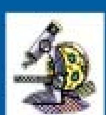
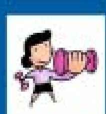
PHYSICS

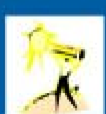
THEORY



EXAMPLES



EXERCISES



SOLUTION & TIPS

UNIT & DIMENSION

CAREER POINT

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UNIT & DIMENSION

Preface

IIT - JEE Syllabus : Unit & Dimension
Unit & Dimensions, Dimensional analysis, Least count, Significant figure, Methods of measurement and Error analysis for physical quantities.

Fundamental concepts of the Physics start from this chapter. Basically the terms & concepts which are illustrated in this topic will be used in so many ways because all Physical quantities have units. It is must to measure all Physical quantities so that we can use them. In this chapter we will have an over view of different units of different Physical quantities. We will learn the dimension and dependence of the unit of any Physical quantity on fundamental quantities or unit. Entire topic is illustrated very systematically with respective examples so that the students can understand the fundamentals very easily & quickly. Students are advised to read every point of supplementary very carefully which is given at the end of the topic. Generally, students are not able to find out the Dimension of unseen or new quantity as their basic concepts are not clear & then they read the dimensions like a parrot. It should be avoided & they should develope themselves, so that they can find out the dimensions of any given quantity.

Total number of Questions in **Units & Dimension** are :

In chapter Examples 27

Physics :

Physics is the study of the laws of nature from the observed events.

1. PHYSICAL QUANTITIES

The quantities by means of which we describe the laws of physics are called physical quantities. There are two type of physical quantities.

1.1 Fundamental quantities

1.2 Derived quantities

1.1 Fundamental quantities:

Physical quantities which are independent of each other and cannot be further resolved into any other physical quantity are known as fundamental quantities. There are seven fundamental quantities.

Fundamental quantity	Units	Symbol
(a) Length	Metre	m
(b) Mass	Kilogram	kg
(c) Time	Second	s
(d) Electric current	Ampere	A
(e) Thermodynamic temperature	Kelvin	K
(f) Luminous intensity	Candela	Cd
(g) Amount of substance	Mole	Mol.

1.2 Derived Quantities :

Physical quantities which depend upon fundamental quantities or which can be derived from fundamental quantities are known as derived quantities.

2. UNITS

Definition : Things in which quantity is measured are known as units.

Measurement of physical quantity

= (Magnitude) × (Unit)

Ex.1 A physical quantity is measured and the result is expressed as nu where u is the unit used and n is the numerical value. If the result is expressed in various units then :

(A) $n \propto$ size of u (B) $n \propto u^2$

(C) $n \propto \sqrt{u}$ (D) $n \propto \frac{1}{u}$

Answer : (D)

There are three types of units

2.1 Fundamental or base units

2.2 Derived units

2.3 Supplementary units

2.1 Fundamental or base units:

Units of fundamental quantities are called fundamental units.

2.1.1 Characteristics of fundamental units:

(i) they are well defined and are of a suitable size

(ii) they are easily reproducible at all places

(iii) they do not vary with temperature, time pressure etc. i.e. invariable.

(iv) there are seven fundamental units.

2.1.2 Definitions of fundamental units:

2.1.2.1 Metre :

The distance travelled by light in Vacuum in $\frac{1}{299,792,458}$ second is called 1m.

2.1.2.2 Kilogram :

The mass of a cylinder made of platinum iridium alloy kept at international bureau of weights and measures is defined as 1kg.

2.1.2.3 Second :

Cesium -133 atom emits electromagnetic radiation of several wavelengths. A particular radiation is selected which corresponds to the transistion between the two hyperfine levels of the ground state of Cs - 133. Each radiation has a time period of repetition of certain characteristics. The time duration in 9, 192, 631, 770 time periods of the selected transition is defined as 1s.

2.1.2.4 Ampere :

Suppose two long straight wires with negligible cross-section are placed parallel to each other in vacuum at a seperation of 1m and electric currents are established in the two in same direction. The wires attract each other. If equal currents are maintained in the two wires so that the force between them is 2×10^{-7} newton per meter of the wire, then the current in any of the wires is called 1A. Here, newton is the SI unit of force.

2.1.2.5 Kelvin :

The fraction $\frac{1}{273.16}$ of the thermodynamic temperature of triple point of water is called 1K.

2.1.2.6 Mole :

The amount of a substance that contains as many elementary entities (Molecules or atoms if the substance is monoatomic) as there are number of atoms in .012 kg of carbon - 12 is called a mole. This number (number of atoms in 0.012 kg of carbon-12) is called Avogadro constant and its best value available is 6.022045×10^{23} .

2.1.2.7 Candela:

The S.I. unit of luminous intensity is 1cd which is the luminous intensity of a blackbody of surface area $\frac{1}{600,000}\,m^2$ placed at the temperature of freezing platinum and at a pressure of $101,325\ N/m^2$, in the direction perpendicular to its surface.

Examples based on **Definition of fundamental Units**

Ex.2 A man seeing a lighting starts counting seconds until he hears thunder. He then claims to have found an approximate but simple rule that if the count of second is divided by an integer, the result directly gives in km, the distance of the lighting source. What is the integer if the velocity of sound is 330 m/s

Sol. If n is the integer then according to the

problem = dist in km.

$$\frac{t \text{ in s}}{n} = (v)\ t$$

$$n = \frac{1}{v} = \frac{1}{330 \times 10^{-3}} = 3$$

Ex.3 In defining the standard of length we have to specify the temperature at which the measurement should be made. Are we justified in calling length a fundamental quantity if another physical. quantity, temperature, has to be specified in choosing a standard.

Sol. Yes, length is a fundamental quantity. One metre is the distance that contains 1650 763.73 wavelength of orange-red light of Kr - 86. Hence, the standard metre is independent of temperature. But the length of object varies with temperature and is given by the relation .

$$L_t = L_0\ (1 + \alpha t)$$

$\therefore$ We usually specify the temperature at which measurement is made.

Ex.4 Which of the following sets cannot enter into the list of fundamental quantities in any system of units

(A) length ; mass ; velocity

(B) length ; time ; velocity

(C) mass ; time; velocity

(D) length ; time, mass

Sol.[B] Since velocity = $\frac{\text{length}}{\text{time}}$ i.e. in this set a quantity is dependent on the other two quantities Where as fundamental quantities are independent.

2.2 Derived units :

Units of derived quantities are called derived units.

	Physical quantity	units
Illustration	Volume = $(\text{length})^3$	m^3
	Speed = length/time	m/s

$\frac{t \text{ in s}}{n}$

2.3 Supplementary units :

The units defined for the supplementary quantities namely plane angle and solid angle are called the supplementary units. The unit for plane angle is rad and the unit for the solid angle is steradian.

Note :

The supplemental quantities have only units but no dimensions (will be discussed later)

3. PRINCIPAL SYSTEM OF UNITS

3.1 C.G.S. system [centimetre (cm) ; gram (g) and second (s)]

3.2 F.P.S system [foot ; pound ; second]

3.3 M.K.S. system [meter ; kilogram ; second]

3.4 S.I. (system of international)

In 1971 the international Bureau of weight and measures held its meeting and decided a system of units. Which is known as the international system of units.

Examples based on Units

Ex.5 The acceleration due to gravity is 9.80 m/s^2. What is its value in ft/s^2 ?

Sol. Because 1 m = 3.28 ft, therefore

$9.80\ m/s^2 = 9.80 \times 3.28\ ft/s^2$

$= 32.14\ ft/s^2$

Ex.6 A cheap wrist watch loses time at the rate of 8.5 second a day. How much time will the watch be off at the end of a month ?

Sol. Time delay = 8.5 s/day

= 8.5 × 30 s/ (30 day)

= 255 s/month = 4.25 min/month.

4. DIMENSIONS

Dimensions of a physical quantity are the powers to which the fundamental quantities must be raised to represent the given physical quantity.

Illustration :

Force (Quantity) = mass × acceleration

$$= \text{mass} \times \frac{\text{velocity}}{\text{time}} = \text{mass} \times \frac{\text{length}}{(\text{time})^2}$$

$$= \text{mass} \times \text{length} \times (\text{time})^{-2}$$

So dimensions of force : 1 in mass

1 in length

–2 in time

and Dimensional formula : $[MLT^{-2}]$

5. DIMENSIONAL FORMULA

It is an expression which shows how and which of the fundamental units are required to represent the unit of physical quantity.

Different quantities with units. symbol and dimensional formula,

Quantity	Symbol	Formula	S.I. Unit	D.F.
Displacement	s	—	Metre or m	M^0LT^0
Area	A	$\ell \times b$	$(\text{Metre})^2$ or m^2	$M^0L^2T^0$
Volume	V	$\ell \times b \times h$	$(\text{Metre})^3$ or m^3	$M^0L^3T^0$
Velocity	v	$v = \frac{\Delta s}{\Delta t}$	m/s	M^0LT^{-1}
Momentum	p	p = mv	kgm/s	MLT^{-1}
Acceleration	a	$a = \frac{\Delta v}{\Delta t}$	m/s^2	M^0LT^{-2}
Force	F	F = ma	Newton or N	MLT^{-2}
Impulse	–	F × t	N.sec	MLT^{-1}
Work	W	F. d	N.m	ML^2T^{-2}
Energy	KE or U	$K.E. = \frac{1}{2}mv^2$ P.E. = mgh	Joule or J	ML^2T^{-2}
Power	P	$P = \frac{W}{t}$	watt or W	ML^2T^{-3}
Density	d	d = mass/volume	kg/m^3	$ML^{-3}T^0$

Pressure	P	P = F/A	Pascal or Pa	$ML^{-1}T^{-2}$
Torque	τ	$\tau = r \times F$	N.m.	ML^2T^{-2}
Angular displacement	θ	$\theta = \frac{\text{arc}}{\text{radius}}$	radian or rad	$M^0L^0T^0$
Angular velocity	ω	$\omega = \frac{\theta}{t}$	rad/sec	$M^0L^0T^{-1}$
Angular acceleration	α	$\alpha = \frac{\Delta\omega}{\Delta t}$	rad/sec^2	$M^0L^0T^{-2}$
Moment of Inertia	I	$I = mr^2$	kg-m^2	ML^2T^0
Angular momentum	J or L	J = mvr	$\frac{kgm^2}{s}$	ML^2T^{-1}
Frequency	ν or f	$f = \frac{1}{T}$	hertz or Hz	$M^0L^0T^{-1}$
Stress	—	F/A	N/m^2	$ML^{-1}T^{-2}$
Strain	—	$\frac{\Delta\ell}{\ell};\frac{\Delta A}{A};\frac{\Delta V}{V}$	—	$M^0L^0T^0$
Youngs modulus (Bulk modulus)	Y	$Y = \frac{F/A}{\Delta\ell/\ell}$	N/m^2	$ML^{-1}T^{-2}$
Surface tension	T	$\frac{F}{\ell}$ or $\frac{W}{A}$	$\frac{N}{m};\frac{J}{m^2}$	ML^0T^{-2}
Force constant (spring)	k	F = kx	N/m	ML^0T^{-2}
Coefficient of viscosity	η	$F = \eta\left(\frac{dv}{dx}\right)A$	kg/ms(poise in C.G.S)	$ML^{-1}T^{-1}$
Gravitational constant	G	$F = \frac{Gm_1m_2}{r^2}$ $\Rightarrow G = \frac{Fr^2}{m_1m_2}$	$\frac{N-m^2}{kg^2}$	$M^{-1}L^3T^{-2}$
Gravitational potential	V_g	$V_g = \frac{PE}{m}$	$\frac{J}{kg}$	$M^0L^2T^{-2}$
Temperature	θ	—	Kelvin or K	$M^0L^0T^0\theta^{+1}$
Heat	Q	$Q = m \times S \times \Delta t$	Joule or Calorie	ML^2T^{-2}
Specific heat	S	$Q = m \times S \times \Delta t$	$\frac{\text{Joule}}{\text{kg.Kelvin}}$	$M^0L^2T^{-2}\theta^{-1}$
Latent heat	L	Q = mL	$\frac{\text{Joule}}{\text{kg}}$	$M^0L^2T^{-2}$
Coefficient of thermal conductivity	K	$Q = \frac{KA(\theta_1-\theta_2)t}{d}$	$\frac{\text{Joule}}{\text{m sec K}}$	$MLT^{-3}\theta^{-1}$

Universal gas constant	R	PV = nRT	$\frac{\text{Joule}}{\text{mol.K}}$	$ML^2T^{-2}\theta^{-1}$
Mechanical equivalent of heat	J	W = JH	—	$M^0L^0T^0$
Charge	Q or q	$I = \frac{Q}{t}$	Coulomb or C	M^0L^0TA
Current	I	—	Ampere or A	$M^0L^0T^0A$
Electric permittivity	ε_0	$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1q_2}{r^2}$	$\frac{(\text{coul.})^2}{\text{N.m}^2}$ or $\frac{C^2}{N-m^2}$	$M^{-1}L^{-3}A^2T^4$
Electric Potential	V	$V = \frac{\Delta W}{q}$	Joule/coul	$ML^2T^{-3}A^{-1}$
Intensity of electric field	E	E =	N/coul.	$MLT^{-3}A^{-1}$
Capacitance	C	Q = CV	Farad	$M^{-1}L^{-2}T^4A^2$
Dielectric constant or relative permittivity	ε_r	$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$	—	$M^0L^0T^0$
Resistance	R	V = IR	Ohm	$ML^2T^{-3}A^{-2}$
Conductance	S	$S = \frac{1}{R}$	Mho	$M^{-1}L^{-2}T^{-3}A^2$
Specific resistance or resistivity	ρ	$\rho = \frac{RA}{\ell}$	Ohm × meter	$ML^3T^{-3}A^{-2}$
Conductivity or specific conductance	σ	$\sigma = \frac{1}{\rho}$	Mho/meter	$M^{-1}L^{-3}T^3A^2$
Magnetic induction	B	$F = qvB\sin\theta$ or F = BIL	Tesla or weber/m^2	$MT^{-2}A^{-1}$
Magnetic flux	ϕ	$e = \frac{d\phi}{dt}$	Weber	$ML^2T^{-2}A^{-1}$
Magnetic intensity	H	$B = \mu H$	A/m	$M^0L^{-1}T^0A$
Magnetic permeability of free space or medium	μ_0	$B = \frac{Idl\sin\theta}{r^2}$	$\frac{N}{\text{amp}^2}$	$MLT^{-2}A^{-2}$
Coefficient of self or Mutual inductance	L	$e = L \cdot \frac{dI}{dt}$	Henery	$ML^2T^{-2}A^{-2}$
Electric dipole moment	p	$p = q \times 2\ell$	C.m.	M^0LTA
Magnetic dipole moment	M	M = NIA	amp.m^2	$M^0L^2AT^0$

Examples based on Dimensions

Ex.7 (a) Can there be a physical quantity which has no unit and dimensions

(b) Can a physical quantity have unit without having dimensions

Sol. (a) Yes, strain

(b) Yes, angle with units radians

Ex.8 Fill in the blanks

(i) Three physical quantities which have same dimensions are ....

(ii) Mention a scalar and a vector physical quantities having same dimensions ....

Sol. (i) Work, energy, torque

(ii) Work, torque

Ex.9 Choose the correct statement (s)

(A) all quantities may be represented dimensionally in terms of the base quantities

(B) all base quantity cannot be represented dimensionally in terms of the rest of the base quantities

(C) the dimension of a base quantity in other base quantities is always zero.

(D) the dimension of a derived quantity is never zero in any base quantity.

Sol. **[A,B,C]**

(B) all the fundamental base quantities are independent of any other quantity

(C) same as above

Ex.10 If velocity (V), time (T) and force (F) were chosen as basic quantities, find the dimensions of mass.

Sol. Dimensionally :

Force = mass × acceleration

Force = mass ×

$$\text{Mass} = \frac{\text{Force} \times \text{time}}{\text{velocity}}$$

mass = FTV^{-1}

Ex.11 In a particular system, the unit of length, mass and time are chosen to be 10cm, 10gm and 0.1s respectively. The unit of force in this system will be equivalent to

(A) $\frac{1}{10}$ N (B) 1N

(C) 10N (D) 100 N

Sol. Dimensionally

$F = MLT^{-2}$

In C.G.S system

1 dyne = 1g 1 cm $(1s)^{-2}$

In new system

1x = (10g) (10 cm) $(0.1s)^{-2}$

$$\frac{1\text{dyne}}{1\text{x}} = \frac{1\text{g}}{10\text{g}} \times \frac{1\text{cm}}{10\text{cm}}\left(\frac{10\text{s}}{1\text{s}}\right)^{-2}$$

$$1 \text{ dyne} = \frac{1}{10{,}000} \times 1\text{x}$$

10^4 dyne = 1x

10 x = 10^5 dyne = 1 N

$$x = \frac{1}{10}\text{N}$$

Ex.12 If the units of length and force are increased four times, then the unit of energy will

(A) increase 8 times

(B) increase 16 times

(C) decreases 16 times

(D) increase 4 times

Sol. Dimensionally

$E = ML^2T^{-2}$

$E = (MLT^{-2})\ (L)$

$E' = (4)\ (MLT^{-2})\ (4L)$

$E' = 16\ (ML^2T^{-2})$

veloc

time

Note :

5.1 Two physical quantities having same dimensions can be added or subtracted but there is no such restriction in division and multiplication. (Principle of homogeneity)

Illustration : Using the theory of dimensions, determine the dimensions of constants 'a' and 'b' in Vander Wall's equation. $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

Sol. $\frac{a}{V^2}$ must have the same dimensions as that of P (because it is added to P)

Dimension of b must be same as that of V.

$$\frac{[a]}{L^6} = ML^{-1}T^{-2}$$

$[a] = ML^5T^{-2}$

$[b] = L^3$

5.2 Expressions such as sinx ; cosx (trigonometric functions) e^x, a^x, log x, ℓnx , have no dimensions. In these quantities 'x' has also no dimensions.

Examples based on **Remarks**

Ex.13 The time dependence of physical quantity P is found to be of the form

$$P = P_0 e^{-\alpha t^2}$$

Where't' is the time and α is some constant. Then the constant α will

(A) be dimensionless

(B) have dimensions of T^{-2}

(C) have dimensions of P

(D) have dimensions of P multiplied by T^{-2}

Sol. Since in e^x, x is dimensionless

$\therefore$ In $e^{-\alpha t^2}$; αt^2 should be dimensionless

$\alpha t^2 = M^0L^0T^0$

$\alpha = M^0L^0T^{-2}$

6. APPLICATION OF DIMENSIONAL ANALYSIS

6.1 To find the unit of a given physical quantity in a given system of units

Illustration F = $[MLT^{-2}]$

6.2 In finding the dimensions of physical constants or coefficients.

Examples based on **Application of dimensional analysis**

Ex.14 To find the dimensions of physical constants, G, h, η etc.

Sol. Dimension of (Gravitational constant)

$$G : F = \frac{Gm_1m_2}{r^2} \Rightarrow [MLT^{-2}] = \frac{[G][M^2]}{L^2}$$

$G = M^{-1}L^3T^{-2}$

Dimensions of h : Plank's constant

E = hν

$$ML^2T^{-2} = h \cdot \frac{1}{T}$$

$h = ML^2T^{-1}$

Dimension of η : Coefficient of viscosity.

$F = 6\pi\eta vr$

$[MLT^{-2}] = \eta[LT^{-1}][L]$

$\eta = [ML^{-1}T^{-1}]$

6.2.1 To convert a physical quantity from one system to the other

Example based on **Conversion of units from one system to other system of units**

Ex.15 Conversion of Newton to Dyne

(MKS) (C.G.S.)

Dimensional formula of F = MLT^{-2}

$$1N = \frac{1kg \times 1m}{(|\sec|^2)} = \frac{1000\,g \times 100\,cm}{(|\sec|)^2}$$

$$= 10^5 \frac{g.cm}{sec^2}$$

$1N = 10^5$ Dyne

Ex.16 Conversion of G from SI system to C.G.S.

Dimensional formula = $M^{-1}L^3T^{-2}$

$$G = 6.67 \times 10^{-11} \times \frac{m^3}{kg.s^2}$$

$$G = 6.67 \times 10^{-11} \times \frac{(100cm)^3}{1000g.(1sec)^2}$$

$$G = 6.67 \times 10^{-11} \times \frac{10^6}{10^3} \frac{cm^3}{g.sec^2}$$

$$G = 6.67 \times 10^{-8} \frac{cm^3}{g.sec^2}$$

Ex.17 If the units of force, energy and velocity in a new system be 10 N, 5J and 0.5 ms^{-1} respectively, find the units of mass, length and time in that system.

Sol. Let M_1, L_1 and T_1 be the units of mass, length and time in SI and M_2, L_2 and T_2 the corresponding units in new system.

The dimensional formula for force is

$(M^1 L^1 T^{-2})$

Hence the conversion formula for force becomes

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^1 \left[\frac{L_1}{L_2}\right]^1 \left[\frac{T_1}{T_2}\right]^{-2}$$

Here n_1= 10 N, n_2 = 1, Substituting we get

$$1 = 10 \left[\frac{M_1}{M_2}\right]^1 \left[\frac{L_1}{L_2}\right]^1 \left[\frac{T_1}{T_2}\right]^{-2} \quad ...(1)$$

The dimensional formula for work is $(M^1 L^2 T^{-2})$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^1 \left[\frac{L_1}{L_2}\right]^2 \left[\frac{T_1}{T_2}\right]^{-2}$$

$n_1 = 5$ J, $n_2 = 1$

Substituting values we get

$$1 = 5 \left[\frac{M_1}{M_2}\right]^1 \left[\frac{L_1}{L_2}\right]^2 \left[\frac{T_1}{T_2}\right]^{-2} \quad ...(2)$$

Similarly the dimensional formula for velocity is $(M^o L^1 T^{-1})$.

Hence, conversion formula for velocity is

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^0 \left[\frac{L_1}{L_2}\right]^1 \left[\frac{T_1}{T_2}\right]^{-1}$$

Here $n_1 = 0.5\ ms^{-1}$, $n_2 = 1$,

Substituting values we get

$$1 = 0.5 \left[\frac{L_1}{L_2}\right]^1 \left[\frac{T_1}{T_2}\right]^{-1} \quad ...(3)$$

Dividing (2) by (1), $1 = \left[\frac{L_1}{L_2}\right]$,

$$L_2 = \frac{L_1}{2} = \frac{1}{2}\ m = 0.5\ m$$

Substituting value of $\left[\frac{L_1}{L_2}\right]$ in (3), we get

$$1 = 0.5 \times 2 \left[\frac{T_1}{T_2}\right]^{-1}, \quad \frac{T_1}{T_2} = 1,\ T_2 = 1s$$

Substituting value of $\left[\frac{L_1}{L_2}\right]$ and $\left[\frac{T_1}{T_2}\right]$ in (1)

$$1 = 10 \left[\frac{M_1}{M_2}\right] \times 2 \times 1,$$

$$1 = 20 \left[\frac{M_1}{M_2}\right],$$

$M_2 = 20\ M_1$ as $M_1 = 1$kg, $M_2 = 20$ kg. Hence units of mass, length and time are 20 kg, 0.5 m and 1 sec respectively

6.2.2 Conversion of a quantity from a given system to new hypothetical system

Examples

Ex.18 The density of a substance is 8 g/cm^3. Now we have a new system in which unit of length is 5cm and unit of mass 20g. Find the density in this new system

Sol. In the new system ; Let the symbol of unit of length be La and mass be Ma.

Since 5cm = 1 La $\Rightarrow$ 1cm = $\frac{1\,La}{5}$

20g = 1Ma $\Rightarrow$ 1g = $\frac{1\,Ma}{20}$

$$D = 8\ g/cm^3 = \frac{\left(8 \times \frac{1}{20}\right)Ma}{\left(\frac{1\,La}{5}\right)^3}$$

D = 50 Ma/(La)^3

= 50 units in the new system

6.3 To check the dimensional correctness of a given relation

Examples based on **Dimensional correctness**

Ex.19 Find the correct relation

$F = \frac{mv^2}{r^2}$ or $\frac{mv^2}{r}$

Sol. Checking the dimensionally correctness of relation

$$F = \frac{mv^2}{r^2}$$

L.H.S. = MLT^{-2}

R.H.S. = $\frac{M(LT^{-1})^2}{L^2} = ML^0T^{-2}$; LHS $\neq$ RHS

$$F = \frac{Mv^2}{r}$$

LHS = MLT^{-2}

RHS = $\frac{M(LT^{-1})^2}{L} = MLT^{-2}$; LHS = RHS

Hence dimensionally second relation is correct

Limitation :

It is not necessary that every dimensionally correct relation, physically may be correct

6.4 As a research tool to derive new relation

Examples based on **Deriving new relation**

Ex.20 To derive the Einstein mass - energy relation

Sol. $E = f(m, c)$

$E = k\, M^x C^y$

$ML^2T^{-2} = M^x (LT^{-1})^y$

$ML^2T^{-2} = M^x L^y T^{-y}$

Comparing the coefficients

$x = 1$; $y = +2$

Through experiments ; $k = 1$

$\therefore\ E = mc^2$

Ex.21 When a small sphere moves at low speed through a fluid, the viscous force F opposing the motion, is found experimentaly to depend on the radius 'r', the velocity v of the sphere and the viscosity η of the fluid. Find the force F (Stoke's law)

Sol. $F = f(\eta\ ;\ r\ ;\ v)$

$F = k\,.\,\eta\,.\,r.\,v$

$MLT^{-2} = (ML^{-1}T^{-1})^x\,(L)^y\,(LT^{-1})^z$

$MLT^{-2} = M^x\,L^{-x+y+z}\,T^{-x-z}$

comparing coefficients

$x = 1$, $-x + y + z = 1$; $-x - z = -2$

$x = y = z = 1$

$F = k\eta vr$

$F = 6\pi\eta vr$

As through experiments : $k = 6\pi$

Ex.22 A gas bubble from an explosion under water, oscillates with a period T proportional to $p^a d^b E^c$ where p is the static pressure, d is the density of water and E is the total energy of explosion. Find the values of a,b, and c.

Sol. $a = -$; $b = \frac{1}{2}$ and $c = \frac{1}{3}$

7. LIMITATIONS OF THE APPLICATION OF DIMENSIONAL ANALYSIS

7.1 If the dimensions are given, then the physical quantity may not be unique as many physical quantities can have same dimensions.

Examples based on **Limitation of dimensional analysis**

Ex.23 If there is a physical quantity whose dimensional formula is $[ML^2T^{-2}]$. Determine the physical quantity.

Sol. It may be torque, work or energy.

7.2 Since numerical constant have no dimensions.

Such as , 1, 6π etc, hence these can't be deduced by the methods of dimensions.

7.3 The method of dimensions cannot be used to derive relations other than product of power functions.

Illustration : $S = ut + \frac{1}{2}at^2$, $y = a\sin\omega t$

Note :

However the dimensional correctness of these can be checked

7.4 The method of dimensions cannot be applied to derive formula if in mechanics a physical quantity depends on more than three physical quantities. As then there will be less number (=3) of equations than the unknowns. However the dimensional correctness of the equation can be checked

Illustration : $T = 2\pi\sqrt{\frac{I}{mg\ell}}$ cannot be derived by theory of dimensions.

7.5 Even if a physical quantity depends on three physical quantities, out of which two have same dimensions, the formula cannot be derived by theory of dimensions.

Illustration : Formula of the frequency of a tunning fork $f = \left(\frac{d}{L^2}\right)v$

Note :

However the dimensional correctness can be checked.

8. SIGNIFICANT DIGITS

8.1 Normally decimal is used after first digit using powers of ten,

Illustration : 3750 m will be written as 3.750×10^3 m

8.2 The order of a physical quantity is expressed in power of 10 and is taken to be 1 if $\le (10)^{1/2} = 3.16$ and 10 if > 3.16

Illustration : speed of light $= 3 \times 10^8$, order $= 10^8$

Mass of electron $= 9.1 \times 10^{-31}$, order $= 10^{-30}$

8.3 **Significant digits :** In a multiplication or division of two or more quantities, the number of significant digits in the answer is equal to the number of significant digits in the quantity which has the minimum number of significant digit

Illustration : 12.0/7.0 will have two significant digits only.

8.4 The insignificant digits are dropped from the result if they appear after the decimal point. They are replaced by zeroes if they appear to the left of two decimal point. The least significant digit is rounded according to the rules given below.

Rounding off : If the digit next to one rounded as more then 5, the digit to be rounded is increased by 1; if the digit next to the one rounded is less than 5, the digit to be rounded is left unchanged, if the digit next to one rounded is 5, then the digit to be rounded is increased by 1 if it odd and is left unchanged if it is even.

8.5 For addition and subtraction write the numbers one below the other with all the decimal points in one line now locate the first column from left that has doubtful digits. All digits right to this column are dropped from all the numbers and rounding is done to this column. The addition and subtraction is now performed to get the answer.

8.6 Number of 'Significant figure' in the magnitude of a physical quantity can neither be increased nor decreased.

Illustration :: If we have 3.10 kg than it can not be written as 3.1 kg or 3.100 kg.

Examples based on significant digits

Ex.24 Round off the following numbers to three significant digits

(a) 15462

(b) 14.745

(c)14.750

(d) 14.650×10^{12}.

Sol.(a) The third significant digit is 4. This digit is to be rounded. The digit next to it is 6 which is greater than 5. The third digit should, therefore , be increased by 1. The digits to be dropped should be replaced by zeros because they appear to the left of the decimal. Thus, 15462 becomes 15500 on rounding to three significant digits.

(b) The third significant digit in 14.745 is 7. The number next to it is less than 5. So 14.745 becomes 14.7 on rounding to three significant digits.

(c) 14.750 will become 14.8 because the digit to be rounded is odd and the digit next to it is 5.

(d) 14.650×10^{12} will become 14.6×10^{12} because the digit to be rounded is even and the digit next to it is 5.

Ex.25 Evaluate $\frac{25.2 \times 1374}{33.3}$. All the digits in this expression are significant.

Sol. We have $\frac{25.2 \times 1374}{33.3}$ = 1039.7838 .....

Out of the three numbers given in the expression 25.2 and 33.3 have 3 significant digits and 1374 has four. The answer should have three significant digits. Rounded 1039.7838 .... to three significant digits, it becomes 1040.

Thus , we write.

$$\frac{25.2 \times 1374}{33.3} = 1040.$$

Ex.26 Evaluate 24.36 + 0.0623 + 256.2

Sol.

24.36
0.0623
256.2

Now the first column where a doubtful digit occurs is the one just next to the decimal point (256.2). All digits right to this column must be dropped after proper rounding. The table is rewritten and added below

24.4
0.1
256.2
———
280.7 The sum is 280.7

SUPPLEMENTRY

9. FRACTIONAL AND PERCENTAGE ERRORS

9.1 Absolute error

= |experimental value – standard value|

9.2 If Δx is the error in measurement of x, then

Fractional error = $\frac{\Delta x}{x}$

Percentage error = × 100%

Percentage error in experimental measurement

= × 100%

9.3 Propagation of error (Addition and Subtraction) :

Let error in x is ± Δx, and error in y is ± Δy, then the error in x + y or x – y is ± (Δx + Δy). The errors add.

9.4 Multiplication and Division :

Let errors in x, y, z are respectively ± Δx, ± Δy and ± Δz. Then error in a quantity f (defined as)

f =   is obtained from the relation

$\frac{\Delta f}{f}$ = | a | + | b | $\left|\frac{\Delta y}{y}\right|$ + | c | $\left|\frac{\Delta z}{z}\right|$. The fraction errors (with proper multiples of exponents) add. The error in f is ± Δf.

9.5 Important Points :

9.5.1 When two quantities are added or subtracted the absolute error in the result is the sum of the absolute error in the quantities.

9.5.2 When two quantities are multiplied or divided, the fractional error in the result is the sum of the fractional error in the quantities to be multiplied or to be divided.

9.5.3 If the same quantity x is multiplied together n times (i.e. x^n), then the fractional error in x^n is n times the fractional error in x,

i.e. $\pm n \frac{\Delta x}{x}$

Examples based on **Errors**

Ex.27 In an experiment to determine acceleration due to gravity by simple pendulum, a student commit positive error in the measurement of length and 3% negative error in the measurement of time period. The percentage error in the value of g will be-

(A) 7% (B) 10%
(C) 4% (D) 3%

Sol. We know T = k

$\therefore T^2 = k'\left(\frac{\ell}{g}\right) \Rightarrow g = k' \frac{\ell}{T^2}$

$\frac{\Delta g}{g}$ × 100 =   × 100 +   × 100

= 1% + 2 x 3% = 7%

Hence correct answer is **(A)**

10. MEASURMENTS OF LENGTH, MASS & TIME

10.1 Distance of a hill :

To find the distance of a hill, a gun is fired towards the hill and the time interval t between the instant of firing the gun and the instant of hearing the echo of the gun is determined. Clearly, during this time interval sound first travels towards the hill from the place of firing and then back from the hill to the place of firing. If v be the velocity of sound, and s the distance of hill from the place of firing , then

$$2s = v \times t$$

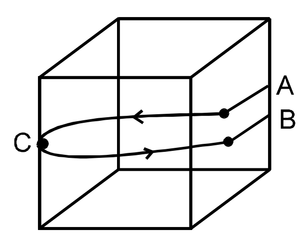
$$\text{or } s = \frac{v \times t}{2}$$

10.2 Distance of moon :

A laser beam is a source of very intense, monochromatic and unidirectional beam. By sending a laser beam towards the moon instead of sound waves, the echo method becomes useful in finding the distance of moon from the earth. If t is the total time taken by laser beam in going towards moon and back, then distance of moon from the earth's surface is given by : $S = \frac{c \times t}{2}$

Where c = 3 × 10^8 m/s ; is the velocity of laser beam.

10.3 Thickness of matter sheet :



For finding the thickness of some matter sheet, a signal from point A on the front surface of sheet is sent to the back surface. The signal gets reflected from point C on the back surface and is again received back at point B on the front surface. If the time interval between the instants of sending the signal from point A and receiving the signal back at B, is t, then thickness of sheet

$$S = \frac{C \times t}{2} \text{ (where C is the velocity of signal)}$$

10.4 Distance of submerged objects or submarines in sea (Sound navigation and ranging - Sonar)

Sonar in a instrument which uses ultrasonic waves (waves having frequency > 20,000 Hz) to detect and locate the submerged objects, submarines etc. in sea. Ultrasonic waves produced from a transmitter are sent towards the distant objects under water. When the object comes in the direction of ultrasonic waves, then the waves are reflected back from it. Measuring the time interval t between the instants the ultrasonic waves are sent and received back, the distance S of the object can be calculated by the relation.

$S = \frac{C \times t}{2}$ (where C is the velocity of ultrasonic waves)

10.5 Distance of aeroplane (Radio detection and Ranging - Radar). Radar is an instrument which uses. radiowaves for detecting and locating an aeroplane. Radiowaves produced by a transmitter at the radar station, are sent towards the aeroplane in space. These waves are reflected from the aeroplane. The reflected waves are received by a receiver at the radar station. By noting the time interval between the instants of transmission of waves and their detection, distance of aeroplane can be measured. If t is the required time interval and C the velocity of light (=equal to velocity of radio waves) then distance of aeroplane

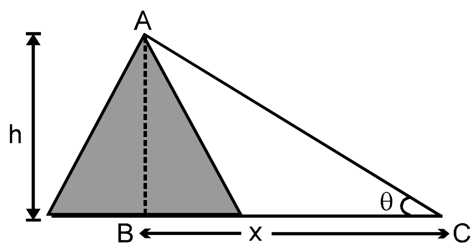
$$S = \frac{C \times t}{2}$$

10.6 Triangulation method :

This method uses the geometry of the triangle and is useful for measuring the heights in following cases

10.6.1

Height of a tower or height of an accessible object.

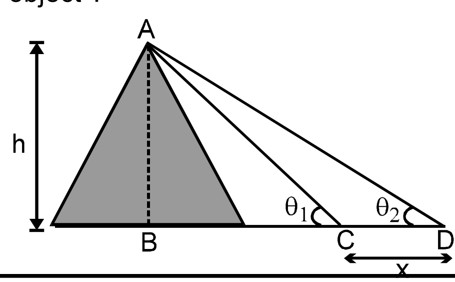


$$\tan\theta = \frac{AB}{BC} = \frac{h}{x}$$

$$h = x \tan\theta$$

10.6.2

Height of a mountain or height of an accessible object :



This method is useful in cases when it becomes impossible to measure the distance between the object and the observation point.

$$\tan\theta_1 = \frac{AB}{BC} = \frac{h}{BC}$$

$$BC = h \cot\theta_1$$

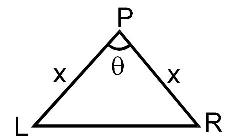
Similarly $\tan\theta_2 = \frac{AB}{BD}$

$$BD = h \cot\theta_2$$

$$BD - BC = x = h(\cot\theta_2 - \cot\theta_1)$$

10.7 Parallax method :

Parallax (Definition) : When we observe the object P by closing our right and left eye alternately, we observe a shift in the position of object w.r.t the background. This is known as parallax.



$\theta = \frac{LR}{x} = \frac{b}{x}$ (assuming distance LR as a circular arc of radius x)

$$x = \frac{b}{\theta}$$

10.7.1

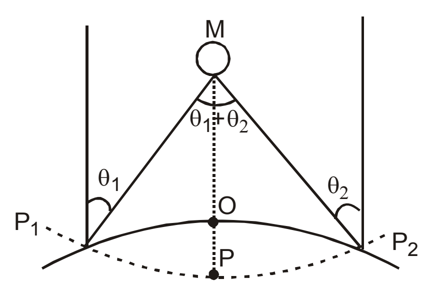
Determination of distance of moon from earth.

$\theta = \theta_1 + \theta_2$

Because $\theta = \frac{P_1PP_2}{PM}$

Hence $PM = \frac{P_1PP_2}{\theta}$

As astronomical bodies are at very large distances from earth



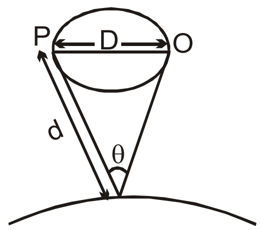
P_1PP_2 P_1P_2 PM MO

OM =

$\frac{\tilde{P}_1P_2}{\theta}$

10.7.2

Determination of size of an astronomical object (Moon)



$\theta = \qquad = \frac{D}{d}$

$D = \theta d$

10.8 Measurement of very small distances :

Various devices are used to measure very small distances like vernier calliper, screw gauge, spherometer, optical microscopes, electron-microscopes, X-ray diffraction etc.

POINTS TO REMEMBER

1. Relation between some practical units of the standard of length

(i) 1 light year = 9.46×10^{15} m

$\frac{PO}{d}$ (ii) 1 par sec = 3.06×10^{16} m

= 3.26 light year

(iii) 1 AU = 1.496×10^{11} m

(iv) 1 X-ray unit = 10^{-3} m

2. Relation between some practical units of the standard mars

(i) 1 C.S.L. (chandra shekhar limit)

= 1.4 time the mass of sun

= 2.8×10^{30} kg

(ii) 1 amu = 1.67×10^{-27} kg

(iii) 1 slug = 14.57 kg

3. Relation between some practical unit of standards of time

(i) 1 solar day's = 86400 sec

(ii) 1 Lunar Month = 27.3 days

(iii) 1 solar year = 365.25 average solar day
= 366.25 sedrial day

(iv) 1 shake = 10^{-8} sec

4. In mechanics, dimensions of a quantity are given in terms of powers of mass (M), length (L) and time (T). In heat and thermodynamics, power of temperature (θ) comes in addition to powers of M, L and T. In electricity and magnetism, dimensions are given in terms of M, L, T and I, the current.

5. Only like quantities having the same dimensions can be added to or subtracted from each other.

6. The dimensional formula of a physical quantity does not depend upon the system of units used to represent that quantity.

7. The value (magnitude) of a physical quantity remains the same in all systems of measurement. However, the numerical value changes.

In general, $n_1u_1 = n_2u_2 = n_3u_3 = ...$

8. All of the following have the same dimensional formula $[M^0L^0T^{-1}]$. Frequency, angular frequency, angular velocity and velocity gradient.

9. All of the following are dimensionless. Angle, Solid angle, T-ratios, Strain, Poisson's ratio, Relative density, Relative permittivity, Refractive index and Relative permeability.

10. Following three quantities have the same dimensional formula $[M^0L^2T^{-2}]$. Square of velocity, gravitational potential, latent heat.

11. Following quantities have the same dimensional formula $[ML^2T^{-2}]$. Work, energy, torque, heat.

12. Force, weight, thrust and energy gradient have the same dimensional formula $[MLT^{-2}]$.

13. Entropy, gas constant, Boltzmann constant and thermal capacity have the same dimensions in mass, length and time.

14. Light year, radius of gyration and wavelength have the same dimensional formula $[M^0L^1T^0]$

15. Rydberg constant and propagation constant have the same dimensional formula $[M^0L^{-1}T^0]$.

16. The decimal point does not separate the certain and uncertain digit, only last digit may be uncertain.

17. Significant figures indicate the precision of measurement which depend on least count of the measuring instruments.

18. So far as significant figures are concerned, in mathematical operations like addition and subtraction, the result would be correct upto minimum number of decimal places in any of the quantities involved. However, in multiplication and division, number of significant figures in the result will be limited corresponding to the minimum number of significant figures in any of the quantities involved.

To represent the result to a correct number of significant figures, we round off as per the rules already stated.

19. Whenever two measured quantities are multiplied or divided, the maximum possible relative or percentage error in the computed result is equal to the sum of relative or percentage errors in the observed quantities. Therefore maximum possible error in

$Z = \dfrac{A^m B^n}{C^p}$ is :

$$\frac{\Delta Z}{Z} \times 100 = m\frac{\Delta A}{A} \times 100 + n\frac{\Delta B}{B} \times 100 + p \times \frac{\Delta C}{C} \times 100$$