The sphere is moulded in the form of disc of radius 'r' and thickness 't'. The moment of inertia of disc about the axis shown in figure 2 is I. **[IIT – 2006]**



The radius of disc is -

(A) $\frac{2R}{\sqrt{15}}$ (B) $\frac{2R}{\sqrt{5}}$
(C) $\frac{R}{\sqrt{15}}$ (D) $\frac{R}{\sqrt{5}}$

Q.4 A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is - **[IIT –2007]**

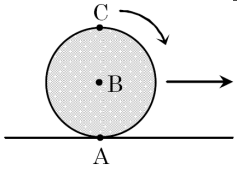


(A) ring (B) solid sphere
(C) hollow sphere (D) disc

Q.5 If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that - **[IIT – 2009]**

(A) linear momentum of the system does not change in time
(B) kinetic energy of the system does not change in time
(C) angular momentum of the system does not change in time
(D) potential energy of the system does not change in time

Q.6 A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then - **MCQ** **[IIT – 2009]**



(A) $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B - \vec{V}_C)$

(B) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

(C) $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$

(D) $|\vec{V}_C - \vec{V}_A| = 4|\vec{V}_B|$

Q.7 **Statement –1**
If there is no external torque on a body about its centre of mass, then the velocity of the center of mass remains constant.
Statement –2
The linear momentum of an isolated system remains constant. **[IIT – 2007]**

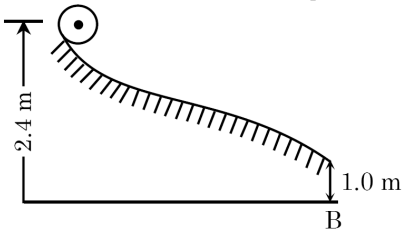
(A) Statement–1 is True, Statement–2 is True; Statement–2 **is** a correct explanation for Statement–1
(B) Statement–1 is True, Statement–2 is True; Statement–2 **is NOT** a correct explanation for Statement–1
(C) Statement–1 is True, Statement–2 is False
(D) Statement–1 is False, Statement–2 is True.

Exercise Level - 6 : Advanced level a bit complex questions for students for solid rock prepertion for Top Rankers.

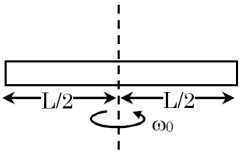
EXERCISE (Level-6)

Review Exercise

Q.1 A small sphere rolls down without slipping from the top of a track in a vertical plane. The track has an elevated section and a horizontal part, The horizontal part is 1.0 metre above the ground level and the top of the track is 2.4 metres above the ground. Find the distance on the ground with respect to the point B (which is vertically below the end of the track as shown in figure) where the sphere lands. During its flight as a projectile, does the sphere continue to rotate about its centre of mass ? Explain. **[IIT–1987]**



Q.2 A smooth uniform rod of length L and mass M has two identical beads of negligible size, each of mass m, which can slide freely along the rod. initially the two beads are at the centre of the rod and the system is rotating with a an angular velocity ω_0 about an axis perpendicular to the rod and passing through the midpoint of the rod. There are no external forces. When the beads reach the ends of the rod, find the angular velocity of the system. **[IIT–1988]**

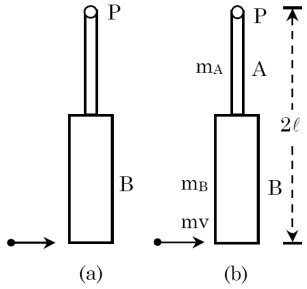


Q.3 A cylinder of mass M and radius R is resting on a horizontal platform (which is parallel to the x–y plane) with its axis fixed along the y–axis and free to rotate about its axis. The platform is given a motion in the x–direction given by $x = A\cos(\omega t)$.
There is no slipping between the cylinder and platform. Find the maximum torque acting on the cylinder during its motion. **[IIT–1988]**

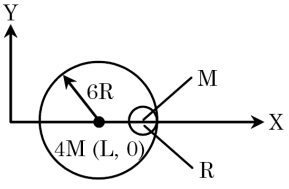
Q.4 A uniform thin rod of mass M and length L is standing vertically along the y–axis on a smooth horizontal surface, with its lower end at the origin (0, 0). A slight disturbance at t = 0 causes the lower end to slip on the smooth surface along the positive x–axis, and the rod starts falling. **[IIT-1993]**

(i) What is the path followed by the centre of mass of the rod during its fall ?
(ii) Find the equation to the trajectory of a point on the rod located at a distance r from the lower end. What is the shape of the path of this point ?

Q.5 Two uniform thin rods A and B of length 0.6 m each and of masses 0.01 kg and 0.02 kg respectively are rigidly jointed, end to end. The combination is pivoted at the lighter end P as shown in figure such that it can freely rotate about the point P in a vertical plane. A small object of mass 0.05 kg, moving horizontally hits the lower end of the combination and sticks to it. What should be the velocity of the object so that the system could just be raised to the horizontal position. **[IIT–1994]**



Q.6 A small sphere of radius R is held against the inner surface of a larger sphere of radius 6R. (shown in figure). The masses of large and small spheres are 4M and M, respectively. This arrangement is placed on a horizontal table. There is no friction between any surfaces of contact. The small sphere is now released. Find the coordinates of the centre of the larger sphere when the smaller sphere reaches the other extreme position. **[IIT–1996]**



Q.7 A uniform disc of mass m and radius R is projected horizontally with velocity v_0 on a rough horizontal floor so that it starts off with a purely sliding motion at t = 0. After t_0 seconds, it acquires a purely rolling motion as shown in figure. **[IIT–1997]**

Answer key

Ankswer key is provided at the end of the exercise sheets.

ANSWER KEY

EXERCISE (Level-1)

1. (B)	**2.** (D)	**3.** (A)	**4.** (B)	**5.** (D)	**6.** (D)	**7.** (A)	**8.** (D)	**9.** (D)	**10.** (A)
11. (D)	**12.** (B)	**13.** (B)	**14.** (A)	**15.** (B)	**16.** (B)	**17.** (D)	**18.** (C)	**19.** (D)	**20.** (B)
21. (A)	**22.** (C)	**23.** (D)	**24.** (B)	**25.** (B)	**26.** (A)	**27.** (C)	**28.** (C)	**29.** (B)	**30.** (C)
31. (D)	**32.** (D)	**33.** (A)	**34.** (D)	**35.** (B)	**36.** (D)	**37.** (B)	**38.** (C)	**39.** (B)	**40.** (A)
41. (D)	**42.** (C)	**43.** (B)	**44.** (A)						

Revision Plan

We emphasis that every student should prepare his/her own revision plan. For this purpose there is Revision Plan Section in each chapter which student should prepare while going thorugh the study material. This will be useful at the time of final revision before final exam for quick & effective revision.

Revision Plan

Prepare Your Revision plan today!

After attempting Exercise Sheet, please fill below table as per the instruction given.

A. Write Question Number (QN) which you are unable to solve at your own in **column A**.

B. After discussing the Questions written in **column A** with faculty, strike off them in the manner so that you can see at the time question number during Revision, to solve such questions again.

C. Write down the Question Number you feel are important or good in the **column B**.

EXERCISE	COLUMN A	COLUMN B
	Questions unable to solve in first attempt	Good or Important questions
Topic wise practice questions		
Level-1		
Level-2		
Level-3		
Level-4		
Level-5		
Level-6		

Revision Strategy:

Whenever you wish to revision this chapter, follow the following steps-

Step-1: Review your theory notes.

Step-2: Solve Questions of column A

Online Solutions

Self explanatory and detailed soltuion of all excercises above are available on Career Point website **www.careerpoint.ac.in**

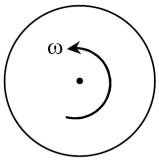
ROTATIONAL MOTION

EXERCISE (Level-1)

Answer Key & Solution

Question Number	Solution	Question Number	Solution	Question Number	Solution	Question Number	Solution
1	Click Here	12	Click Here	23	Click Here	34	Click Here
2	Click Here	13	Click Here	24	Click Here	35	Click Here
3	Click Here	14	Click Here	25	Click Here	36	Click Here
4	Click Here	15	Click Here	26	Click Here	37	Click Here
5	Click Here	16	Click Here	27	Click Here	38	Click Here
6	Click Here	17	Click Here	28	Click Here	39	Click Here
7	Click Here	18	Click Here	29	Click Here	40	Click Here
8	Click Here	19	Click Here	30	Click Here	41	Click Here
9	Click Here	20	Click Here	31	Click Here	42	Click Here
10	Click Here	21	Click Here	32	Click Here	43	Click Here
11	Click Here	22	Click Here	33	Click Here	44	Click Here

Sol.1 **[B]**



When body is spinning. Anticlockwise about the axis perpendicular to page then $\vec{\omega}$ is out of page (Ans. B)

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