CAREER POINT

KVPY QUESTION PAPER-2015 (STREAM SX)

Part – I

One - Mark Questions

Date : 01 / 11 / 2015

MATHEMATICS

1. The number of ordered pairs (x, y) of real numbers that satisfy the simultaneous equations $x + y^2 = x^2 + y = 12$ is

(A) 0 (B) 1 (C) 2 (D) 4

Ans. **[D]**

Sol. $x + y^2 = x^2 + y = 12$

curve (1) $x + y^2 = 12$

$y^2 = -(x - 12)$

Intersection on x-axis (12, 0)

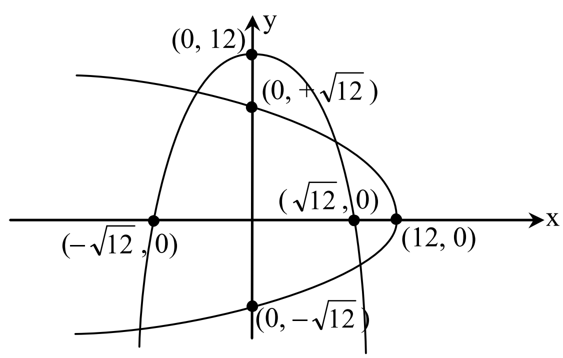
Intersection on y-axis $(0, \pm\sqrt{12})$

curve (2) $x^2 + y = 12$

$x^2 = -(y - 12)$

Intersection on x-axis = $(\pm\sqrt{12}, 0)$

Intersection on y-axis = (0, 12)



four intersection

2. If z is a complex number satisfying $|z^3 + z^{-3}| \le 2$, then the maximum possible value of $|z + z^{-1}|$ is -

(A) 2 (B) $\sqrt[3]{2}$ (C) $2\sqrt{2}$ (D) 1

Ans. **[A]**

Sol. $|z^3 + z^{-3}| \le 2$

$$\left|z^3 + \frac{1}{z^3}\right| \le 2$$

$$\left|\left(z + \frac{1}{z}\right)\left(z^2 + \frac{1}{z^2} - 1\right)\right| \le 2$$

$$\left|\left(z+\frac{1}{z}\right)\left(\left(z+\frac{1}{z}\right)^2-3\right)\right| \le 2$$

$$\left|z+\frac{1}{z}\right|\left|\left(z+\frac{1}{z}\right)^2-3\right| \le 2$$

$$\left|z+\frac{1}{z}\right|\left\{\left|\left|z+\frac{1}{z}\right|^2-3\right|\right\} \le 2 \qquad \{\because |z_1 - z_2| \ge ||z_1| - |z_2||\}$$

$t\,|t^2 - 3| \le 2$ $\qquad$ $(t \ge 0)$ where $t = \left|z+\frac{1}{z}\right|$

$t \ge \sqrt{3}$	$0 \le t < \sqrt{3}$
$t(t^2 - 3) \le 2$	$t(3 - t^2) \le 2$
$t^3 - 3t - 2 \le 0$	$3t - t^3 \le 2$
$(t - 2)\,(t + 1)^2 \le 0$	$t^3 - 3t + 2 \ge 0$
	$(t - 1)^2\,(t + 2) \ge 0$
$t - 2 \le 0$	$t \ge -2$
$t \le 2$	$t \in [0, \sqrt{3}\,)$

$$\left|z+\frac{1}{z}\right|_{max} = 2$$

3. The largest perfect square that divides $2014^3 - 2013^3 + 2012^3 - 2011^3 + ... + 2^3 - 1^3$ is -

(A) 1^2 (B) 2^2 (C) 1007^2 (D) 2014^2

Ans. **[C]**

Sol. $2\{(2014)^3 + (2012)^2 + \ldots + 2^3\} - \{(2014)^3 + (2013)^3 + \ldots + 1^3\}$

$= 2 \times 8\ \{(1007)^2 + (1006)^2 + \ldots + 1^3\} - \{(2014)^3 + (2013)^2 + \ldots + 1^3\}$

$$= 2 \times 8 \times \left(\frac{(1007)(1008)}{2}\right)^2 - \left(\frac{(2014)(2015)}{2}\right)^2$$

$$= 2 \times 8 \times \frac{(1007)^2(1008)^2}{4} - \frac{(2014)^2(2015)^2}{4}$$

$= (1007)^2\,(2016)^2 - (1007)^2\,(2015)^2$

$= (1007)^2\ \{2016 - 2015\}\ \{2016 + 2015\}$

$= (1007)^2\,(4031)$

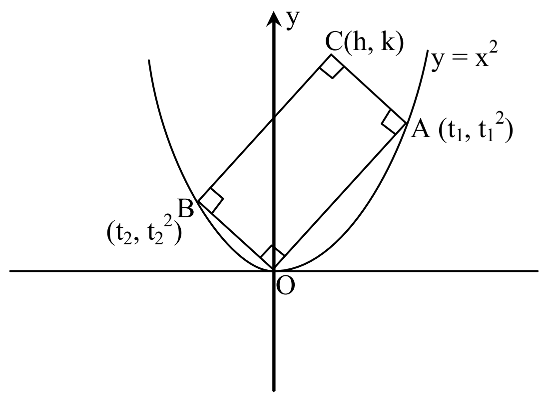
$=$ divisible by $(1007)^2$

4. Suppose OABC is a rectangle in the xy-plane where O is the origin and A, B lie on the parabola $y = x^2$. Then C must lie on the curve -

(A) $y = x^2 + 2$ (B) $y = 2x^2 + 1$ (C) $y = -x^2 + 2$ (D) $y = -2x^2 + 1$

Ans. **[A]**

Sol.



$\because$ OB $\perp$ OA

So, $t_1t_2 = -1$

Now $\dfrac{h}{2} = \dfrac{t_1 + t_2}{2}$

$t_1 + t_2 = h$ ...(1)

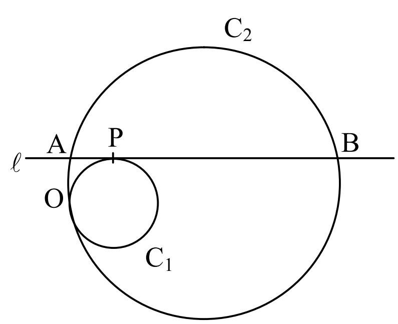
also $t_1^2 + t_2^2 = k$

$(t_1 + t_2)^2 - 2t_1t_2 = k$

$h^2 + 2 = k$

locus is $x^2 + 2 = y$

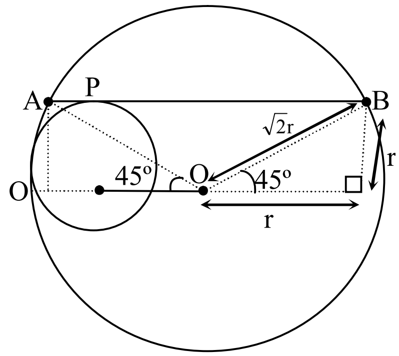
5. Circles C_1 and C_2, of radii r and R respectively, touch each other as shown in the figure. The line ℓ, which is parallel to the line joining the centres of C_1 and C_2, is tangent to C_1 at P and intersects C_2 at A, B. If $R^2 = 2r^2$, then $\angle AOB$ equals -



(A) $22\frac{1}{2}^o$ (B) 45^o (C) 60^o (D) $67\frac{1}{2}^o$

Ans. **[B]**

Sol.



Chose AB subtend 90º at centre.

so that AB subtend 45º at O (circumference of circle)

6. The shortest distance from the origin to a variable point on the sphere $(x-2)^2 + (y-3)^2 + (z-6)^2 = 1$ is -

(A) 5 (B) 6 (C) 7 (D) 8

Ans. **[B]**

Sol. Sphere $x^2 + y^2 + z^2 - 4x - 6x - 12z + 48 = 0$

Centre (2, 3, 6)

radius = $\sqrt{4+9+36-48} = 1$

distance between centre and origin = $\sqrt{4+9+36} = 7$

shortest distance $= 7 - 1 = 6$ (Origin lies outside the sphere)

7. The number of real numbers λ for which the equality

$$\frac{\sin(\lambda\alpha)}{\sin\alpha} - \frac{\cos(\lambda\alpha)}{\cos\alpha} = \lambda - 1,$$

holds for all real α which are not integral multiples of $\pi/2$ is -

(A) 1 (B) 2 (C) 3 (D) Infinite

Ans. **[B]**

Sol. $$\frac{\sin(\lambda\alpha)}{\sin\alpha} - \frac{\cos(\lambda\alpha)}{\cos\alpha} = \lambda - 1$$

By observation

$\sin(\lambda\alpha)\cos\alpha - \cos(\lambda\alpha)\sin\alpha = (\lambda - 1)\sin\alpha\cos\alpha$

$\sin(\lambda - 1)\alpha = (\lambda - 1)\sin\alpha\cos\alpha$

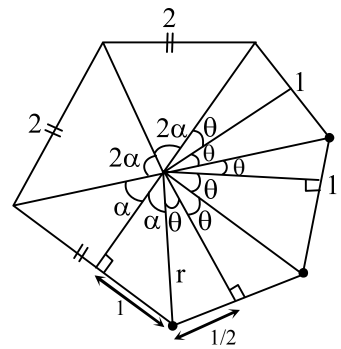
clearly $\lambda = 1, \lambda = 3$ is solution

8. Suppose ABCDEF is a hexagon such that AB = BC = CD = 1 and DE = EF = FA = 2. If the vertices A, B, C, D, E, F are concylic, the radius of the circle passing through them is -

(A) $\sqrt{\frac{5}{2}}$ (B) $\sqrt{\frac{7}{3}}$ (C) $\sqrt{\frac{11}{5}}$ (D) $\sqrt{2}$

Ans. **[B]**

Sol.



From the figure:

$\sin\theta = \frac{1}{2r}$ & $\sin\alpha = \frac{1}{r}$

$3 \times (2\theta) + (2\alpha) \times 3 = 360^\circ$

$\theta + \alpha = 60^\circ$

Now, $\cos(\theta + \alpha) = \frac{1}{2}$

$\Rightarrow \cos\theta.\cos\alpha - \sin\theta.\sin\alpha = \frac{1}{2}$

$\Rightarrow \sqrt{1-\frac{1}{4r^2}}\sqrt{1-\frac{1}{r^2}} - \frac{1}{2r}\cdot\frac{1}{r} = \frac{1}{2}$

$\Rightarrow \sqrt{4r^2-1}\ \sqrt{r^2-1}\ - 1 = r^2$

$\Rightarrow (4r^2 - 1)(r^2 - 1) = (r^2 + 1)^2$

$\Rightarrow 4r^4 - 5r^2 + 1 = r^4\backslash + 2r^2 + 1$

$\Rightarrow 3r^4 = 7r^2$

$\Rightarrow r^2 = \frac{7}{3}$

$\Rightarrow r = \sqrt{\frac{7}{3}}$

9. Let p(x) be a polynomial such that $p(x) - p'(x) = x^n$, where n is a positive integer. Then p(0) equals -

(A) $n!$ (B) $(n-1)!$ (C) $\frac{1}{n!}$ (D) $\frac{1}{(n-1)!}$

Ans. **[A]**

Sol. Let $P(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n$

$P'(x) = na_0x^{n-1} + (n-1)a_1x^{n-2} + \ldots + a_{n-1}$

$P(x)P'(x) = a_0x^n + (a_1 - na_0)\, x^{n-1} + (a_2 - (n-1)a_1)\, x^{n-2} + \ldots + (a_n - a_{n-1})$

given $P(x) - P'(x) = a_0x^n$.

so that

$a_1 - na_0 = 0 \qquad \frac{a_1}{a_0} = n$

$a_2 - (n-1)a_1 = 0 \qquad \frac{a_2}{a_1} = (n-1)$

$a_n - a_{n-1} = 0 \qquad \frac{a_n}{a_{n-1}} = 1$

$P(0) = a_n = \left(\frac{a_n}{a_{n-1}}\right)\left(\frac{a_{n-1}}{a_{n-2}}\right)\ldots\left(\frac{a_1}{a_0}\right)$

$= 1 \times 2 \times 3 \times \ldots \times n$

$= n!$

10. The value of the limit

$\lim_{x\to 0}\left(\frac{x}{\sin x}\right)^{6/x^2}$ is -

(A) e (B) e^{-1} (C) $e^{-1/6}$ (D) e^6

Ans. **[A]**

Sol. $\lim_{x\to 0}\left(\frac{x}{\sin x}\right)^{6/x^2} \quad (1)^{\infty}$

$e^{\lim_{x\to 0}\frac{6}{x^2}\left(\frac{x}{\sin x}-1\right)}$

$e^{\lim_{x\to 0}\frac{6}{x^2}\left(\frac{x-\sin x}{\sin x}\right)}$

$e^{\lim_{x\to 0}\frac{6}{x^2}\left\{\frac{x-\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}.....\right)}{\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}.....\right)}\right\}}$

$e^{\lim_{x\to 0}\frac{6}{x^2}\frac{\left\{\frac{x^3}{3!}-\frac{x^5}{5!}....\right\}}{\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}....\right)}}$

$e^{\lim_{x\to 0}\frac{6x^3}{x^3}\frac{\left\{\frac{1}{3!}-\frac{x^2}{5!}....\right\}}{\left\{1-\frac{x^2}{3!}....\right\}}} = e^1$

11. Among all sectors of a fixed perimeter, choose the one with maximum area. Then the angle at the center of this sector (i.e., the angle between the bounding radii) is -

(A) $\frac{\pi}{3}$ (B) $\frac{3}{2}$ (C) $\sqrt{3}$ (D) 2

Ans. **[D]**

Sol. Given that $2r + r\theta = P$ $r = \frac{P}{2+\theta}$

$$\text{area} = \frac{1}{2}r^2\theta = \frac{1}{2}\theta\left(\frac{P}{2+\theta}\right)^2 = \frac{1}{2}\frac{P^2\theta}{(2+\theta)^2}$$

$$\frac{dA}{d\theta} = 0 \qquad \theta = 2^C$$

12. Define a function $f : R \to R$ by

$f(x) = \max\ \{|x|, |x-1|, \ldots, |x-2n|\}$

where n is a fixed natural number. Then $\int\limits_0^{2n} f(x)dx$ is -

(A) n (B) n^2 (C) 3n (D) $3n^2$

Ans. **[D]**

Sol. $f(x) = \max\ \{|x|, |x-1|, \ldots, |x-2n|\}$

$x \geq n$ $x < n$

$f(x) = |x|$ $|x - 2n|$

$$\int_0^{2n} f(x)dx = \int_0^{n} f(x)dx + \int_n^{2n} f(x)dx$$

$$= \int_0^{n} |x-2n|dx + \int_n^{2n} |x|dx$$

$$= \int_0^{n} (2n-x)\ dx + \int_n^{2n} x.dx$$

$$= \left[2nx - \frac{x^2}{2}\right]_0^n + \left[\frac{x^2}{2}\right]_n^{2n}$$

$$= \left(2n^2 - \frac{n^2}{2}\right) + \left(\frac{4n^2}{2} - \frac{n^2}{2}\right)$$

$$= \frac{3n^2}{2} + \frac{3n^2}{2} = 3n^2$$

13. If p(x) is a cubic polynomial with p(1) = 3, p(0) = 2 and p(–1) = 4, then $\int_{-1}^{1} p(x)dx$ is -

(A) 2 (B) 3 (C) 4 (D) 5

Ans. **[D]**

Sol. Let $P(x) = ax^3 + bx^2 + cx + d$

$a + b + c + d = 3$

$d = 2$

$-a + b - c + d = 4$

$2b + 2d = 7$

$2b + 4 = 7$

$2b = 3$

$b = \frac{3}{2}$

$\int_{-1}^{1}(ax^3 + bx^2 + cx + d)\,dx$

$2\int_{0}^{1}(bx^2 + d)dx$

$= 2\left[b\frac{x^3}{3} + dx\right]_0^1$

$= 2\left(\frac{b}{3} + d\right)$

$= 2\left(\frac{1}{2} + 2\right) = 5$

14. Let x > 0 be a fixed real number. Then the integral $\int_{0}^{\infty} e^{-1}|x - t|\,dt$ is equal to -

(A) $x + 2e^{-x} - 1$ (B) $x - 2e^{-x} + 1$ (C) $x + 2e^{-x} + 1$ (D) $-x - 2e^{-x} + 1$

Ans. **[A]**

Sol. $f(x) = \int_{0}^{\infty} e^{-t}|x - t|dt$

$f(x) = \int_{0}^{x} e^{-t}(x - t)dt + \int_{x}^{\infty} e^{-t}(t - x)dt$

$f'(x) = e^{-x}(x - x) - e^{-0}(x - 0).\,0 + \int_{0}^{x} e^{-t}(1)\,dt + 0 - e^{-x}(x - x).1 + \int_{x}^{\infty} -e^{-t}\,dt$

$= [-e^{-t}]_0^x + [e^{-t}]_x^{\infty}$

$= -e^{-x} + 1 + 0 - e^{-x}$

$f'(x) = 1 - 2e^{-x}$

$dy = 1 - 2e^{-x}\,dx$

$y = x + 2e^{-x} + c$

$f(x) = x + 2e^{-x} + c$

$$f(0) = \int_0^\infty e^{-t}\ t\ dt$$

$$= [(-e^{-t})t]_0^\infty + \int_0^\infty e^{-t}dt$$

$$= [0 - e^{-t}]_0^\infty$$

$= 0 + 1$

$f(0) = 1$

$f(0) = 1 = 0 + 2e^{-0} + c$

$c = -1$

$f(x) = x + 2e^{-x} - 1$

15. An urn contains marbles of four colours : red, white, blue and green. When four marbles are drawn without replacement, the following events are equally likely :
(1) the selection of four red marbles
(2) the selection of one white and three red marbles
(3) the selection of one white, one blue and two red marbles
(4) the selection of one marble of each colour
The smallest total number of marbles satisfying the given condition is
(A) 19 (B) 21 (C) 46 (D) 69

Ans. **[B]**

Sol. Let Red Balls = x
White Balls = y
Blue Balls = z
Green Balls = w

$$\frac{{}^xC_4}{{}^{x+y+z+w}C_4} = \frac{{}^xC_3 \cdot {}^yC_1}{{}^{x+y+z+w}C_4} = \frac{{}^xC_2\,{}^yC_1\,{}^zC_1}{{}^{x+y+z+w}C_4} = \frac{{}^xC_1 \times {}^yC_1 \times {}^zC_1 \times {}^wC_1}{{}^{x+y+z+w}C_4}$$

${}^xC_4 = {}^xC_3\,{}^yC_1$ $\quad x - 3 = 4y \quad x = 4y + 3$

${}^xC_3\,{}^yC_1 = {}^xC_2\,{}^yC_1\,{}^zC_1$ $\quad x - 2 = 3z \quad x = 3z + 2$

${}^xC_2\,{}^yC_1\,{}^zC_1 = {}^xC_1\,{}^yC_1\,{}^zC_1\,{}^wC_1$ $\quad x - 1 = 2w \quad x = 2w + 1$

Cleary for y = 1not possible
at y = 2 x = 11
z = 3 x = 11
w = 5 x = 11

so, minimum number of Ball = 11 + 2 + 3 + 5 = 21

16. There are 6 boxes labelled $B_1, B_2, \ldots, B_6$. In each trial, two fair dice D_1, D_2 are thrown. If D_1 shows j and D_2 shows k, then j balls are put into the box B_k. After n trials, what is the probability that B_1 contains at most one ball ?

(A) $\left(\frac{5^{n-1}}{6^{n-1}}\right) + \left(\frac{5^n}{6^n}\right)\left(\frac{1}{6}\right)$ (B) $\left(\frac{5^n}{6^n}\right) + \left(\frac{5^{n-1}}{6^{n-1}}\right)\left(\frac{1}{6}\right)$

(C) $\left(\frac{5^n}{6^n}\right) + n\left(\frac{5^{n-1}}{6^{n-1}}\right)\left(\frac{1}{6}\right)$ (D) $\left(\frac{5^n}{6^n}\right) + n\left(\frac{5^{n-1}}{6^{n-1}}\right)\left(\frac{1}{6^2}\right)$

Ans. **[D]**

Sol.

B_1…………………….. B_6.

Required probability

D_1 never Shows '1' D_2 Shows '1' (one time) Then D_1 shows '1'

$\left(\frac{5}{6}\right)^n$ $\left\{{}^nC_1\left(\frac{5}{6}\right)^{n-1}\left(\frac{1}{6}\right)\right\}\left\{\frac{1}{6}\right\}$

Required probability $= \left(\frac{5}{6}\right)^n + n\left(\frac{5^{n-1}}{6^{n-1}}\right)\left(\frac{1}{6^2}\right)$

17. Let $\vec{a} = 6\vec{i} - 3\vec{j} - 6\vec{k}$ and $\vec{d} = \vec{i} + \vec{j} + \vec{k}$. Suppose that $\vec{a} = \vec{b} + \vec{c}$ where $\vec{b}$ is parallel to $\vec{d}$ and $\vec{c}$ is perpendicular to $\vec{d}$. Then $\vec{c}$ is -

(A) $5\vec{i} - 4\vec{j} - \vec{k}$ (B) $7\vec{i} - 2\vec{j} - 5\vec{k}$

(C) $4\vec{i} - 5\vec{j} + \vec{k}$ (D) $3\vec{i} + 6\vec{j} - 9\vec{k}$

Ans. **[B]**

Sol. $\vec{b} = \lambda\,(\hat{i}+\hat{j}+\hat{k})$

$\vec{a} = \vec{b} + \vec{c}$

$\vec{c} = \vec{a} - \lambda\,(\hat{i}+\hat{j}+\hat{k})$

$\vec{c} = (6\hat{i}-3\hat{j}-6\hat{k}) - \lambda\,(\hat{i}+\hat{j}+\hat{k})$

$\vec{c} = (6-\lambda)\hat{i}+(-3-\lambda)\hat{j}+(-6-\lambda)\hat{k}$

$\vec{c}\,.\,\vec{\lambda} = 6-\lambda-3-\lambda-6-\lambda = 0$

$3\lambda = 3$

$\lambda = -1$

$\vec{c} = 7\hat{i}-2\hat{j}-5\hat{k}$

18. If $\log_{(3x-1)}(x-2) = \log_{(9x^2-6x+1)}(2x^2-10x-2)$, then x equals -

(A) $9-\sqrt{15}$ (B) $3+\sqrt{15}$ (C) $2+\sqrt{5}$ (D) $6-\sqrt{5}$

Ans. **[B]**

Sol. $\log_{(3x-1)}(x-2) = \log_{(3x-1)^2}(2x^2-10x-2)$

$\log_{(3x-1)}(x-2)^2 = \log_{(3x-1)}(2x^2-10x-2)$

$(x-2)^2 = 2x^2-10x-2$

$x^2-4x+4 = 2x^2-10x-2$

$x^2-6x-6=0$

$x = 3\pm\sqrt{15}$

$x = 3-\sqrt{15}$ $\quad$ $x = 3+\sqrt{15}$

at $3-\sqrt{15}$

$(x-2)$ is negative

19. Suppose a, b, c are positive integers such that $2^a + 4^b + 8^c = 328$. Then $\frac{a+2b+3c}{abc}$ is equal to -

(A) $\frac{1}{2}$ (B) $\frac{5}{8}$ (C) $\frac{17}{24}$ (D) $\frac{5}{6}$

Ans. **[C]**

Sol. c = 3 not possible

Equation is possible if

a = 3 b = 4 c = 2

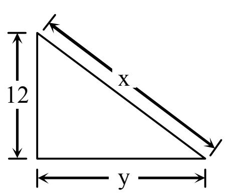
$\frac{a+2b+3c}{abc} = \frac{17}{24}$.

20. The sides of a right-angled triangle are integers. The length of one of the sides is 12. The largest possible radius of the incircle of such a triangle is -

(A) 2 (B) 3 (C) 4 (D) 5

Ans. **[D]**

Sol.



Clearly

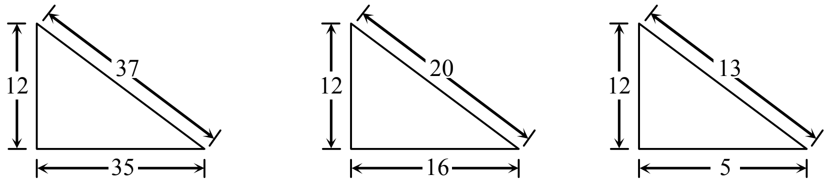
$x^2 - y^2 = 144$

$(x-y)(x+y) = 144$

$x, y \in I$

So factorsize 144 into two even factors

$x + y = 72$	$x + y = 36$	$x + y = 18$
$x - y = 2$	$x - y = 9$	$x - y = 8$
$x = 37$	$x = 20$	$x = 13$
$y = 35$	$y = 16$	$y = 5$



$r = \Delta/s$	$r = \Delta/s$	$r = \Delta/s$
$r = \dfrac{1/2 \times 12 \times 35}{\left(\dfrac{12+35+37}{2}\right)}$	$r = \dfrac{1/2 \times 12 \times 16}{\left(\dfrac{16+20+12}{2}\right)}$	$r = 2$
$r = \dfrac{12 \times 35}{84}$	$r = \dfrac{12 \times 16}{48}$	
$r = 5$	$r = 4$	

PHYSICS

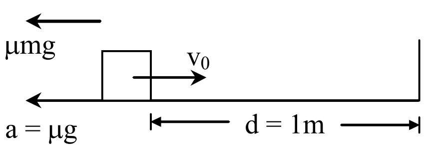
21. A small box resting on one edge of the table is struck in such a way that it slides off the other edge, 1 m away, after 2 seconds. The coefficient of kinetic friction between the box and the table -

(A) must be less than 0.05 (B) must be exactly zero

(C) must be more than 0.05 (D) must be exactly 0.05

Ans. **[A]**

Sol.



Velocity after time t

$v = v_0 - \mu g t$

$v \geq 0 \Rightarrow v_0 \geq \mu g t$ ….(i)

Displacement in time 't'

$\Delta X = v_0 t - \dfrac{1}{2}\mu g t^2$

$1 = 2v_0 - \dfrac{1}{2}\mu g(2)^2$

$2v_0 = (1 + 2\mu g)$

$v_0 = \left(\frac{1}{2} + \mu g\right)$ ....(ii)

$\frac{1}{2} + \mu g > \mu gt$ (t = 2 sec)

$\frac{1}{2} + \mu g > 2\mu g$

$\mu g < \frac{1}{2}$

$\mu < \frac{1}{2g}$

$\mu < 0.05$

22. Carbon-II decays to boron-II according to the following formula.

$$^{11}_{6}C \rightarrow {}^{11}_{5}B + e^{+} + \nu_e + 0.96 \text{ MeV}$$

Assume that positrons (e^+) produced in the decay combine with free electrons in the atmosphere and annihilate each other almost immediately. Also assume that the neutrinos (ν_e) are massless and do not interact with the environment. At t = 0 we have 1 μg of $^{12}_{6}C$. If the half-life of the decay process is t_0, the net energy produced between time t = 0 and t = $2t_0$ will be nearly -

(A) 8×10^{18} MeV (B) 8×10^{16} MeV (C) 4×10^{18} MeV (D) 4×10^{16} MeV

Ans. **[B]**

Sol. $^{12}_{6}C \longrightarrow {}^{11}_{5}B + \beta^{\oplus} + \nu + 0.96 \text{ MeV}$

$M = \frac{M_0}{2^n} \Rightarrow M = \frac{1\ \mu g}{2^2}$ [Half life = t_0 ; n = t/t_0 $\Rightarrow$ n = $\frac{2t_0}{t}$ $\Rightarrow$ n = 2]

M = 0.25 μg (remained)

Carbon used $\Rightarrow M_0 - M$

$\Rightarrow 0.75$ μg

Number of moles = $\left(\frac{0.75 \times 10^{-6}}{12}\right)$

Number of reaction = $\frac{0.75 \times 10^{-6}}{12} \times 6.023 \times 10^{23}$

$= 0.37 \times 10^{17}$ reaction

Energy from reaction $= 0.376 \times 10^{17} \times 0.96$ MeV

$= 0.36 \times 10^{17}$

$= 3.6 \times 10^{16}$ MeV

$\approx 4 \times 10^{16}$ MeV

Energy from annihilation = $2\ m_0c^2\ (0.376 \times 10^{17})$

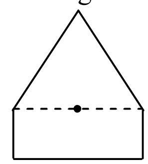
$\approx 1.02\ (0.376 \times 10^{17})$ MeV

$\approx 4 \times 10^{16}$ MeV

Total energy = $E_{reaction} + E_{anhillation}$

$E_T \approx 8 \times 10^{16}$ MeV

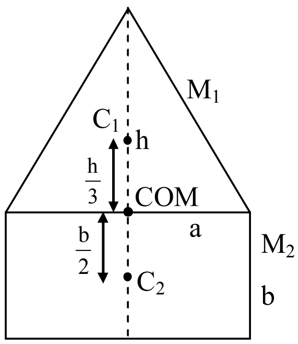
23. Two uniform plates of the same thickness and area but of different materials, one shaped like an isosceles triangle and the other shaped like a rectangle are joined together to form a composite body as shown in the figure. If the centre of mass of the composite body is located at the midpoint of their common side, the ratio between masses of the triangle to that of the rectangle is -



(A) 1 : 1 (B) 4 : 3 (C) 3 : 4 (D) 2 : 1

Ans. **[C]**

Sol.



Equal are

$\frac{1}{2}ah = ab$

$h = 2b$

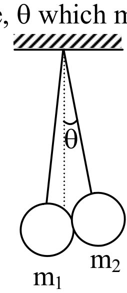
$M_1\frac{h}{3} = M_2\frac{b}{2}$ [centre of mass of combination at the mid-point of their common edge]

$\frac{M_1}{M_2} = \frac{3}{2}\frac{b}{h}$

$\frac{M_1}{M_2} = \frac{3}{2}\left[\frac{1}{2}\right]$

$\frac{M_1}{M_2} = \frac{3}{4}$

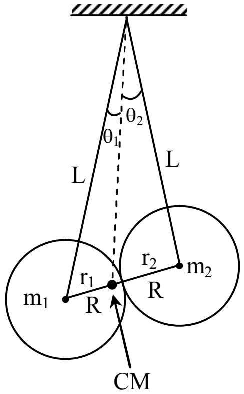
24. Two spherical objects each of radii R and masses m_1 and m_2 are suspended using two strings of equal length L as shown in the figure (R << L). The angle, θ which mass m_2 makes with the vertical is approximately -



(A) $\frac{m_1R}{(m_1+m_2)L}$ (B) $\frac{2m_1R}{(m_1+m_2)L}$ (C) $\frac{2m_2R}{(m_1+m_2)L}$ (D) $\frac{m_2R}{(m_1+m_2)L}$

Ans. **[B]**

Sol.



Using concept of COM

$m_1r_1 = m_2r_2$

$r_1 + r_2 = 2R$

$$\left(\frac{m_2}{m_1}+1\right)r_2 = 2R$$

$$r_2 = \frac{2m_1R}{m_1+m_2}$$

$L \sin \theta_2 = r_2$ [R < < L]

$$\theta_2 = \frac{2m_1R}{(m_1+m_2)L}$$

25. A horizontal disk of moment of inertia 4.25 kg-m^2 with respect to its axis of symmetry is spinning counter clockwise at 15 revolutions per second about its axis, as viewed from above. A second disk of moment of inertia 1.80 kg-m^2 with respect to its axis of symmetry is spinning clockwise at 25 revolutions per second as viewed from above about the same axis and is dropped on top of the first disk. The two disks stick together and rotate as one about their axis of symmetry. The new angular velocity of the system as viewed from above is close to -

(A) 18 revolutions/second and clockwise

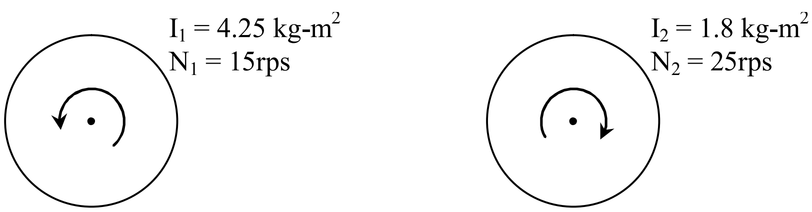
(B) 18 revolutions/second and counter clockwise

(C) 3 revolutions/second and clockwise

(D) 3 revolutions/second and counter clockwise

Ans. **[D]**

Sol.



Their axis of rotation is common.

Angular momentum conservation $I_1\omega_1 - \omega_2I_2 = (I_1 + I_2)\ \omega$

$2\pi\ (4.25)\ N_1 - 2\pi\ (1.8)\ N_2 = (4.25 + 1.80)\ N\ (2\pi)$

$(4.25 \times 15 - 1.8 \times 25) = (6.05)\ N$

$63.75 - 45 = 6.05\ N$

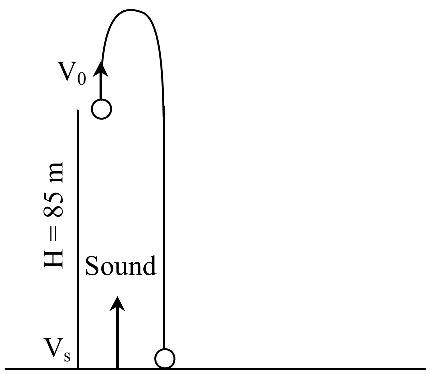
$N = 3$ rev/s.

26. A boy is standing on top of a tower of height 85 m and throws a ball in the vertically upward direction with a certain speed. If 5.25 seconds later he hears the ball hitting the ground, then the speed with which the boy threw the ball is (take g = 10 m/s^2, speed of sound in air = 340 m/s)

(A) 6 m/s (B) 8 m/s (C) 10 m/s (D) 12 m/s

Ans. **[B]**

Sol.



Time taken to reach sound after hit $t_s = \dfrac{H}{V_s}$

$t_s = \left(\dfrac{85}{340}\right)$ sec ; $t_s = 0.25$ sec

For ball time of flight T_f

$T_f + t_s = 5.25$ sec

$T_f = 5.25 - 0.25$

$T_f = 5$ sec

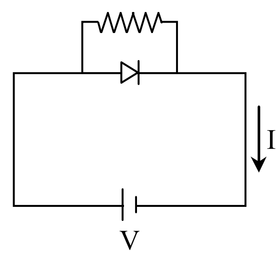
For ball

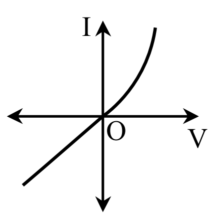
$V_0t - \dfrac{1}{2}gt^2 = -H \;\; \Rightarrow V_0\,(5) - \dfrac{1}{2}10(5)^2 = -85$

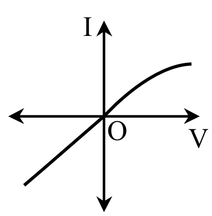
$5V_0 = 40$

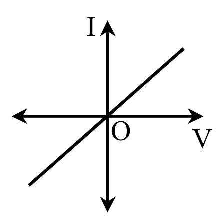
$V_0 = 8$ m/s

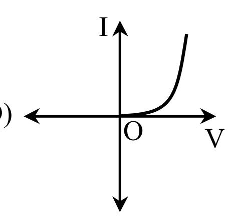
27. For a diode connected in parallel with a resistor, which is the most likely current (I) – voltage (V) characteristic ?



(A) 

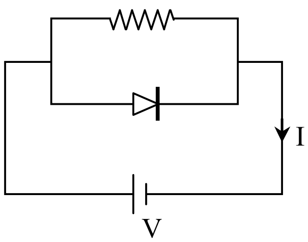
(B) 

(C) 

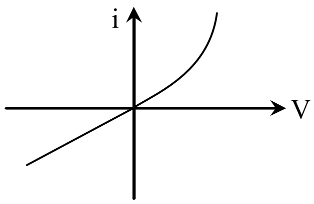
(D) 

Ans. **[A]**

Sol.



When V is positive in forward bias the potential drop on diode is low. When V is negative current will pass through the resister as V = IR. (in diode current almost zero because of reverse bias diode)



28. A beam of monoenergetic electrons, which have been accelerated from rest by a potential U, is used to form an interference pattern in a Young's Double slit experiment. The electrons are now accelerated by potential 4U. The fringe width -

(A) remains the same
(B) is half the original fringe width
(C) is twice the original fringe width
(D) is one-fourth the original fringe width

Ans. **[B]**

Sol. Debroglie wavelength of electron

$\lambda = \frac{h}{\sqrt{2mK}}$ Kinetic energy $\boxed{K = qV}$

$\lambda \propto \frac{1}{\sqrt{V}}$

Potential becomes four time so wavelength becomes half.

Fringe width (β) = $\frac{\lambda D}{d}$

$\boxed{\beta \propto \lambda}$

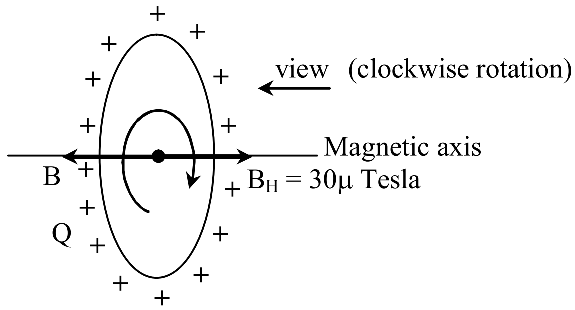
Fringe width becomes half.

29. A point charge Q(= 3×10^{-12}C) rotates uniformly in a vertical circle of radius R =1 mm. The axis of the circle is aligned along the magnetic axis of the earth. At what value of the angular speed ω, the effective magnetic field at the center of the circle will be reduced to zero ? (Horizontal component of Earth's magnetic field is 30 micro Tesla)

(A) 10^{11} rad/s (B) 10^{9} rad/s (C) 10^{13} rad/s (D) 10^{7} rad/s

Ans. **[A]**

Sol.



$B = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{Q\omega}{R}\right)$

$B = B_H$ (at centre effective magnetic field become zero)

$\frac{\mu_0 Q\omega}{4\pi R} = B_H$

$\omega = \frac{B_H(4\pi R)}{\mu_0 Q}$ ($B_H = 30 \times 10^{-6}$ T; R = 1 mm; $Q = 3 \times 10^{-12}$C)

$\omega = 10^{11}$ rad/s

30. A closed bottle containing water at 30ºC is open on the surface of the moon. Then -

(A) the water will boil
(B) the water will come as a spherical ball
(C) the water will freeze
(D) the water will decompose into hydrogen and oxygen

Ans. **[A]**

Sol. Because on earth there is no atmosphere. So water will boil.
(At Boiling point vapour pressure = Atmospheric pressure, in open vessel)

31. A simple pendulum of length ℓ is made to oscillate with an amplitude of 45 degrees. The acceleration due to gravity is g. Let $T_0 = 2\pi\sqrt{\ell/g}$. The time period of oscillation of this pendulum will be -

(A) T_0 irrespective of the amplitude
(B) slightly less than T_0
(C) slightly more than T_0
(D) dependent on whether it swings in a plane aligned with the north-south or east-west directions

Ans. **[C]**

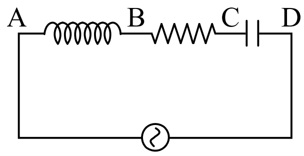
Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}\left(1+\frac{\theta^2}{16}\right)$ (This is valid when θ is not small)

$T_0 = 2\pi\sqrt{\frac{\ell}{g}}$ (for small θ)

$T = T_0\left(1+\frac{\theta^2}{16}\right)$

$T > T_0$

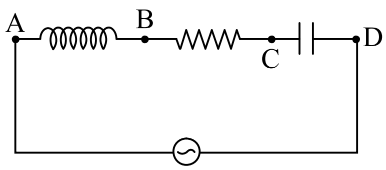
32. An ac voltmeter connected between points A and B in the circuit below reads 36 V. If it is connected between A and C, the reading is 39 V. The reading when it is connected between B and D is 25 V. What will the voltmeter read when it is connected between A and D ? (Assume that the voltmeter reads true rms voltage values and that the source generates a pure ac)



(A) $\sqrt{481}$ V (B) 31V (C) 61 V (D) $\sqrt{3361}$ V

Ans. **[A]**

Sol.



Voltmeter between A & B $V_L = 36$ V ...(1)

between A & C $\sqrt{V_L^2 + V_R^2} = 39$ ...(2)

between B & D $\sqrt{V_L^2 + V_R^2} = 25$ ...(3)

from equation (1) & (2) $V_R^2 = 39^2 - 36^2$

$V_R = 15$ V ...(4)

From Eq. (3) & (4) $V_C^2 = 25^2 - 15^2$

$V_C = 20$ V ...(5)

When connected through AD

$V_{rms} = \sqrt{(V_L - V_C)^2 + V_R^2}$

$\Rightarrow \sqrt{16^2 + 15^2}$

$\Rightarrow \sqrt{481}$ V

33. A donor atom in a semiconductor has a loosely bound electron. The orbit of this electron is considerably affected by the semiconductor material but behaves in many ways like an electron orbiting a hydrogen nucleus. Given that the electron has an effective mass of 0.07 m_e, (where m_e is mass of the free electron) and the space in which it moves has a permittivity $13\varepsilon_0$, then the radius of the electron's lowermost energy orbit will be close to (The Bohr radius of the hydrogen atom is 0.53 Å)

(A) 0.53 Å (B) 243 Å (C) 10 Å (D) 100 Å

Ans. **[D]**

Sol. From Bohr postulates

$$\frac{kze^2}{r^2} = \frac{mv^2}{r} \quad \text{....(i)}$$

$$mvr = \frac{nh}{2\pi} \quad \text{....(ii)}$$

$$\Rightarrow v = \frac{e^2}{2\varepsilon_0 h}\,\frac{z}{n}$$

$$r = \frac{nh}{2\pi mv}$$

$$r = \frac{nh}{2\pi m\left(\frac{e^2}{2\varepsilon_0 h}\right)\left(\frac{z}{n}\right)}$$

$$r = \left(\frac{\varepsilon_0 h^2}{\pi m e^2}\right)\left(\frac{n^2}{z}\right)$$

$$r = r_0\frac{n^2}{z}$$

Because the medium of permittivity $\varepsilon = 13\ \varepsilon_0$

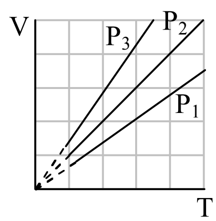
effective mass $m = 0.07\ m_e$

$$r = \frac{13 r_0}{0.07}\ \frac{n^2}{z}$$

At ground state (n = 1, assuming like H atom, z = 1)

$$r = \frac{13}{0.07}\ (0.53)\ \text{Å} \quad \Rightarrow r \approx 100\ \text{Å}$$

34. The state of an ideal gas was changed isobarically. The graph depicts three such isobaric lines. Which of the following is true about the pressures of the gas ?



(A) $P_1=P_2=P_3$ (B) $P_1>P_2>P_3$ (C) $P_1<P_2<P_3$ (D) $P_1/P_2=P_3/P_1$

Ans. **[B]**

Sol. Ideal gas equation PV = nRT

For isobaric process

$$V = \left(\frac{nR}{P}\right) T\ (V \propto T \text{ (straight line)})$$

$$\text{Slope of line} = \left(\frac{nR}{P}\right)$$

$$\text{slope} \propto \frac{1}{P}$$

$\text{slope}_3 > \text{slope}_2 > \text{slope}_1$

$P_3 < P_2 < P_1$

35. A metallic ring of radius a and resistance R is held fixed with its axis along a spatially uniform magnetic field whose magnitude is $B_0 \sin(\omega t)$. Neglect gravity. Then,

(A) the current in the ring oscillates with a frequency of 2ω.

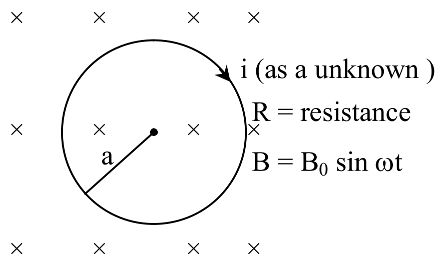
(B) the joule hearting loss in the ring is proportional to a^2,

(C) the force per unit length on the ring will be proportional to B_0^2.

(D) the net force on the ring is non-zero

Ans. [C]

Sol.



$$\text{Emf} = \frac{-d\phi}{dt} \Rightarrow \varepsilon = -\frac{d}{dt}(BA) \Rightarrow \varepsilon = -\frac{AdB}{dt}$$

$$\Rightarrow \varepsilon = -A B_0\omega \cos \omega t \Rightarrow i = \frac{\varepsilon}{R}$$

$$i = -\frac{B_0\omega A}{R}\cos \omega t$$

current oscillates with "ω".

Heating loss = i^2R

$H \propto i^2$ $\qquad \left[i = -\dfrac{B_0\omega(\pi a^2)}{R}\cos \omega t\right]$

$H \propto B_0^2\,\omega^2 a^4$

Force on $d\ell$ length

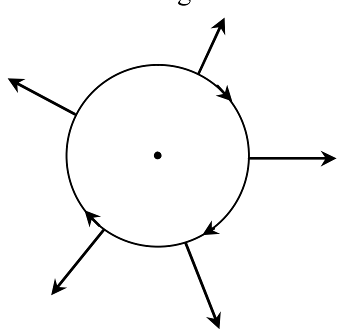
$|F| = Bid\ell$

$$|F| = B_0 \sin \omega t \left(\frac{B_0\omega\pi\, a^2}{R}\right)\cos \omega t \,.\, d\ell$$

$$\text{Force per unit length} = \frac{|F|}{d\ell} = \frac{B_0^2\omega\pi\, a^2}{R}\sin \omega t \cos \omega t$$

Force per unit length $\propto B_0^2$

Net force on ring will be zero.



(Force cancel)

36. The dimensions of the area A of a black hole can be written in terms of the universal gravitational constant G, its mass M and the speed of light c as $A = G^{\alpha}M^{\beta}c^{\gamma}$. Here -

(A) $\alpha = -2, \beta = -2$, and $\gamma = 4$ (B) $\alpha = 2, \beta = 2$, and $\gamma = -4$

(C) $\alpha = 3, \beta = 3$, and $\gamma = -2$ (D) $\alpha = -3, \beta = -3$, and $\gamma = 2$

Ans. **[B]**

Sol. $A = G^{\alpha} M^{\beta} C^{\gamma}$

$[M^0L^2T^0] = [M^{-1}L^3T^{-2}]^{\alpha} [M]^{\beta}[LT^{-1}]^{\gamma}$

$-\alpha + \beta = 0 \Rightarrow \alpha = \beta$

$3\alpha + \gamma = 2$

$-2\alpha - \gamma = 0$

on solving

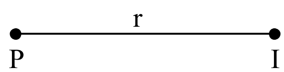
$\alpha = 2, \beta = 2, \gamma = -4$

37. A 160 watt infrared source is radiating light of wavelength 50000Å uniformly in all directions. The photon flux at a distance of 1.18 m is of the order of -

(A) $10\ m^{-2}s^{-1}$ (B) $10^{10}m^{-2}s^{-1}$ (C) $10^{15}\ m^{-2}s^{-1}$ (D) $10^{20}m^{-2}s^{-1}$

Ans. **[D]**

Sol. $P = 160$ watt, $\lambda = 50000$Å



$$I = \frac{P}{4\pi r^2}$$

$$nh\nu = \frac{P}{4\pi r^2}$$

$$\Rightarrow n = \frac{P}{4\pi r^2 (h\nu)}$$ (n = no. of photons per sec per m^2)

$$n = \frac{P\lambda}{4\pi r^2 hc}$$

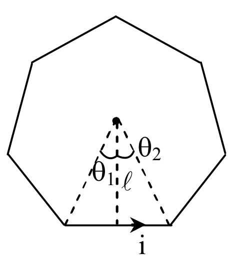
$n = 10^{20}\ m^{-2}\ s^{-1}$

38. A wire bent in the shape of a regular n-polygonal loop carries a steady current I. Let ℓ be the perpendicular distance of a given segment and R be the distance of vertex both from the centre of the loop. The magnitude of the magnetic field at the centre of the loop is given by -

(A) $\frac{n\mu_0 I}{2\pi\ell}\sin(\pi/n)$ (B) $\frac{n\mu_0 I}{2\pi R}\sin(\pi/n)$ (C) $\frac{n\mu_0 I}{2\pi\ell}\cos(\pi/n)$ (D) $\frac{n\mu_0 I}{2\pi R}\cos(\pi/n)$

Ans. **[A]**

Sol.



n sides, n wires

$$\theta_1 = \theta_2 = \frac{\pi}{n}$$

B_{net} at centre = n × B due to one side

$$B_{net} = \frac{n \times \mu_0 I}{4\pi\ell}[\sin\theta_1 + \sin\theta_2] \Rightarrow \frac{n\mu_0 I}{2\pi\ell}\sin\frac{\pi}{n}$$

39. The intensity of sound during the festival season increased by 100 times. This could imply a decibel level rise from -

(A) 20 to 120 dB (B) 70 to 72 dB (C) 100 to 10000 dB (D) 80 to 100 dB

Ans. **[D]**

Sol. Loudness of sound in decibel dB = 10 $\log_{10}\left(\frac{I}{I_0}\right)$

when intensity of sound become 100 I then new decibel level = dB' = 10 $\log_{10}\left(\frac{100\,I}{I_0}\right)$

dB' – dB = 10 $\log_{10}$ 100

dB' – dB = 20

$\therefore$ decibel rise by 20 dB

only one option i.e. 80 to 100 dB match with it.

40. One end of a slack wire (Young's modulus Y, length L and cross-section area A) is clamped to rigid wall and the other end to a block (mass m) which rests on a smooth horizontal plane. The block is set in motion with a speed ν. What is the maximum distance the block will travel after the wire becomes taut ?

(A) $\nu\sqrt{\frac{mL}{AY}}$ (B) $\nu\sqrt{\frac{2mL}{AY}}$ (C) $\nu\sqrt{\frac{mL}{2AY}}$ (D) $L\sqrt{\frac{m\nu}{AY}}$

Ans. **[A]**

Sol. Initially wire is slack so it do not have any deformation energy. When block is given some velocity it move due to kinetic energy, one wire get taut. Internal force get develop in wire and KE start decreases and deformation energy of wire increase. Till block come at rest using energy conservation

$$\frac{1}{2}mv^2 = \frac{1}{2}Y \times (\text{strain})^2 \times A \times L$$

$$\frac{1}{2}mv^2 = \frac{1}{2}Y \times \left(\frac{x}{L}\right)^2 \times A \times L$$

$$x = v\sqrt{\frac{mL}{AY}}$$

CHEMISTRY

41. The Lewis acid strength of BBr_3, BCl_3 and BF_3 is in the order

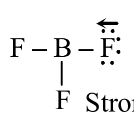
(A) $BBr_3<BCl_3<BF_3$ (B) $BCl_3<BF_3<BBr_3$ (C) $BF_3<BCl_3<BBr_3$ (D) $BBr_3<BF_3<BCl_3$

Ans. **[C]**

Sol. Lewis acid strength of BBr_3, BCl_3 and BF_3 is in the order of

$$BF_3 < BCl_3 < BBr_3$$

Due to back bonding



Strong 2p – 2p Back bonding

42. O^{2-} is isoelectronic with

(A) Zn^{2+} (B) Mg^{2+} (C) K^{+} (D) Ni^{2+}

Ans. **[B]**

Sol. O^{-2} is isoelectronic with Mg^{+2}

$O^{-2} \longrightarrow 8 + 2 = 10\ e^{-}$

$Mg^{+2} \longrightarrow 12 - 2 = 10\ e^{-}$

43. The H-C-H, H-N-H, and H-O-H bond angles (in degrees) in methane, ammonia and water are respectively, closest to

(A) 109.5, 104.5, 107.1 (B) 109.5, 107.1, 104.5

(C) 104.5, 107.1, 109.5 (D) 107.1, 104.5, 109.5

Ans. **[B]**

Sol.

	H–C–H	H–$\ddot{N}$–H	H–$\ddot{O}$–H
	CH_4	NH_3	H_2O
	B.P. = 4	B.P. = 3	B.P. = 2
	L.P. = 0	L.P. = 1	L.P. = 1
Hybridization	sp^3	sp^3	sp^3
Bond angle	109°, 28'	107°.1	104.5°

44. In alkaline medium, the reaction of hydrogen peroxide with potassium permanganate produces a compound in which the oxidation state of Mn is

(A) 0 (B) +2 (C) +3 (D) +4

Ans. **[D]**

Sol. $KMnO_4 + H_2O_2 \xrightarrow{OH^-} MnO_2 + H_2O$

45. The rate constant of a chemical reaction at a very high temperature will approach

(A) Arrhenius frequency factor divided by the ideal gas constant

(B) activation energy

(C) Arrhenius frequency factor

(D) activation energy divided by the ideal gas constant

Ans. **[C]**

Sol. $K = Ae^{-Ea/RT}$

$T \to \infty \quad k = A$

46. The standard reduction potentials (in V) of a few metal ion/metal electrodes are given below.
$Cr^{3+}/Cr = -0.74$; $Cu^{2+}/Cu = +0.34$; $Pb^{2+}/Pb = -0.13$; $Ag^{+}/Ag = +0.8$. The reducing strength of the metals follows the order

(A) Ag > Cu > Pb > Cr (B) Cr > Pb > Cu > Ag (C) Pb > Cr > Ag > Cu (D) Cr > Ag > Cu > Pb

Ans. **[B]**

Sol. SRP↓ Reducing power↑

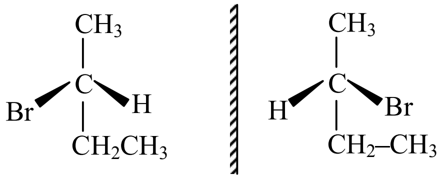
47. Which of the following molecules can exhibit optical activity?

(A) 1-bromopropane (B) 2-bromobutane (C) 3-bromopentane (D) bromocyclohexane

Ans. **[B]**

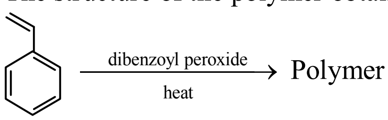
Sol.
$$\underset{*}{CH_3{-}\overset{\overset{\displaystyle Br}{|}}{C}H{-}CH_2{-}CH_3}$$ 2 Bromo butane

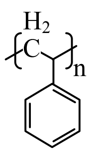
This molecule contain 1 chiral centre and molecule having one chiral carbon do not have any type of symmetry so it is optically active



Non superimposable on mirror image

48. The structure of the polymer obtained by the following reaction is





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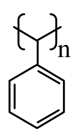

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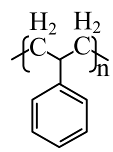

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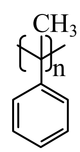
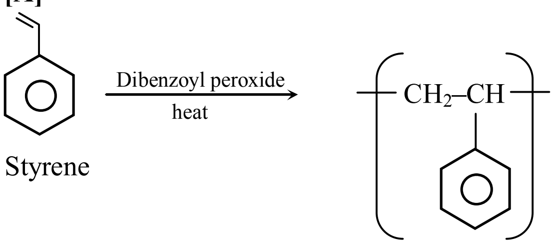


IV

(A) I (B) II (C) III (D) IV

Ans. **[A]**

Sol.



Styrene

Polystyrene

49. The major product of the reaction between CH_3CH_2ONa and $(CH_3)_3CCl$ in ethanol is

(A) $CH_3CH_2OC(CH_3)_3$ (B) $CH_2 = C(CH_3)_2$

(C) $CH_3CH_2C(CH_3)_3$ (D) $CH_3CH{=}CHCH_3$

Ans. **[B]**

Sol.
$$CH_3{-}\underset{\underset{\displaystyle CH_3}{|}}{\overset{\overset{\displaystyle CH_3}{|}}{C}}{-}Cl \xrightarrow[\text{Alkoxide (base)}]{CH_3{-}CH_2{-}\overset{\ominus}{O}\ \overset{\oplus}{Na}} CH_3{-}\overset{\overset{\displaystyle CH_3}{|}}{C}{=}CH_2$$

3° (halide)

Alkoxide ion is strong nucleophile and strong base & with 3° Alkyl halide Alkenes is the major product [E_2 Elimination]

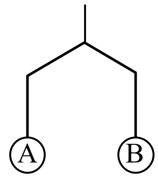
50. When H_2S gas is passed through a hot acidic aqueous solution containing Al^{3+}, Cu^{2+}, Pb^{2+} and Ni^{2+}, a precipitate is formed which consists of

(A) CuS and Al_2S_3 (B) PbS and NiS (C) CuS and NiS (D) PbS and CuS

Ans. [D]

Sol. H_2S gas is passed through a hot acidic aqueous solution containing Al^{+3}, Cu^{+2}, Pb^{+2} and Ni^{+2}

II group elements give ppt $\longrightarrow$ CuS, PbS



Pb^{+2} Cu^{+2} Cd^{+2} Bi^{+2} Hg^{+2} As^{+3} Sb^{+3} Sn^{+2} Sn^{+4}

51. The electronic configuration of an element with the largest difference between the 1st and 2nd ionization energies is

(A) $1s^2\ 2s^2\ 2p^6$ (B) $1s^2\ 2s^2\ 2p^6\ 3s^1$ (C) $1s^2\ 2s^2\ 2p^6\ 3s^2$ (D) $1s^2\ 2s^2\ 2p^1$

Ans. [B]

Sol. $Na = 1s^2\ 2s^2\ 2p^6\ 3s^1$

$Na^+ \rightarrow 1s^2\ 2s^2\ 2p^6$ [Ne]

[Ne] is inert gas, so, electron removal is very difficult so I.P. is very high.

$$Na^{+2} \longrightarrow 1s^2\ 2s^2\ 2p^5$$

52. The order of electronegativity of carbon in sp, sp^2 and sp^3 hybridized states follows

(A) $sp > sp^2 > sp^3$ (B) $sp^3 > sp^2 > sp$ (C) $sp > sp^3 > sp^2$ (D) $sp^2 > sp > sp^3$

Ans. [A]

Sol. Electronegativity $\propto$ % s character

$CH_3 - CH_3$	<	$CH_2 = CH_2$	<	$CH \equiv CH$
sp^3		sp^2		sp
25% s character		33.3% s character		50% s character

53. The most abundant transition metal in human body is

(A) copper (B) iron (C) zinc (D) manganese

Ans. [B]

Sol. Fact

54. The molar conductivities of HCl, NaCl, CH_3COOH, and CH_3COONa at infinite dilution follow the order

(A) HCl > CH_3COOH > NaCl > CH_3COONa (B) CH_3COONa > HCl > NaCl > CH_3COOH

(C) HCl > NaCl > CH_3COOH > CH_3COONa (D) CH_3COOH > CH_3COONa > HCl > NaCl

Ans. [A]

Sol. Since conductance of H^+ is highest so molar conductivity of HCl will be highest and after that conductance of CH_3COOH will come

$\therefore$ order HCl > CH_3COOH > NaCl > CH_3COONa

55. The spin only magnetic moment of $[ZCl_4]^{2-}$ is 3.87 BM where Z is

(A) Mn (B) Ni (C) Co (D) Cu

Ans. **[C]**

Sol. $[ZCl_4]^{-2}$ is 3.87

$x + (-1)\,4 = 2 \qquad u = \sqrt{n(n+2)}$

$x = +4 - 2$

$x = +2$

$Mn^{+2} \longrightarrow 4s^{\circ}3d^{5}$ $\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$

$Ni^{+2} \longrightarrow 4s^{\circ}3d^{8}$ $\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}$

$Co^{+2} \longrightarrow 4s^{\circ}3d^{7}$ $\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$

$Cu^{+2} \longrightarrow 4s^{\circ}3d^{9}$ $\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}$

56. If α–D–glucose is dissolved in water and kept for a few hours, the major constituent(s) present in the solution is (are)

(A) α–D-glucose

(B) mixture of β–D-glucose and open chain D-glucose

(C) open chain D-glucose

(D) mixture of α–D-glucose and β–D-glucose

Ans. **[D]**

Sol. α-D Glucose $\rightleftharpoons$ Open chain Structure Glucose $\rightleftharpoons$ β-D Glucose

35% 65%

57. The pH of 1N aqueous solutions of HCl, CH_3COOH and HCOOH follows the order

(A) HCl > HCOOH > CH_3COOH

(B) HCl = HCOOH > CH_3COOH

(C) CH_3COOH > HCOOH > HCl

(D) CH_3COOH = HCOOH > HCl

Ans. **[C]**

Sol. $pH = -\log[H^+]$

$pH \propto \dfrac{1}{[H^+]}$

Order of pH $CH_3COOH > HCOOH > HCl$

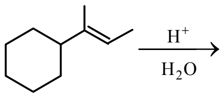
Acidic strength order $CH_3-\underset{\underset{O}{\|}}{C}-OH < H-\underset{\underset{O}{\|}}{C}-OH < HCl$

Acidic strength ∝ stability of Anion

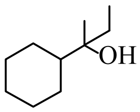
$Cl^{\ominus} > H-\underset{\underset{O}{\|}}{C}-\overset{\ominus}{O} > \overset{+I}{CH_3}-\underset{\underset{O}{\|}}{C}-\overset{\ominus}{O}$

Size large

58. The major product of the reaction



Products is



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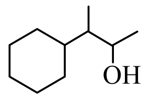

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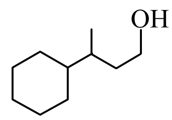

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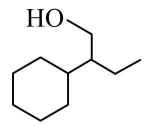
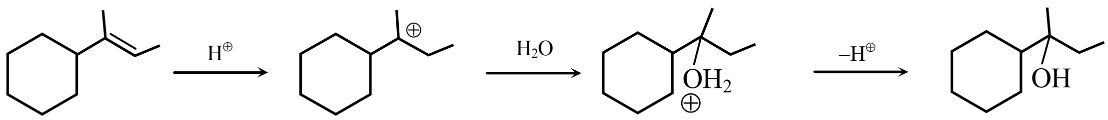


IV

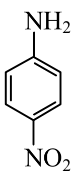
(A) I (B) II (C) III (D) IV

Ans. **[A]**

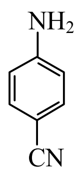
Sol.



59. Reaction of aniline with $NaNO_2$ + dil. HCl at 0 ºC followed by reaction with CuCN yields



I



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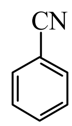

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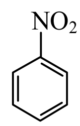
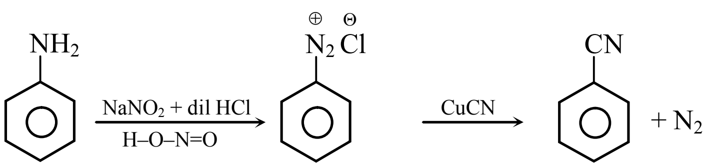


IV

(A) I (B) II (C) III (D) IV

Ans. **[C]**

Sol.



Benzene diazonium chloride

Cynobenzene (III)

60. Schottky defect in a crystal arises due to

(A) creation of equal number of cation and anion vacancies

(B) creation of unequal number of cation and anion vacancies

(C) migration of cations to interstitial voids

(D) migration of anions to interstitial voids

Ans. **[A]**

Sol. By definition

BIOLOGY

61. Immunosuppressive drugs like cyclosporin delay the rejection of graft post organ transplantation by

(A) inhibiting T cell infiltration (B) killing B cells

(C) killing macrophages (D) killing dendrite cells

Ans. **[A]**

Sol. Immunosuppressive Drug inhibits T-cell infiltration

62. Which one of these substances will repress the *lac* operon?

(A) Arabinose (B) Glucose (C) Lactose (D) Tryptophan

Ans. **[B]**

Sol. Presence of glucose inhibit the lac operon

63. Assume a spherical mammalian cell has a diameter of 27 microns. If a polypeptide chain with alpha helical conformation has to stretch across the cell, how many amino acids should it be comprised of?

(A) 18000 (B) 1800 (C) 27000 (D) 12000

Ans. **[A]**

Sol. No. of amino acid in one term in α helix = 3.6

Pitch length for α Helix = 5.4 A°

$\therefore$ The No. of Amino acid in polypeptide

$$= \frac{3.6}{5.4 \times 10^{-10}} \times 27 \times 10^{-6}$$

$= 1.8 \times 10^4$ Amino acid = 18000 amino acid

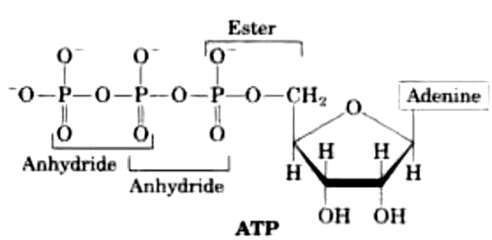
64. Which one of the following has phosphoric acid anhydride bonds?

(A) Deoxy ribonucleic acid (B) Ribonucleic acid

(C) dNTPs (D) Phospholipids

Ans. **[C]**

Sol.



65. The two components of autonomous nervous system have antagonistic actions. But in certain cases their effects are mutually helpful. Which of the following statement is correct?

(A) At rest, the control of heart beat is not by the vagus nerve

(B) During exercise the sympathetic control decreases

(C) During exercise the parasympathetic control decreases

(D) Stimulation of sympathetic system results in constriction of the pupil

Ans. **[C]**

Sol. Parasympathetic control decreased during exercise.

66. In a random DNA sequence, what is the lowest frequency of encountering a stop codon?

(A) 1 in 20 (B) 1 in 3 (C) 1 in 64 (D) 1 in 10

Ans. **[A]**

Sol. Total no. of codon = 64

Functional codon = 61

Stop codon = 3

$\therefore$ frequency of encountering stop codon $= \frac{3}{61} \approx \frac{1}{20}$

67. The two alleles that determine the blood group AB of an individual are located on

(A) two different autosomes

(B) the same autosome

(C) two different sex chromosomes

(D) one on sex chromosome and the other on an autosome

Ans. **[B]**

Sol. Allele of the same gene are present at same gene locus on homologous chromosome

68. In biotechnology applications, a selectable marker is incorporated in a plasmid

(A) to increase its copy number (B) to increase the transformation efficiency

(C) to eliminate the non-transformants (D) to increase the expression of the gene of interest

Ans. **[C]**

Sol. Selectable marker is used to eliminate the non transformant from the transformants

69. Spermatids are formed after the second meiotic division from secondary spermatocytes. The ploidy of the secondary spermatocytes is

(A) n (B) 2n (C) 3n (D) 4n

Ans. **[A]**

Sol. Secondary spermatocyte formed by meiosis-I .

70. Phospholipids are formed by the esterification of

(A) three ethanol molecules with three fatty acid molecules

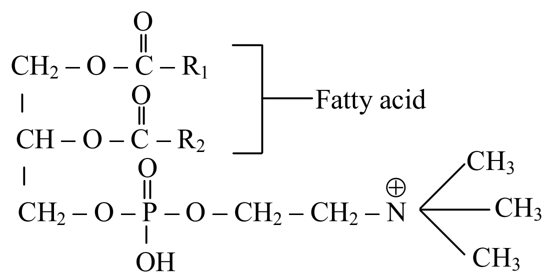
(B) one glycerol and two fatty acid molecules

(C) one glycerol and three fatty acid molecules

(D) one ethylene glycol and two fatty acids molecules

Ans. **[B]**

Sol. Phospholipid : two fatty acid and one phosphorylated nitrogenous organic compound attached to glycerol



71. Given the fact that histone binds DNA, it should be rich in

(A) arginine, lysine (B) cysteine, methionine (C) glutamate, aspartate (D) isoleucine, leucine

Ans. **[A]**

Sol. Histone is basic protein and it is rich in lysine and arginine

72. If molecular weight of a polypeptide is 15.3 kDa, what would be the minimum number of nucleotides in the mRNA that codes for this polypeptide? Assume that molecular weight of each amino acid is 90 Da.

(A) 510 (B) 663 (C) 123 (D) 170

Ans. **[A]**

Sol. Molecular weight of polypeptide = 15.3 kda

Molecular weight of amino acid = 90 Da

$\therefore$ No. of Amino acid in polypeptide $= \dfrac{15.3 \times 10^3}{90}$

$= 170$

$\because$ One amino acid is coded by = 3 Nitrogen base

$\therefore$ 170 amino acid would be coded by = 170 × 3 = 510 Nitrogen base

73. Melting temperature for double stranded DNA is the temperature at which 50% of the double stranded molecules are converted into single stranded molecules. Which one of the following DNA will have the highest melting temperature?

(A) DNA with 15% guanine (B) DNA with 30% cytosine

(C) DNA with 40% thymine (D) DNA with 50% adenine

Ans. **[B]**

Sol. Melting temperature of DNA $\propto$ GC content

74. Following are the types of immunoglobulin and their functions. Which one of the following is INCORRECTLY paired?

(A) IgD : viral pathogen (B) IgG : phagocytosis

(C) IgE : allergic reaction (D) IgM : complement fixation

Ans. **[A]**

Sol. IgD activates , B- lymphocyte

75. Which one of the following can be used to detect amino acids?

(A) Iodine vapour (B) Ninhydrin (C) Ethidium bromide (D) Bromophenol blue

Ans. **[B]**

Sol. Ninhydrin stain is used to detect primary or secondary amino acid and ammonia.

76. Mutation in a single gene can lead to changes in multiple traits. This is an example of

(A) Heterotrophy (B) Co-dominance (C) Penetrance (D) Pleiotropy

Ans. **[D]**

Sol. Pleiotrophy : A single gene affects more than one phenotype.

77. Which one of the following is used to treat cancers?

(A) Albumin (B) Cyclosporin A (C) Antibodies (D) Growth hormone

Ans. **[C]**

Sol. Monoclonal antibodies are used for treatment of cancer.

78. Which of the following processes leads to DNA ladder formation?

(A) Necrosis (B) Plasmolysis (C) Apoptosis (D) Mitosis

Ans. **[C]**

Sol. In apoptosis or programmed cell death DNA degradation occur which form DNA ladder.

79. Co-enzymes are components of an enzyme complex which are necessary for its function. Which of these is a known co-enzyme?

(A) Zinc (B) Vitamin B_{12} (C) Chlorophyll (D) Heme

Ans. **[B]**

Sol. Co-enzymes are the organic compound which are necessary for function of enzyme.

80. The peptidoglycans of bacteria consist of

(A) sugars, D-amino acids and L-amino acids (B) sugars and only D-amino acids

(C) sugars and only L-amino acids (D) sugars and glycine

Ans. **[A]**

Part – II

Two - Mark Questions

MATHEMATICS

81. Let $x = (\sqrt{50} + 7)^{1/3} - (\sqrt{50} - 7)^{1/3}$. Then -

(A) $x = 2$

(B) $x = 3$

(C) x is a rational number, but not an integer

(D) x is an irrational number

Ans. **[A]**

Sol. $x = (\sqrt{50}+7)^{1/3} - (\sqrt{50}-7)^{1/3}$

$x^3 = (\sqrt{50}+7) - (\sqrt{50}-7) - 3(\sqrt{50}+7)\ (\sqrt{50}-7)\ ((\sqrt{50}+7)^{1/3} - (\sqrt{50}-7)^{1/3})$

$x^3 = 14 - 3\,(1)\,(x)$

$x^3 = 14 - 3x$

$x^3 + 3x - 14 = 0$

$x = 2$

82. Let

$(1 + x + x^2)^{2014} = a_0 + a_1x + a_2x^2 + a_3x^3 + ... + a_{4028}x^{4028}$, and let

$A = a_0 - a_3 + a_6 - .... + a_{4026}$,

$B = a_1 - a_4 + a_7 - .... - a_{4027}$,

$C = a_2 - a_5 + a_8 - .... + a_{4028}$,

Then -

(A) $|A| = |B| > |C|$ (B) $|A| = |B| < |C|$ (C) $|A| = |C| > |B|$ (D) $|A| = |C| < |B|$

Ans. **[D]**

Sol. $(1 + x + x^2)^{2014} = a_0 + a_1x + a_2x^2 + a_3x^3 + ... + a_{4028}x^{4028}$

put $x = -1$

$1 = 1 = a_0 - a_1 + a_2 - a_3 + a_4 - a_5 + a_6 - a_7 \ldots.$ ... (1)

put $x = -\omega$

$(2\omega)^{2014} = (1 - \omega + \omega^2)^{2014} = a_0 - a_1\omega + a_2\omega^2 - a_3 + a_4\omega - a_5\omega^2 + a_6\ldots\ldots$ ...(2)

Put $x = -\omega^2$

$(2\omega^2)^{2014} = (1 - \omega^2 + \omega)^{2014} = a_0 - a_1\omega^2 + a_2\omega - a_3 + a_4\omega^2 - a_5\omega\ldots\ldots$ ...(3)

Now, (1) + (2) + (3)

$\Rightarrow 1 + (2\omega)^{2014} + (2\omega^2)^{2014} = 3(a_0 - a_3 + a_6\ldots\ldots\ldots)$

$$\Rightarrow a_0 - a_3 + a_6\ldots\ldots\ldots = \frac{1 + 2^{2014}\omega + 2^{2014}\omega^2}{3}$$

$$A = \frac{1-2^{2014}}{3}$$

$$|A| = \frac{2^{2014}-1}{3}$$

and (1) + (2) × ω + (3)ω^2

$$\Rightarrow \frac{1+2^{2014}.\omega^{2014}.\omega+2^{2014}.(\omega^2)^{2014}.\omega^2}{3} = a_2 - a_5 + a_8\ldots\ldots$$

$$\Rightarrow \frac{1+2^{2014}+\omega^{2015}+2^{2014}.\omega^{4030}}{3} = C$$

$$\Rightarrow C = \frac{1-2^{2014}}{3} \Rightarrow |C| = \frac{2^{2014}-1}{3}$$

and similarly (1) + (2) × ω^2 + (3) × ω

$$B = \frac{1+2^{2014}.\omega^{2014}.\omega^2+2^{2014}.(\omega^2)^{2014}.\omega}{3}$$

$$= \frac{1+2^{2014}.\omega^{2016}+2^{2014}.\omega^{4029}}{3}$$

$$|B| = \frac{1+2^{2015}}{3}$$

$\therefore$ |B| > |A| = |C|

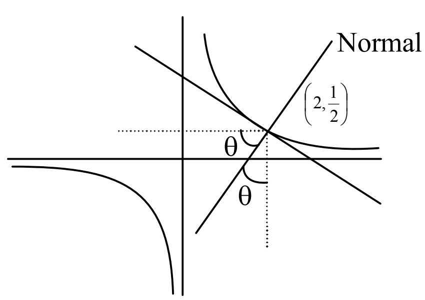
83. A mirror in the first quadrant is in the shape of a hyperbola whose equation is xy = 1. A light source in the second quadrant emits a beam of light that hits the mirror at the point (2, 1/2). If the reflected ray is parallel to the y-axis, the slope of the incident beam is -

(A) $\frac{13}{8}$ (B) $\frac{7}{4}$

(C) $\frac{15}{8}$ (D) 2

Ans. **[C]**

Sol.



Curve $y = \frac{1}{x}$

$$\frac{dy}{dx} = -\frac{1}{x^2}$$

slope of tangent $= -\frac{1}{4}$ at $\left(2, \frac{1}{2}\right)$

slope of normal = 4

where foot of normal is $\left(2, \frac{1}{2}\right)$

let slope at incident beam is 'm'

$$\left|\frac{4-m}{1+4m}\right| = \left|\frac{\infty - 4}{1 - 4.\infty}\right|$$

$$\frac{4-m}{1+4m} = \pm\frac{1}{4}$$

$$m = \frac{15}{8}$$

84. Let

$$C(\theta) = \sum_{n=0}^{\infty} \frac{\cos(n\theta)}{n!}$$

Which of the following statements is FALSE ?

(A) $C(0).C(\pi) = 1$ (B) $C(0) + C(\pi) > 2$

(C) $C(\theta) > 0$ for all $\theta \in R$ (D) $C'(\theta) \neq 0$ for all $\theta \in R$

Ans. **[D]**

Sol. $C(\theta) = \sum_{n=0}^{\infty} \frac{\cos n\theta}{n!}$

$$= \lim_{n\to\infty}\left(1 + \frac{\cos\theta}{1!} + \frac{\cos 2\theta}{2!} + \frac{\cos 3\theta}{3!} + \ldots + \frac{\cos n\theta}{n!}\right)$$

$C(0) = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots$ up to ∞ term

$= e$

$C(\pi) = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots$ up to ∞ term

$= e^{-1}$

Clearly $C(0) . C(\pi) = 1$

$C(0) . C(\pi) = e + \frac{1}{e} > 2$

$C'(\theta) = -\frac{\sin\theta}{1!} - 2\frac{\sin 2\theta}{2!} - \frac{3\sin 3\theta}{3!} + \ldots$ up to ∞ term

And that value is equal to zero at $\theta = 0$

85. Let a > 0 be a real number. Then the limit

$$\lim_{x\to 2}\frac{a^x + a^{3-x} - (a^2 + a)}{a^{3-x} - a^{x/2}}$$

is -

(A) 2 log a (B) $-\frac{4}{3}$ a

(C) $\frac{a^2 + a}{2}$ (D) $\frac{2}{3}$ (1 – a)

Ans. **[D]**

Sol. $$\lim_{x\to 2}\frac{a^x + a^{3-x} - (a^2 + a)}{a^{3-x} - a^{x/2}}\left(\frac{0}{0}\right)$$

Apply L hospital rule

$$\lim_{x\to 2}\frac{a^x \ell na - a^{3-x}\ell na}{-a^{3-x}\ell na - \frac{1}{2}a^{x/2}\ell na}$$

$$\frac{a^2\ell na - a\ell na}{-a\ell na - \frac{a}{2}\ell na} = \frac{a^2 - a}{-\frac{3a}{2}} = \frac{2}{3}(1 - a)$$

86. Let $f(x) = \alpha\, x^2 - 2 + \frac{1}{x}$ where α is a real constant. The smallest α for which $f(x) \geq 0$ for all $x > 0$ is -

(A) $\frac{2^2}{3^3}$ (B) $\frac{2^3}{3^3}$ (C) $\frac{2^4}{3^3}$ (D) $\frac{2^5}{3^3}$

Ans. **[D]**

Sol. $f(x) = \alpha x^2 - 2 + \frac{1}{x}$

$f(x) = \frac{\alpha x^3 - 2x + 1}{x}$ $\forall$ x (0, ∞)

so $\phi(x) = \alpha x^3 - 2x + 1$ should be positive

$\phi(x) = \alpha x^3 - 2x + 1$

$\phi'(x) = 3\alpha x^2 - 2 = 0$

$x = \pm\sqrt{\frac{2}{3\alpha}}$

Clearly $x = \sqrt{\frac{2}{3\alpha}}$ point of minima

$$\phi\left(\sqrt{\frac{2}{3\alpha}}\right) \geq 0$$

$$\sqrt{\frac{2}{3\alpha}}\left\{\alpha.\frac{2}{3\alpha} - 2\right\} + 1 \geq 0$$

$$\sqrt{\frac{2}{3\alpha}}\left(-\frac{4}{3}\right)+1 \geq 0$$

$$\sqrt{\frac{2}{3\alpha}}\left(\frac{4}{3}\right) \leq 1$$

$$\sqrt{\frac{2}{3\alpha}} \leq \frac{3}{4}$$

$$\frac{2}{3\alpha} \leq \frac{3^2}{4^2}$$

$$\alpha \geq \frac{32}{27}$$

87. Let $f : R \to R$ be a continuous function satisfying

$$f(x) + \int_0^x tf(t)dt + x^2 = 0,$$

for all $x \in R$. Then -

(A) $\lim_{x\to\infty} f(x) = 2$

(B) $\lim_{x\to-\infty} f(x) = -2$

(C) f(x) has more than one point in common with the x-axis

(D) f(x) is an odd functions

Ans. **[B]**

Sol. $f(x) + \int_0^x t\, f(t)dt + x^2 = 0$

$$f'(x) + xf(x) + 2x = 0$$

$$\frac{f'(x)}{f(x)+2} = -x$$

$$\int \frac{f'(x)}{(f(x)+2)}dx = -\int xdx$$

$$\ell n\,(f(x) + 2) = -\frac{x^2}{2} + c$$

$$f(x) + 2 = e^{-x^2/2} + c$$

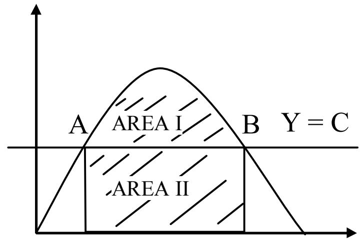
$$f(x) = k\, e^{-x^2/2} - 2$$

where $x = 0$ $\quad f(x) = 0 \quad k = 2$

$$f(x) = 2\,(e^{-x^2/2} - 1)$$

clearly $\lim\limits_{x\to-\infty} f(x) = -2$

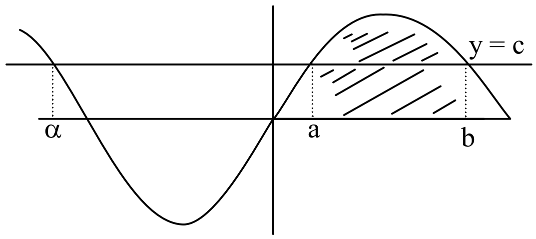
88. The figure shows a portion of the graph $y = 2x - 4x^3$. The line $y = c$ is such that the areas of the regions marked I and II are equal. If a, b are the x-coordinates of A, B respectively, then a + b equals -



(A) $\frac{2}{\sqrt{7}}$ (B) $\frac{3}{\sqrt{7}}$ (C) $\frac{4}{\sqrt{7}}$ (D) $\frac{5}{\sqrt{7}}$

Ans. **[A]**

Sol.



$\int_a^b (2x - 4x^3)dx = 2(b - a)c$

$(x^2 - x^4)_a^b = 2(b - a)c$

$(a + b)(1 - (a^2 + b^2)) = 2c$

$(a + b)(1 - (a + b)^2 + 2ab) = 2c$ ...(1)

again $2x - 4x^3 = c$

$4x^3 - 2x + c = 0 \begin{cases} a \\ b \\ \alpha \end{cases}$

$a + b + \alpha = 0$ clearly $a + b = -\alpha$ ...(2)

$ab + (a + b)\alpha = -\frac{1}{2}$ $ab = \alpha^2 - \frac{1}{2}$ ...(3)

$ab\alpha = -\frac{c}{4}$ $c = -4\alpha\left(\alpha^2 - \frac{1}{2}\right)$ ...(4)

put value of (a + b), ab, c from eq. (2), (3), (4) in equation (1) and solve it

$\left\{1 - \alpha^2 + 2\left(\alpha^2 - \frac{1}{2}\right)\right\} = -8\alpha\left(\alpha^2 - \frac{1}{2}\right)$

$1 - \alpha^2 + 2\alpha^2 - 1 = 8\alpha^2 - 4$

$\alpha^2 = 8\alpha^2 - 4$

$7\alpha^2 = 4$

$\alpha = \frac{2}{\sqrt{7}}$

89. Let $X_n = \{1, 2, 3, ..., n\}$ and let a subset A of X_n be chosen so that every pair of elements of A differ by at least 3. (For example, if n = 5, A can be $\varnothing$, $\{2\}$ or $\{1,5\}$ among others). When n = 10, let the probability that $1 \in A$ be p and let the probability that $2 \in A$ be q. Then -

(A) $p > q$ and $p - q = \frac{1}{6}$ (B) $p < q$ and $q - p = \frac{1}{6}$

(C) $p > q$ and $p - q = \frac{1}{10}$ (D) $p < q$ and $q - p = \frac{1}{10}$

Ans. **[C]**

Sol. when n = 10

Let A_r be no. of ways of selecting r numbers.

No. of selection of A is

$= n(A_0) + n(A_1) + n(A_2) + n(A_3) + n(A_4)$

$= 1 + 10 + (7 + 6 + 5 + \ldots + 1) + (4 + 3 + 2 + 1) + (3 + 2 + 1) + (2 + 1) + 1$

$= 11 + \frac{7.8}{2} + 10 + 6 + 3 + 1 + 1 = 60$

N(p) = n(no. of ways 1 is selected) = 1 + 7 + 4 + 3 + 2 + 1 + 1 = 19

N(q) = n(no. of ways 2 is selected) = 1 + 6 + 3 + 2 + 1 = 13

So $p = \frac{19}{60}$ $q = \frac{13}{60}$

$p - q = \frac{1}{10}$

90. The remainder when the determinant

$$\begin{vmatrix} 2014^{2014} & 2015^{2015} & 2016^{2016} \\ 2017^{2017} & 2018^{2018} & 2019^{2019} \\ 2020^{2020} & 2021^{2021} & 2022^{2022} \end{vmatrix}$$

is divided by 5 is -

(A) 1 (B) 2 (C) 3 (D) 4

Ans. **[4]**

Sol. Determinant = $\begin{vmatrix} (2015-1)^{2014} & (2015)^{2015} & (2015+1)^{2016} \\ (2015+2)^{2017} & (2020-2)^{2018} & (2020-1)^{2019} \\ (2020)^{2020} & (2020+1)^{2021} & (2020+2)^{2022} \end{vmatrix}$

Determinant of remainder = $\begin{vmatrix} (1)^{2014} & 0 & 1 \\ 2^{2017} & 2^{2018} & (-1)^{2019} \\ 0 & 1^{2021} & 2^{2022} \end{vmatrix}$

$= 1\{2^{4040} + 1\} + 1\ \{2^{2017}\}$

$= \{(4)^{2020} + 1\} + 2\ .\ 2^{2016}$

$\Rightarrow (5-1)^{2020} + 1 + 2.4^{1008}$

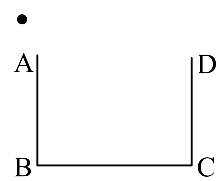
$= (5-1)^{2020} + 1 + 2.\ (5-1)^{1008}$

remainder, $(-1)^{2020} + 1 + 2.\ (-1)^{1008}$

$= 1 + 1 + 2 = 4$

PHYSICS

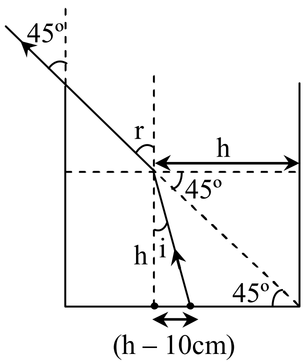
91. A cubical vessel has opaque walls. An observer (dark circle in figure below) is located such that she can see only the wall CD but not the bottom. Nearly to what height should water be poured so that she can see an object placed at the bottom at a distance of 10 cm from the corner C ? Refractive index of water is 1.33.



(A) 10 cm (B) 16 cm (C) 27 cm (D) 45 cm

Ans. **[C]**

Sol.

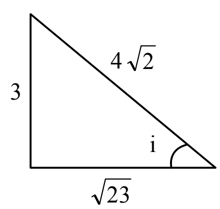


From diagram r = 45°

using snell law

$\frac{4}{3}$ sini = sin r

$\text{sini} = \frac{3}{\sqrt{2} \times 4}$



$\tan i = \frac{3}{\sqrt{23}}$

$\tan i = \frac{h-10}{h}$

h tan i = h – 10

10 = h [1 – tan i]

$h = \frac{10}{1-\tan i}$

⇒ 27 approx.

= 27 cm

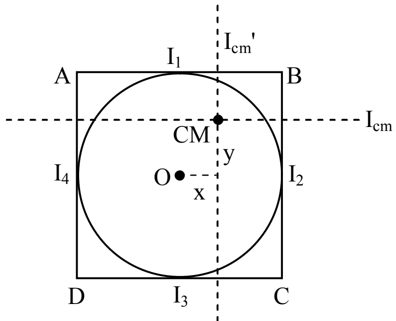
92. The moments of inertia of a non-uniform circular disc (of mass M and radius R) about four mutually perpendicular tangents AB, BC, CD, DA are I_1, I_2, I_3 and I_4, respectively (the square ABCD circumscribes the circle). The distance of the center of mass of the disc from its geometrical center is given by -

(A) $\frac{1}{4MR}\sqrt{(I_1 - I_3)^2 + (I_2 - I_4)^2}$ (B) $\frac{1}{12MR}\sqrt{(I_1 - I_3)^2 + (I_2 - I_4)^2}$

(C) $\frac{1}{3MR}\sqrt{(I_1 - I_2)^2 + (I_3 - I_4)^2}$ (D) $\frac{1}{2MR}\sqrt{(I_1 + I_3)^2 + (I_2 + I_4)^2}$

Ans. **[A]**

Sol.



$I_{cm} + m(R + y)^2 = I_3$ ...(1)
$I_{cm} + m(R - y)^2 = I_1$ ...(2)
from (1) & (2)
$I_1 - I_3 = m[(R - y)^2 - (R + y)^2]$
$I_1 - I_3 = m(2R)(-2y)$ ...(3)
$I_{cm}' + m(R + x)^2 = I_4$ ...(4)
$I_{cm}' + m(R - x)^2 = I_2$ ...(5)
from (4) & (5)
$(I_2 - I_4) = m[(R - x)^2 - (R + x)^2]$
$I_2 - I_4 = m[(2R)(-2x)]$ ...(6)
$(3)^2 + (6)^2$
$\Rightarrow (I_1 - I_3)^2 + (I_2 - I_4)^2 = (m^2 \times 4R^2 \times 4(x^2 + y^2)$
distance of CM from O = $\sqrt{x^2 + y^2} = \frac{1}{4mR}\sqrt{(I_1 - I_3)^2 + (I_2 - I_4)^2}$

93. A horizontal steel railroad track has a length of 100 m when the temperature is 25ºC. The track is constrained from expanding or bending. The stress on the strack on a hot summer day, when the temperature is 40ºC, is (Note : the linear coefficient of thermal expansion for steel is 1.1×10^{-5}/ºC and the Young's modulus of steel is 2×10^{11} Pa)

(A) 6.6×10^7 Pa (B) 8.8×10^7 Pa

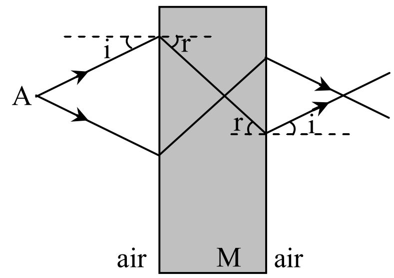
(C) 3.3×10^7 Pa (D) 5.5×10^7 Pa

Ans. **[C]**

Sol. When some body is constrained from expanding or bending then on heating thermal stress get develop in the body.

Stress = $Y \alpha \Delta T$
$= 2 \times 10^{11} \times 1.1 \times 10^{-5} \times (40 - 25)$
$= 3.3 \times 10^7$ N/m^2
$= 3.3 \times 10^7$ Pa

94. Electromagnetic waves emanating from a point A (in air) are incident on a rectangular block of material M and emerge from the other side as shown. The angles i and r are angles of incidence and refraction when the wave travels from air to the medium. Such paths for the rays are possible



(A) if the material has a refractive index very nearly equal to zero.

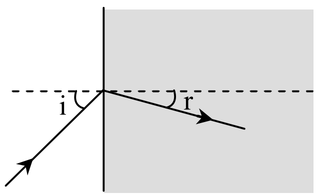
(B) only with gamma rays with a wavelength smaller than the atomic nuclei of the material

(C) if the material has a refractive index less than zero.

(D) only if the wave travels in M with a speed faster than the speed of light in vacuum.

Ans. **[C]**

Sol.



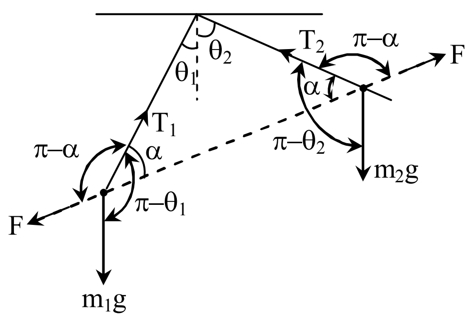
Meta materials are the material for which refractive index is negative for them. Refraction diagram is shown, here. In question same type of diagram is given.

95. Two small metal balls of different mass m_1 and m_2 are connected by strings of equal length to a fixed point. When the balls are given equal charges, the angles that the two strings make with the vertical are 30º and 60º, respectively. The ratio m_1/m_2 is close to -

(A) 1.7 (B) 3.0 (C) 0.58 (D) 2.0

Ans. **[A]**

Sol.



$\theta_1 = 30º$, $\theta_2 = 60º$

using Lami theorem on m_1

$$\frac{F}{\sin(\pi - \theta_1)} = \frac{m_1 g}{\sin(\pi - \alpha)}$$

$$\frac{F}{\sin\theta_1} = \frac{m_1 g}{\sin\alpha} \quad ...(1)$$

using Lami theorem on m_2

$$\frac{F}{\sin(\pi-\theta_2)} = \frac{m_2 g}{\sin(\pi-\alpha)}$$

$$\frac{F}{\sin\theta_2} = \frac{m_2 g}{\sin\alpha} \quad ...(2)$$

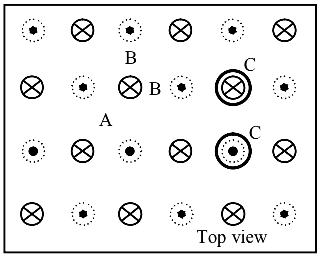
using (1) & (2)

$m_1 \sin\theta_1 = m_2 \sin\theta_2$

$m_1 \times \sin 30^\circ = m_2 \sin 60^\circ$

$$\frac{m_1}{m_2} = \sqrt{3} = 1.7$$

96. Consider the regular array of vertical identical current carrying wires (with direction of current flow as indicated in the figure below) protruding through a horizontal table. If we scatter some diamagnetic particles on the table, they are likely to accumulate -



(A) around regions such as A

(B) around regions such as B

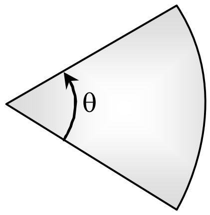
(C) in circular regions around individual wires such as C

(D) uniformly everywhere

Ans. **[A]**

Sol. Diamagnetic material move from high magnetic field to low magnetic field so this material are likely to accumulate over the region such as A as here magnetic field is minimum.

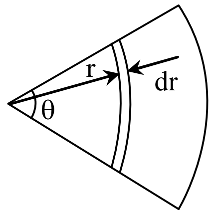
97. The distance between the vertex and the center of mass of a uniform solid planar circular segment of angular size θ and radius R is given by -



(A) $\frac{4}{3}R\frac{\sin(\theta/2)}{\theta}$ (B) $R\frac{\sin(\theta/2)}{\theta}$ (C) $\frac{4}{3}R\cos\left(\frac{\theta}{2}\right)$ (D) $\frac{2}{3}R\cos(\theta)$

Ans. **[A]**

Sol. Planar circular segment can be seen as it consist of Arc element



Mass of element = dm = $\sigma \times r\theta \times dr$

centre of mass of Arc element is at $\dfrac{r\sin\frac{\theta}{2}}{\frac{\theta}{2}}$

$\therefore$ Centre of mass location of segment = $\dfrac{\sum\left(dm\left[\frac{r\sin\theta/2}{\theta/2}\right]\right)}{\Sigma dm}$

$$= \frac{\int_0^R \frac{\sigma r\theta dr \times r\sin(\theta/2)}{\theta/2}}{\int_0^R \sigma r\theta dr}$$

$$\Rightarrow 2\left[\frac{\sin\frac{\theta}{2}}{\theta}\right]\frac{R^3}{3\times\frac{R^2}{2}}$$

$$\Rightarrow \frac{4}{3}R\,\frac{\sin(\theta/2)}{\theta}$$

98. An object is propelled vertically to a maximum height of 4R from the surface of a planet of radius R and mass M. The speed of object when it returns to the surface of the planet is -

(A) $2\sqrt{\dfrac{2GM}{5R}}$ (B) $\sqrt{\dfrac{GM}{2R}}$

(C) $\sqrt{\dfrac{3GM}{2R}}$ (D) $\sqrt{\dfrac{GM}{5R}}$

Ans. **[A]**

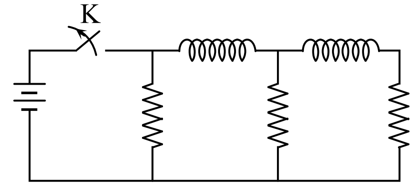
Sol. Using energy conservation

$$-\frac{GMM}{R} + \frac{1}{2}mV^2 = -\frac{GMM}{5R}$$

$$V = 2\sqrt{\frac{2GM}{5R}}$$

V is the velocity by which object is projected. When object return to earth its speed will be V.

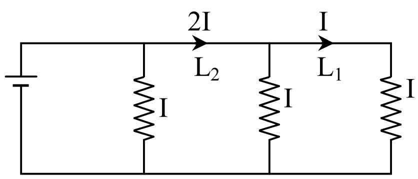
99. In the circuit shown below, all the inductors (assumed ideal) and resistors are identical. The current through the resistance on the right is I after the key K has been switched on for along time. The currents through the three resistors (in order, from left to right) immediately after the key is switched off are -



(A) 2I upwards, I downwards and I downwards
(B) 2I downwards, I downwards and I downwards
(C) I downwards, I downwards and I downwards
(D) 0, I downwards and I downwards

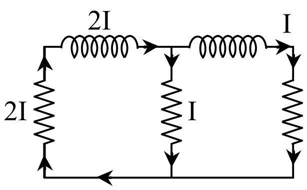
Ans. **[A]**

Sol.



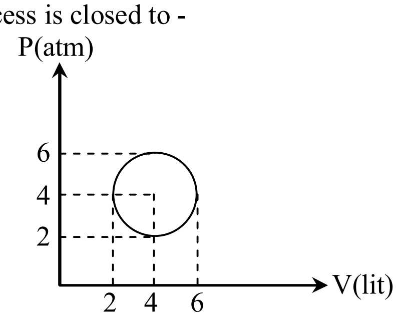
(After long time for switched on)

Initially circuit is in steady state current through each resistor as all are identical & are in parallel combination. When switch is off current through L_1 & L_2 just after remain same.



In right & middle wire current is I downward and in left wire current is 2I upward

100. An ideal gas undergoes a circular cycle centered at 4 atm, 4 lit as shown in the diagram. The maximum temperature attained in this process is closed to -



(A) 30/R (B) 36/R (C) 24/R (D) 16/R

Ans. **[A]** ; [**Note :** No. of mole of gas is not given, we have assumed no. of mole = 1]

Sol. $T = \dfrac{PV}{R}$

T will be maximum when PV is maximum

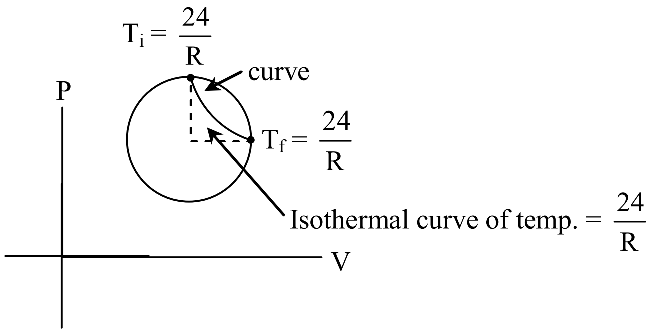
$T = \dfrac{PV}{R} = \dfrac{(4+2\sin\theta)(4+2\cos\theta)}{R}$

As sin θ and cos θ both

can not be equal to 1 for same value of θ

$\therefore$ T can not be $\frac{36}{R}$

T_{max} should be less than $\frac{36}{R}$



curve is above isothermal curve

$\therefore$ temp. is more than $\frac{24}{R}$ on the given process

So T_{max} lie between $\frac{24}{R}$ and $\frac{36}{R}$

only one option is present

CHEMISTRY

101. For the reaction $N_2 + 3X_2 \rightarrow 2NX_3$ where X = F, Cl (the average bond energies are F-F = 155 kJ mol^{-1} N-F = 272 kJ mol^{-1}, Cl-Cl = 242 kJ mol^{-1}, N-Cl = 200 kJ mol^{-1} and N≡N = 941 kJ mol^{-1}), the heats of formation of NF_3 and NCl_3 in kJ mol^{-1}, respectively, are closest to

(A) – 226 and +467 (B) + 226 and – 467 (C) – 151 and + 311 (D) + 151 and – 311

Ans. **[A]**

Sol. $N_2 + 3X_2 \longrightarrow 2NX_3$

$N_2 + 3F_2 \longrightarrow 2NF_3$

$\Delta H_{NF_3} = 941 + 3(155) - 6(272) = -226$

$\Delta H_{NCl_3} = 941 + 3(242) - 6(200) = +467$

102. The equilibrium constants for the reactions X = 2Y and Z = P + Q are K_1 and K_2, respectively. If the initial concentrations and the degree of dissociation of X and Z are the same, the ratio K_1/K_2 is

(A) 4 (B) 1 (C) 0.5 (D) 2

Ans. **[A]**

Sol.
$$\begin{array}{cccccccc} X & \rightleftharpoons & 2Y & k_1 & Z & \rightleftharpoons & P + Q & k_2 \\ 1 & & 0 & & 1 & & 0 \quad 0 & \\ 1-\alpha & & 2\alpha & & 1-\alpha & & \alpha \quad \alpha & \end{array}$$

$$\frac{k_1}{k_2} = \frac{\frac{(2\alpha)^2}{(1-\alpha)}}{\frac{\alpha^2}{(1-\alpha)}} = \frac{4\alpha^2}{\alpha^2} = 4$$

103. The geometry and the number of unpaired electron(s) of $[MnBr_4]^{2-}$, respectively, are

(A) tetrahedral and 1 (B) square planar and 1 (C) tetrahedral and 5 (D) square planar and 5

Ans. **[C]**

Sol. $[Mn\ Br_4]^{-2}$

$x + (-1)4 = -2$

$x = +2$

$Mn \longrightarrow 4s^2 3d^5$

$Mn^{+2} \longrightarrow 4s^\circ 3d^5$

$\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$

We know that Br is weak field ligand so Hund's Rule is applicable

then $\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$ $\underset{Br}{\overset{4s}{\boxed{}}}$ $\underset{Br\ \ Br\ \ Br}{\overset{4p}{\boxed{}\boxed{}\boxed{}}}$

Hybridization is $\longrightarrow sp^3$

and N_0 of unpaired = 5

electron.

104. The standard cell potential for $Zn|Zn^{2+}||Cu^{2+}|Cu$ is 1.10 V. When the cell is completely discharged, log $[Zn^{2+}]/[Cu^{2+}]$ is closest to

(A) 37.3 (B) 0.026 (C) 18.7 (D) 0.052

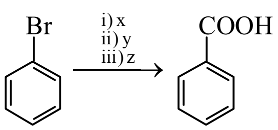
Ans. **[A]**

Sol. $0 = 1.1 - \frac{0.0591}{2}\log\frac{Zn^{+2}}{Cu^{+2}}$

$-1.1 = \frac{0.0591}{2}\log\frac{Zn^{+2}}{Cu^{+2}}$

$\log\frac{Zn^{+2}}{Cu^{+2}} = 37.3$

105. In the reaction



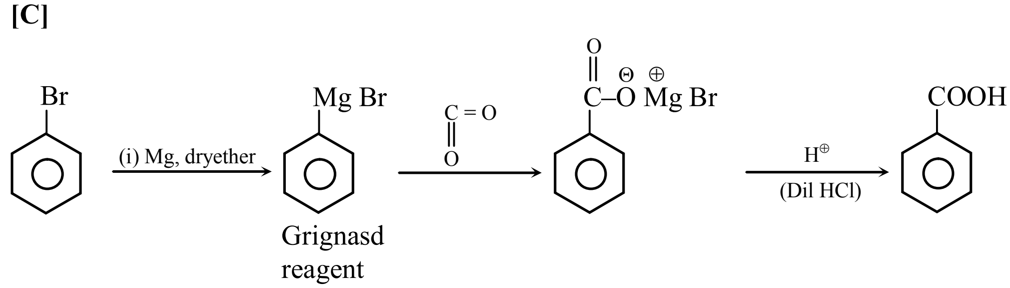
x, y and z are

(A) x = Mg, dry ether; y = CH_3Cl; z = H_2O (B) x = Mg, dry methanol; y = CO_2; z = dil.HCl

(C) x = Mg, dry ether; y = CO_2; z = dil. HCl (D) x = Mg, dry methanol; y = CH_3Cl; z = H_2O

Ans. **[C]**

Sol.



106. An organic compound having molecular formula C_2H_6O undergoes oxidation with $K_2Cr_2O_7/H_2SO_4$ to produce X which contains 40% carbon, 6.7% hydrogen and 53.3% oxygen. The molecular formula of the compound X is

(A) CH_2O (B) $C_2H_4O_2$ (C) C_2H_4O (D) $C_2H_6O_2$

Ans. **[B]**

Sol. $CH_3–CH_2–OH \xrightarrow{K_2Cr_2O_7/H_2SO_4} CH_3–\underset{\underset{O}{||}}{C}–O–H$

(C_2H_{60})
Alcohol
($C_nH_{2n+2}O$)

Carboxylic acid

Total wt = 60

C = 24 (40% carbon)

O = 32 (53.3%)

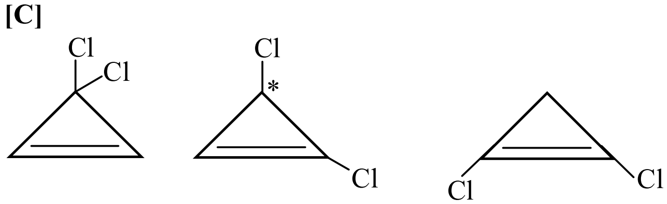
H = 4 (6.7%)

107. The maximum number of cyclic isomers (positional and optical) of a compound having molecular formula $C_3H_2Cl_2$ is

(A) 2 (B) 3 (C) 4 (D) 5

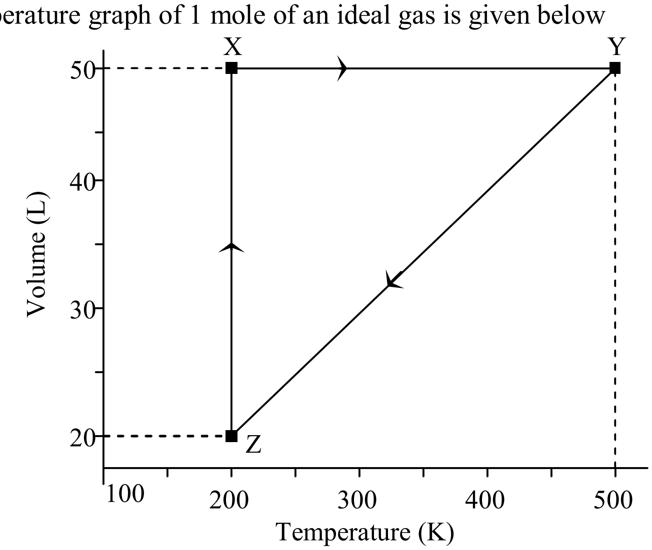
Ans. **[C]**

Sol.



Racemic mixture
(Enantiomer)
pair

108. The volume vs. temperature graph of 1 mole of an ideal gas is given below



The pressure of the gas (in atm) at X, Y and Z, respectively, are

(A) 0.328, 0.820, 0.820 (B) 3.28, 8.20, 3.28

(C) 0.238, 0.280, 0.280 (D) 32.8, 0.280, 82.0

Ans. **[A]**

Sol. At X $V = 50L$ $T = 200$ K

$n = 1$

$$P_X = \frac{nRT}{V} = \frac{1\times 0.0821\times 200}{50}$$

$= 0.328$

At Z $P_Z = \dfrac{1\times 0.0821\times 200}{20} = 0.821$

At Y $P_Y = \dfrac{1\times 0.0821\times 500}{50} = 0.821$

109. MnO_2 when fused with KOH and oxidized in air gives a dark green compound X. In acidic solution, X undergoes disproportionation to give an intense purple compound Y and MnO_2. The compounds X and Y, respectively, are

(A) K_2MnO_4 and $KMnO_4$ (B) Mn_2O_7 and $KMnO_4$

(C) K_2MnO_4 and Mn_2O_7 (D) $KMnO_4$ and K_2MnO_4

Ans. **[A]**

Sol. $MnO_2 + KO_2 \xrightarrow[\text{In Air}]{\text{Oxidized}}$ dark green compound (X) MnO_4^{-2} & K_2MnO_4

$\downarrow$ Disproportionation

Purple colour (Y) + $\overset{+4}{MnO_2}$

$\overset{+7}{KMnO_4}$

$(X) \longrightarrow K_2MnO_4$

$(Y) \longrightarrow KMnO_4$

110. A metal (X) dissolves both in dilute HCl and dilute NaOH to liberate H_2. Addition of NH_4Cl and excess NH_4OH to an HCl solution of X produces Y as a precipitate. Y is also produced by adding NH_4Cl to the NaOH solution of X. The species X and Y, respectively, are

(A) Zn and $Zn(OH)_2$ (B) Al and $Al(OH)_3$ (C) Zn and Na_2ZnO_2 (D) Al and $NaAlO_2$

Ans. **[B]**

Sol.

Al

X (Metal) $\xrightarrow[\text{Dil NaOH}]{\text{Dil HCl}} H_2\uparrow$

X (Metal) $\xrightarrow[\text{NaOH}]{NH_4Cl}$ (Y) $Al(OH)_3$

X (Metal) $\xrightarrow{NH_4Cl \text{, Excess } NH_4OH \text{ In HCl solution}}$ (Y) $Al(OH)_3$ White ppt

$(X) \longrightarrow Al$

$(Y) \longrightarrow Al(OH)_3$

BIOLOGY

111. How many bands are seen when immunoglobulin G molecules are analysed on a sodium dodecyl sulphate polyacrylamide gel electrophoresis (SDS – PAGE) under reducing conditions?

(A) 6 (B) 1 (C) 2 (D) 4

Ans. **[C]**

Sol. On SDS – PAGE of IgG two principal Bands are visible

112. In a mixed culture of slow and fast growing bacteria, penicillin will

(A) kill the fast growing bacteria more than the slow growing

(B) kill slow growing bacteria more than the fast growing

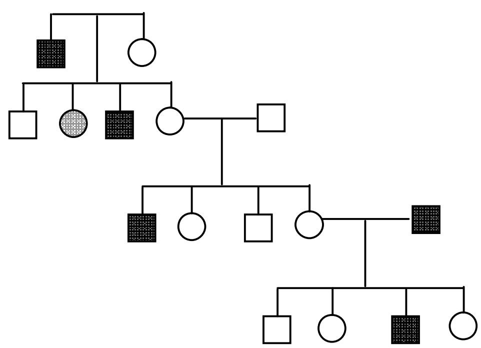
(C) kill both the fast and slow growing bacteria equally

(D) will not kill bacteria at all

Ans. **[A]**

Sol. Penicillin inhibit the cell wall formation in bacteria thus it kill rapidly growing bacteria more than the slow growing bacteria.

113. Consider the following pedigree over four generations and mark the correct answer below about the inheritance of haemophilia.



Normal male Normal female
Haemophilic male Haemophilic female

(A) Haemophilia is X-linked dominant (B) Haemophilia is autosomal dominant

(C) Haemophilia is X-linked recessive (D) Haemophilia is Y-linked dominant

Ans. **[C]**

Sol.

	X^h	Y
X^h	$X^h X^h$	X^hY
X	X^hX	XY

Female parent is carrier and male and female offspring is affected.

114. A person has 400 million alveoli per lung with an average radius of 0.1 mm for each alveolus. Considering the alveoli are spherical in shape, the total respiratory surface of that person is closest to

(A) 500 mm^2 (B) 200 mm^2 (C) 100 mm^2 (D) 1000 mm^2

Ans. **[D]**

Sol. Radius of spherical alveoli = 0.1 mm

Total No. of Alveoli = 400 × 2 = 800 million

Surface area of sphere = $4\pi R^2$

Total respiratory surface Area = $4\pi R^2$ × 800 million

= 1000 mm^2 (approx)

115. A mixture of equal numbers of fast and slow dividing cells is cultured in a medium containing a trace amount of radioactively labeled thymidine for one hour. The cells are then transferred to regular (unlabelled) medium. After 24hrs of growth in regular media

(A) fast dividing cells will have maximum radioactivity

(B) slow dividing cells will have maximum radioactivity

(C) both will have same amount of radioactivity

(D) there will be no radioactivity in either types of cells

Ans. **[A]**

Sol. Radioactivity is mostly incorporated in rapidly dividing cell during S phase. Thus after 24 hour it will be mainly present in few rapidly dividing cell

116. If a double stranded DNA has 15% cytosine, what is the % of adenine in the DNA?

(A) 15% (B) 70% (C) 35% (D) 30%

Ans. **[C]**

Sol. Cytosine = 15%

According to chargaff principle

A = T & G = C

A + T + G + C = 100

Thus A will be 35%

117. The mitochondrial inner membrane consists of a number of infoldings called cristae. The increased surface area due to cristae helps in :

(A) Increasing the volume of mitochondria

(B) Incorporating more of the protein complexes essential for electron transport chain

(C) Changing the pH

(D) Increasing diffusion of ions

Ans. **[B]**

118. The activity of a certain protein is dependent on its phosphorylation. A mutation in its gene changed a single amino acid which affected the function of the molecule. Which amino acid change is most likely to account for this observation?

(A) Tyrosine to Tryptophan (B) Lysine to valine

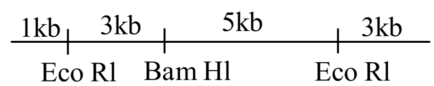
(C) Leucine to isoleucine (D) Valine to alanine

Ans. **[A]**

Sol. Serine, tyrosine, threonine and histidine are most common target for phosphorylation in eukaryotes.

If Tyrosine get mutated in tryptophan than phosphorylation of gene does not occur and gene can not be activated

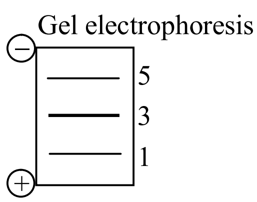
119. Consider the linear double stranded DNA shown below. The restriction enzyme sites and the lengths demarcated are shown. This DNA is completely digested with both *Eco*RI and *Bam*HI restriction enzymes. If the product is analyzed by gel electrophoresis, how many distinct bands would be observed?



(A) 5 (B) 2 (C) 3 (D) 4

Ans. **[C]**

Sol.



There are 4 fragment but 2 fragment of 3 Kb will appear as one band so three band will appear

120. Enzyme X catalyzes hydrolysis of GTP into GDP. The GTP-bound form of X transmits a signal that leads to cell proliferation. The GDP – bound form does not transmit any such signal. Mutations in X are found in many cancers. Which of the following alterations of X are most likely to contribute to cancer?

(A) Mutations that increase the affinity of X for GDP

(B) Mutations that decrease the affinity of X for GTP

(C) Mutations that decrease the rate of GTP hydrolysis

(D) Mutations that prevent expression of enzyme X.

Ans. **[C]**

Sol. GTP –X Complex $\longrightarrow$ Active cell division

GDP –X Complex $\longrightarrow$ No cell division

and GTP $\xrightarrow[\text{X enzyme}]{\text{Hydrolysis}}$ GDP

Mutation cause the decrease of rate of hydrolysis of GTP. Thus GTP –X complex remain present for long time and cell division become uncontrolled.